بسم الله الرحمن الرحیم

فصل چهارم

خصوصیات زبان‌های منظم (۲)

Properties of Regular Languages (2)

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More Applications

of

the Pumping Lemma

The Pumping Lemma:

- Given a infinite regular language L
- there exists an integer m
- for any string $w \in L$ with length $|w| \geq m$
- we can write $w = x\,y\,z$
- with $|x\,y| \leq m$ and $|y| \geq 1$
- such that: $x\,y^i\,z \in L \qquad i = 0, 1, 2, \ldots$

Non-regular languages $L = \{ww^R : w \in \Sigma^*\}$

Regular languages

Theorem: The language

$$L = \{ww^R : w \in \Sigma^*\} \qquad \Sigma = \{a, b\}$$

is not regular

Proof: Use the Pumping Lemma

$$L = \{ww^R : w \in \Sigma^*\}$$

Assume for contradiction
that L is a regular language

Since L is infinite
we can apply the Pumping Lemma

$$L = \{ww^R : w \in \Sigma^*\}$$

Let m be the integer in the Pumping Lemma

Pick a string w such that: $w \in L$

length $|w| \geq m$

pick $w = a^m b^m b^m a^m$

Write $a^m b^m b^m a^m = x\ y\ z$

From the Pumping Lemma

it must be that length $|x\ y| \leq m, \quad |y| \geq 1$

$$a^m b^m b^m a^m = \underbrace{a\ldots a}_{x}\underbrace{a\ldots a}_{y}\underbrace{\ldots ab\ldots bb\ldots ba\ldots a}_{z}$$

$$y = a^k, \quad k \geq 1$$

We have: $x\,y\,z = a^m b^m b^m a^m$

$$y = a^k, \quad k \geq 1$$

From the Pumping Lemma: $x\,y^i\,z \in L$

$$i = 0, 1, 2, \ldots$$

Thus: $x\,y^2\,z \in L$

$$x\,y^2\,z = x\,y\,y\,z = a^{m+k} b^m b^m a^m \quad \in L$$

Therefore: $a^{m+k}b^m b^m a^m \in L$

BUT: $L = \{ww^R : w \in \Sigma^*\}$

$$\Downarrow$$

$$a^{m+k}b^m b^m a^m \notin L$$

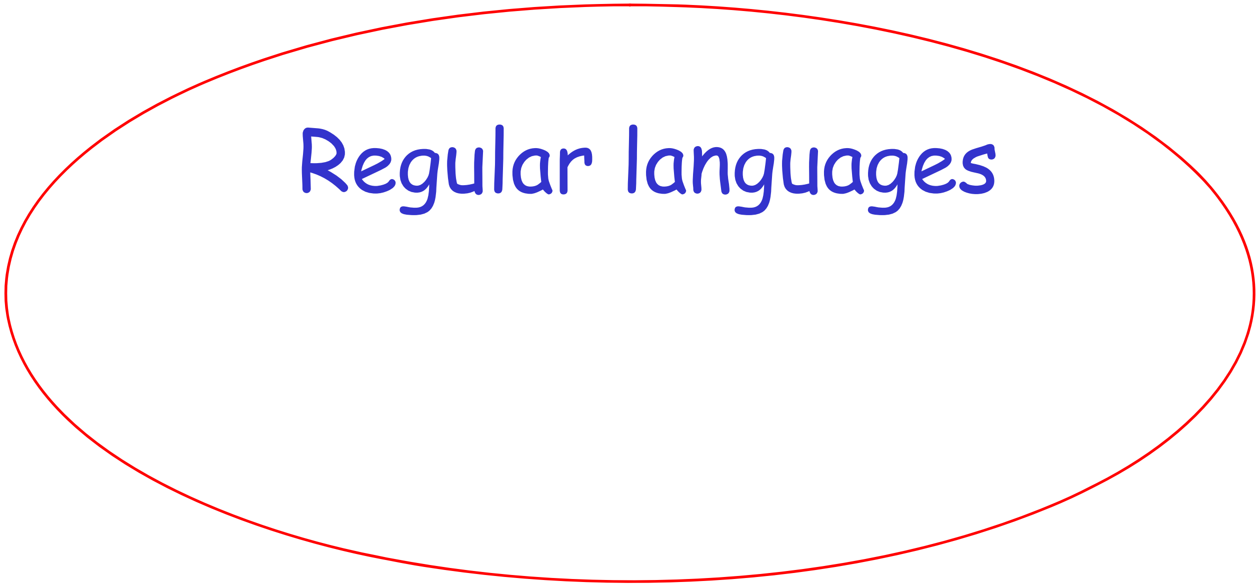
CONTRADICTION!!!

Therefore: Our assumption that L is a regular language is not true

Conclusion: L is not a regular language

Non-regular languages

$$L = \{a^n b^l c^{n+l} : \; n, l \geq 0\}$$



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Assume for contradiction
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$$L = \{a^n b^l c^{n+l} : \; n, l \geq 0\}$$

Let m be the integer in the Pumping Lemma

Pick a string w such that: $w \in L$

length $|w| \geq m$

pick $w = a^m b^m c^{2m}$

Write $\quad a^m b^m c^{2m} = x\, y\, z$

From the Pumping Lemma

it must be that length $\quad |x\, y| \le m, \quad |y| \ge 1$

$$a^m b^m c^{2m} = \underbrace{a...a}_{x}\underbrace{a...a}_{y}\underbrace{a...ab...bc...cc...c}_{z}$$

$$\overbrace{a...aa...aa...a}^{m}\overbrace{b...b}^{m}\overbrace{c...cc...c}^{2m}$$

$y = a^k, \quad k \ge 1$

We have: $$x\,y\,z = a^m b^m c^{2m}$$

$$y = a^k, \quad k \geq 1$$

From the Pumping Lemma: $$x\,y^i\,z \;\in\; L$$

$$i = 0, 1, 2, \ldots$$

Thus: $$x\,y^0\,z \;\in\; L$$

$$x\,y^0\,z = x\,z = a^{m-k} b^m c^{2m} \;\in\; L$$

Therefore: $a^{m-k}b^{m}c^{2m} \in L$

BUT: $L = \{a^{n}b^{l}c^{n+l} : n,l \geq 0\}$

$$\Downarrow$$

$$a^{m-k}b^{m}c^{2m} \notin L$$

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Therefore: Our assumption that L is a regular language is not true

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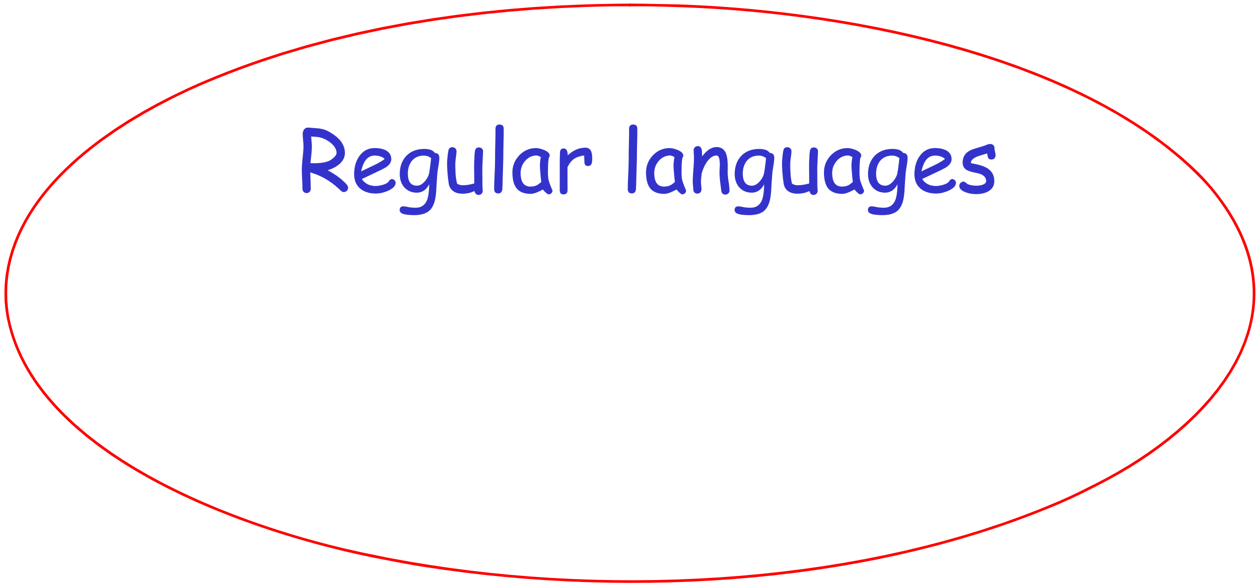
Non-regular languages $L = \{a^{n!} : \; n \geq 0\}$



Theorem: The language $L = \{a^{n!}: \ n \geq 0\}$

is not regular

$$n! \ = \ 1 \cdot 2 \cdots (n-1) \cdot n$$

Proof: Use the Pumping Lemma

$$L = \{a^{n!} : \; n \geq 0\}$$

Assume for contradiction
that L is a regular language

Since L is infinite
we can apply the Pumping Lemma

$$L = \{a^{n!} : \; n \geq 0\}$$

Let m be the integer in the Pumping Lemma

Pick a string w such that: $w \in L$

length $|w| \geq m$

pick $w = a^{m!}$

Write $\quad a^{m!} = x\, y\, z$

From the Pumping Lemma

it must be that length $\quad |x\, y| \leq m, \quad |y| \geq 1$

$$a^{m!} = \overbrace{\underbrace{a\ldots a}_{x}\underbrace{a\ldots a}_{y}a\ldots a}^{m}\overbrace{a\ldots aa\ldots a}^{m!-m}$$

$$z = a\ldots aa\ldots aa\ldots a$$

$$y = a^k, \quad 1 \leq k \leq m$$

We have: $$x\,y\,z = a^{m!}$$

$$y = a^k, \quad 1 \le k \le m$$

From the Pumping Lemma: $$x\,y^i\,z \in L$$

$$i = 0, 1, 2, \ldots$$

Thus: $$x\,y^2\,z \in L$$

$$x\,y^2\,z = x\,y\,y\,z = a^{m!+k} \in L$$

Therefore: $a^{m!+k} \in L \qquad 1 \leq k \leq m$

And since: $L = \{a^{n!} : \; n \geq 0\}$

There is p : $m!+k = p! \qquad 1 \leq k \leq m$

However: $m!+k \leq m!+m$ for $m > 1$

$$\leq m!+m!$$

$$< m!m + m!$$

$$= m!(m+1)$$

$$= (m+1)!$$

$$\Downarrow$$

$$m!+k < (m+1)!$$

$$\Downarrow$$

$m!+k \neq p!$ for any p

Therefore: $a^{m!+k} \in L$

BUT: $L = \{a^{n!}:\ n \geq 0\}$ and $1 \leq k \leq m$

$\Downarrow$

$a^{m!+k} \notin L$

CONTRADICTION!!!

Therefore: Our assumption that L is a regular language is not true

Conclusion: L is not a regular language

Lex

Lex: a lexical analyzer

- A Lex program recognizes strings

- For each kind of string found
  the lex program takes an action

Input

```
Var = 12 + 9;
if (test > 20)
   temp = 0;
else
   while (a < 20)
      temp++;
```

→ **Lex program** →

Output

Identifier: Var
Operand: =
Integer: 12
Operand: +
Integer: 9
Semicolumn: ;
Keyword: if
Parenthesis: (
Identifier: test
....

In Lex strings are described with regular expressions

Lex program

Regular expressions

```
"+"
"-"               /* operators */
"="

"if"
"then"            /* keywords */
```

Lex program

Regular expressions

(0|1|2|3|4|5|6|7|8|9)+ /* integers */

(a|b|..|z|A|B|...|Z)+ /* identifiers */

integers

(0|1|2|3|4|5|6|7|8|9)+

[0-9]+

identifiers

(a|b|..|z|A|B|...|Z)+

[a-zA-Z]+

Each regular expression

has an associated action (in C code)

Examples:

Regular expression	Action
\n	linenum++;
[0-9]+	prinf("integer");
[a-zA-Z]+	printf("identifier");

Default action: ECHO;

Prints the string identified to the output

A small program

```
%%
[ \t\n]         ;    /*skip spaces*/

[0-9]+          printf("Integer\n");

[a-zA-Z]+       printf("Identifier\n");
```

Input

1234	test	
var	566	78
9800		

Output

Integer
Identifier
Identifier
Integer
Integer
Integer

Another program

```
%{
int linenum = 1;
%}

%%

[ \t]           ;    /*skip spaces*/
\n              linenum++;

[0-9]+          prinf("Integer\n");

[a-zA-Z]+       printf("Identifier\n");

.               printf("Error in line: %d\n",
                        linenum);
```

Input

```
1234    test

var  566      78

9800   +

temp
```

Output

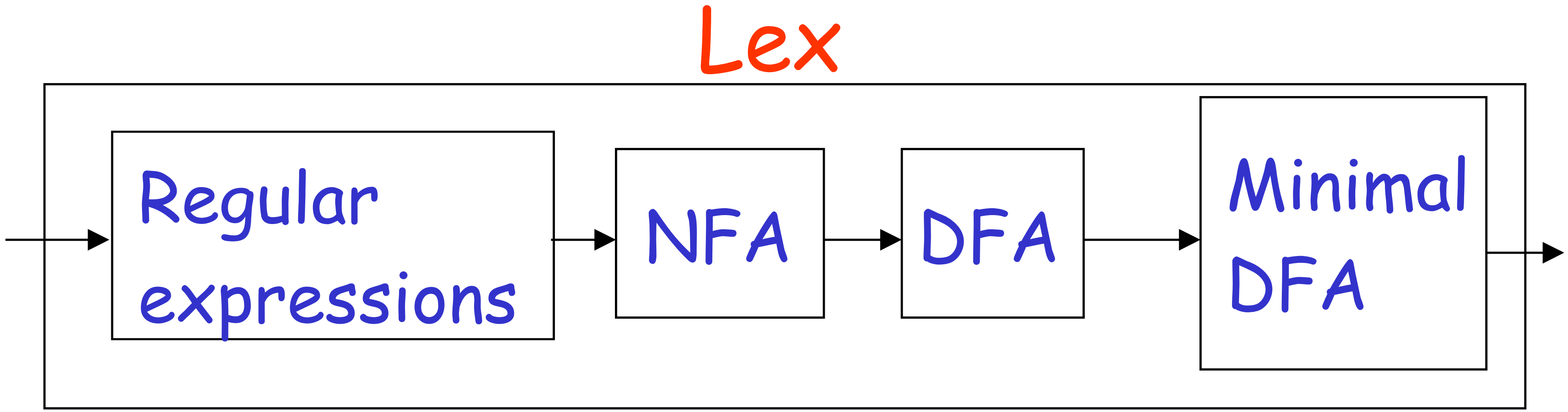
```
Integer
Identifier
Identifier
Integer
Integer
Integer
Error in line: 3
Identifier
```

Lex matches the longest input string

Example: Regular Expressions "if"
"ifend"

Input:	ifend	if	ifn
Matches:	"ifend"	"if"	nomatch

Internal Structure of Lex



The final states of the DFA are associated with actions