بسم الله الرحمن الرحیم

فصل ششم

# ساده‌سازی گرامرهای مستقل از متن

## Simplification of Context-Free Grammars

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# Simplifications of Context-Free Grammars

# A Substitution Rule

$A \to a$

$A \to aaA$

$A \to abBc$

$B \to abbA$

$B \to b$


Equivalent grammar

$A \to a$

$A \to aaA$

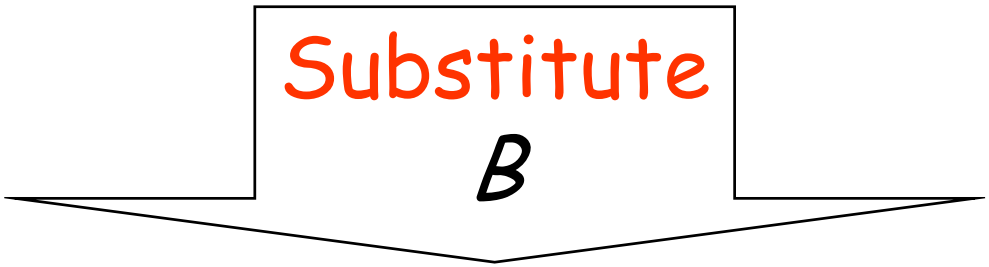
$A \to ababbAc$

$A \to abbc$

In general:

$$A \to xBz$$

$$B \to y_1 \mid y_2 \mid \cdots \mid y_n$$




$$A \to xy_1z \mid xy_2z \mid \cdots \mid xy_nz$$

equivalent grammar

# Useless Productions

$$S \to aSb$$

$$S \to \lambda$$

$$S \to A$$

$$A \to aA$$ Useless Production

Some derivations never terminate…

$$S \Rightarrow A \Rightarrow aA \Rightarrow aaA \Rightarrow \ldots \Rightarrow aa\ldots aA \Rightarrow \ldots$$

## Another grammar:

$$S \to A$$

$$A \to aA$$

$$A \to \lambda$$

$$B \to bA$$ Useless Production

Not reachable from S

In general:

If $S \Rightarrow \ldots \Rightarrow xAy \Rightarrow \ldots \Rightarrow w$

$w \in L(G)$

Then variable $A$ is useful

Otherwise, variable $A$ is useless

A production $A \to x$ is useful
if all its variables are useful

# Removing Useless Productions

Example Grammar:

$$S \to aS \mid A \mid C$$

$$A \to a$$

$$B \to aa$$

$$C \to aCb$$

**First:** find all variables that produce strings with only terminals

$$S \to aS \mid A \mid C$$

$$A \to a$$

$$B \to aa$$

$$C \to aCb$$

Round 1: $\{A, B\}$

Round 2: $\{A, B, S\}$

Keep only the variables
that produce terminal symbols

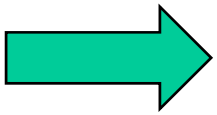
$\{A, B, S\}$

$S \to aS \mid A \mid C$

$A \to a$

$B \to aa$

$C \to aCb$



$S \to aS \mid A$

$A \to a$

$B \to aa$

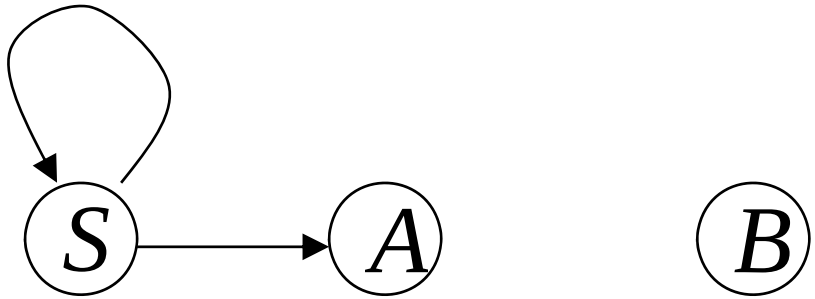
**Second:** Find all variables reachable from $S$

$S \rightarrow aS \mid A$

$A \rightarrow a$

$B \rightarrow aa$

Dependency Graph



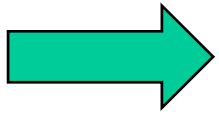

not reachable

Keep only the variables reachable from S

$S \rightarrow aS \mid A$

$A \rightarrow a$

~~$B \rightarrow aa$~~



Final Grammar

$S \rightarrow aS \mid A$

$A \rightarrow a$

# Nullable Variables

$\lambda$ – production : $A \to \lambda$

Nullable Variable: $A \Rightarrow \ldots \Rightarrow \lambda$

# Removing Nullable Variables

Example Grammar:

$$S \rightarrow aMb$$

$$M \rightarrow aMb$$

$$M \rightarrow \lambda$$

Nullable variable

## Final Grammar

$S \rightarrow aMb$

$M \rightarrow aMb$

~~$M \rightarrow \lambda$~~

Substitute

$M \rightarrow \lambda$

$S \rightarrow aMb$

$S \rightarrow ab$

$M \rightarrow aMb$

$M \rightarrow ab$

# Unit-Productions

Unit Production: $A \to B$

# Removing Unit Productions

Observation:

$$A \to A$$

Is removed immediately

# Example Grammar:

$$S \to aA$$

$$A \to a$$

$$A \to B$$

$$B \to A$$

$$B \to bb$$

$S \to aA$

$A \to a$

~~$A \to B$~~

$B \to A$

$B \to bb$

Substitute

$A \to B$

$S \to aA \mid aB$

$A \to a$

$B \to A \mid B$

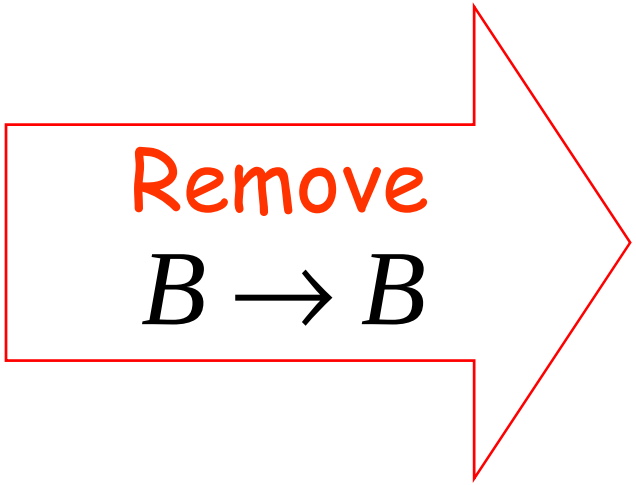
$B \to bb$

$S \rightarrow aA \mid aB$

$A \rightarrow a$

$B \rightarrow A \mid B$

$B \rightarrow bb$




$S \rightarrow aA \mid aB$

$A \rightarrow a$

$B \rightarrow A$

$B \rightarrow bb$

$S \rightarrow aA \mid aB$

$A \rightarrow a$

~~$B \rightarrow A$~~

$B \rightarrow bb$

Substitute

$B \rightarrow A$

$S \rightarrow aA \mid aB \mid aA$

$A \rightarrow a$

$B \rightarrow bb$

# Remove repeated productions

$$S \to aA \mid aB \mid \cancel{aA}$$

$$A \to a$$

$$B \to bb$$

Final grammar

$$S \to aA \mid aB$$

$$A \to a$$

$$B \to bb$$

# Removing All

**Step 1:** Remove Nullable Variables

**Step 2:** Remove Unit-Productions

**Step 3:** Remove Useless Variables