بسم الله الرحمن الرحيم

فصل دوم

آتوماتای متناهی (۱)

Finite Automata (1)

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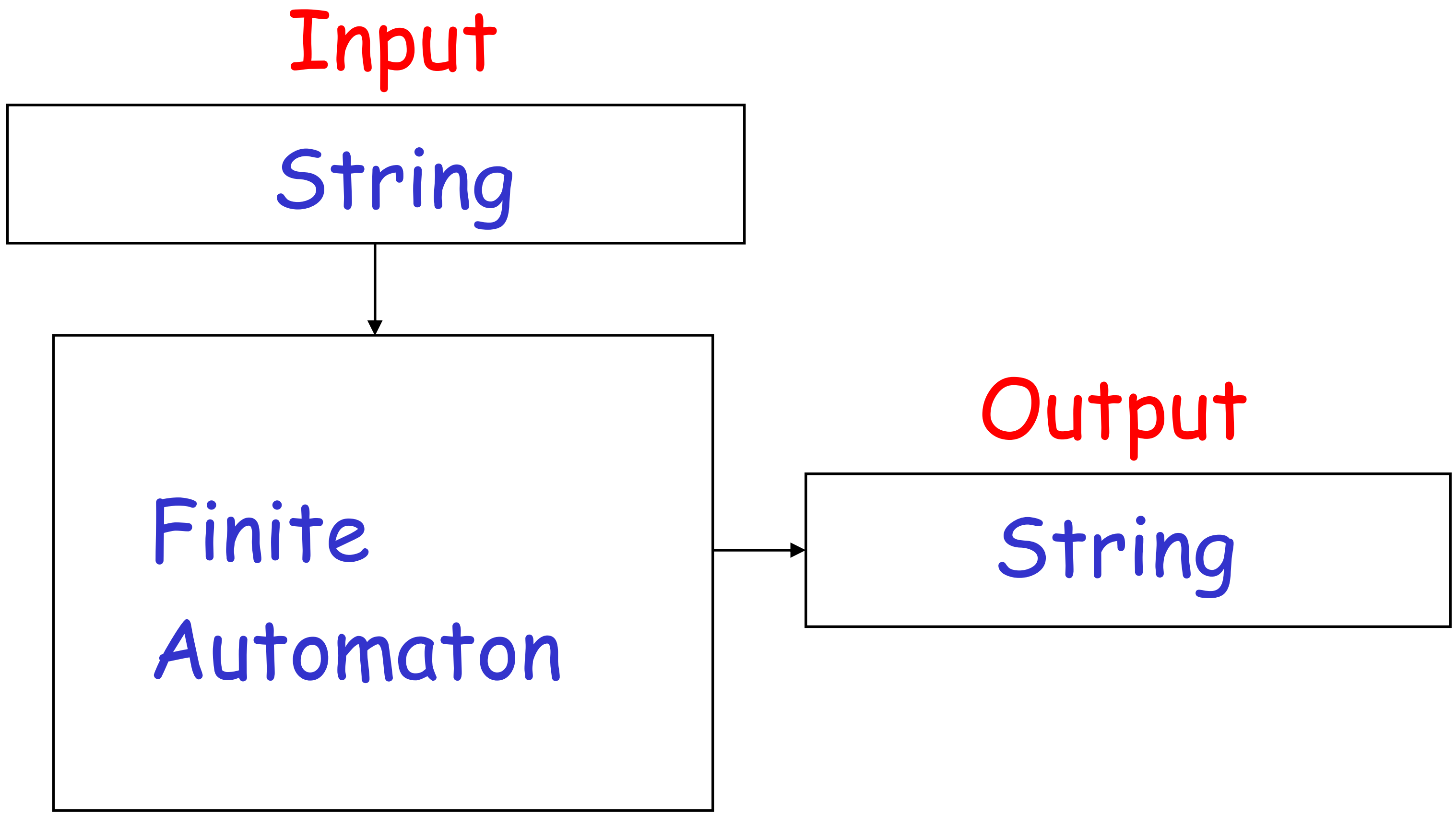
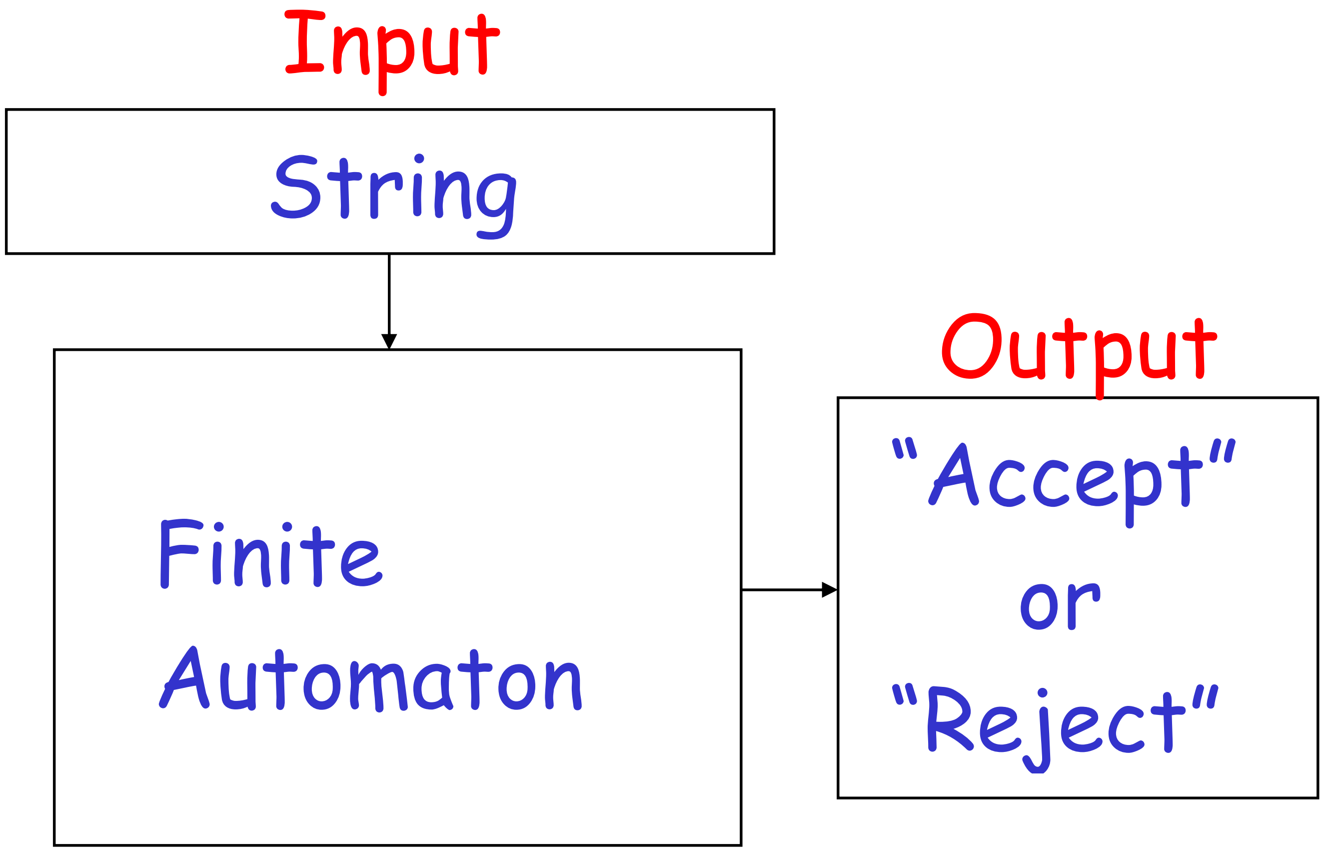
دانشگاه تهران

Finite Automata

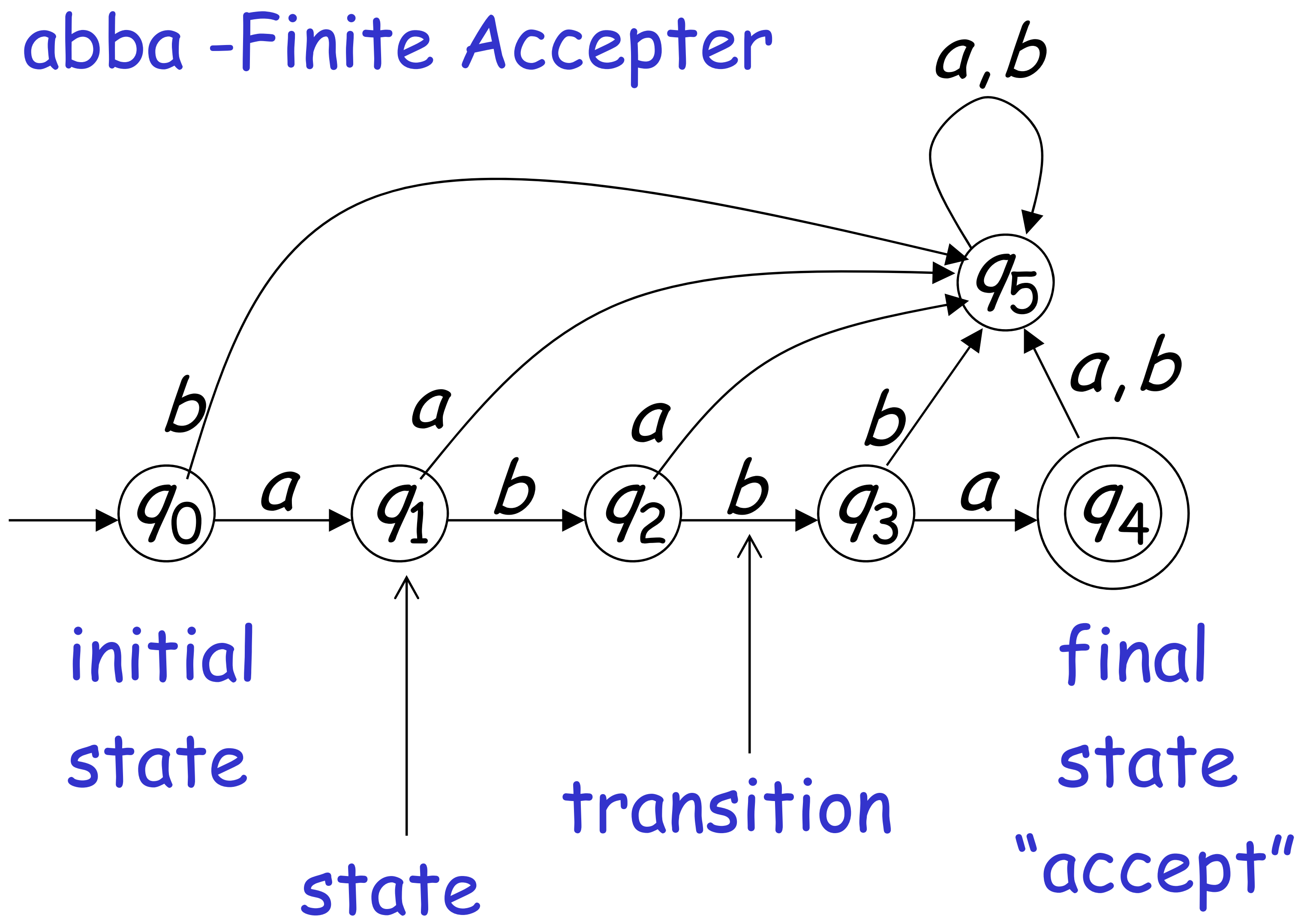
Finite Automaton



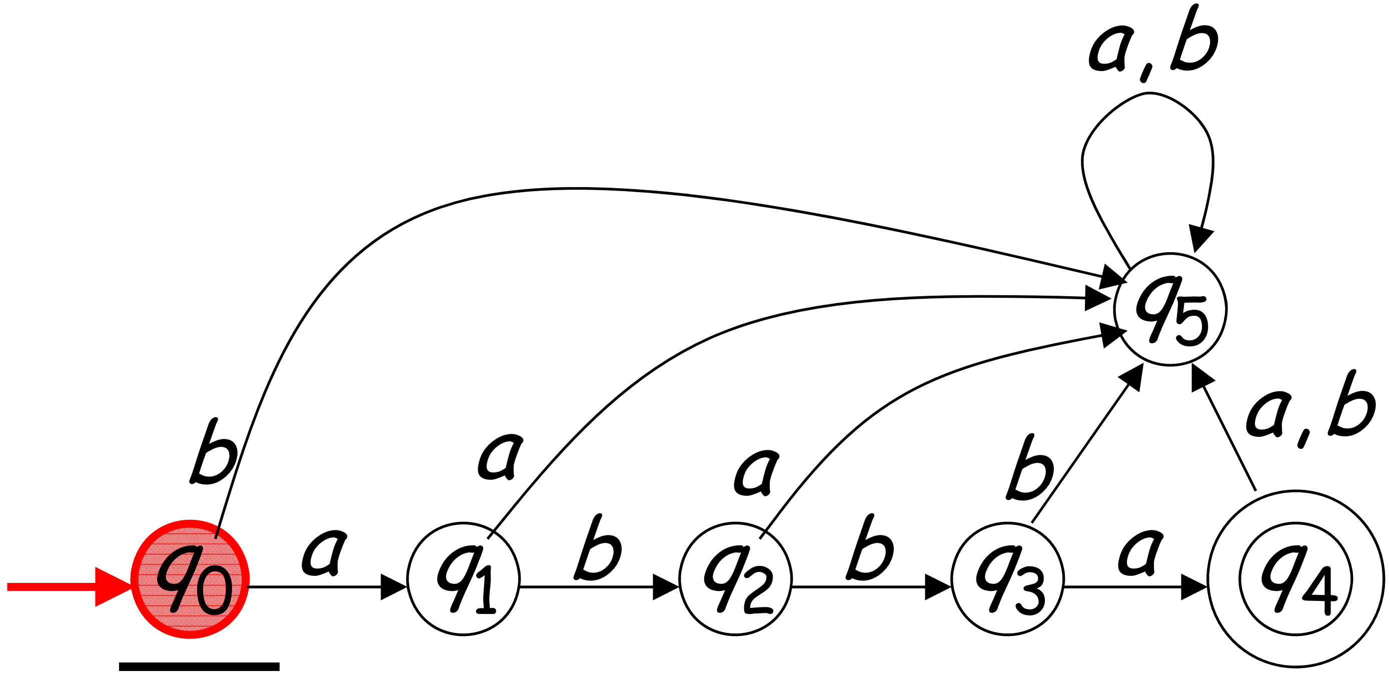
Finite Accepter



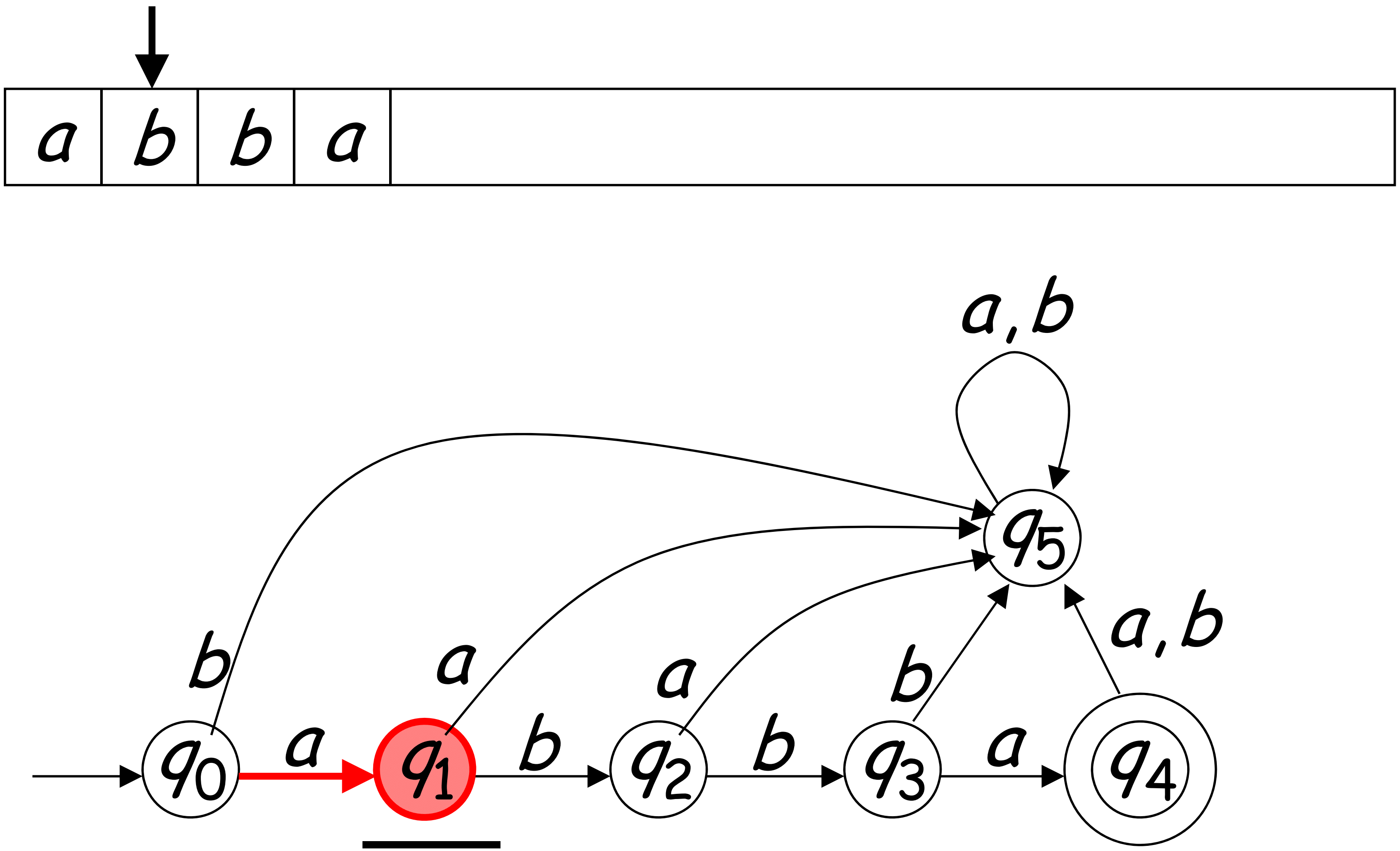
Transition Graph

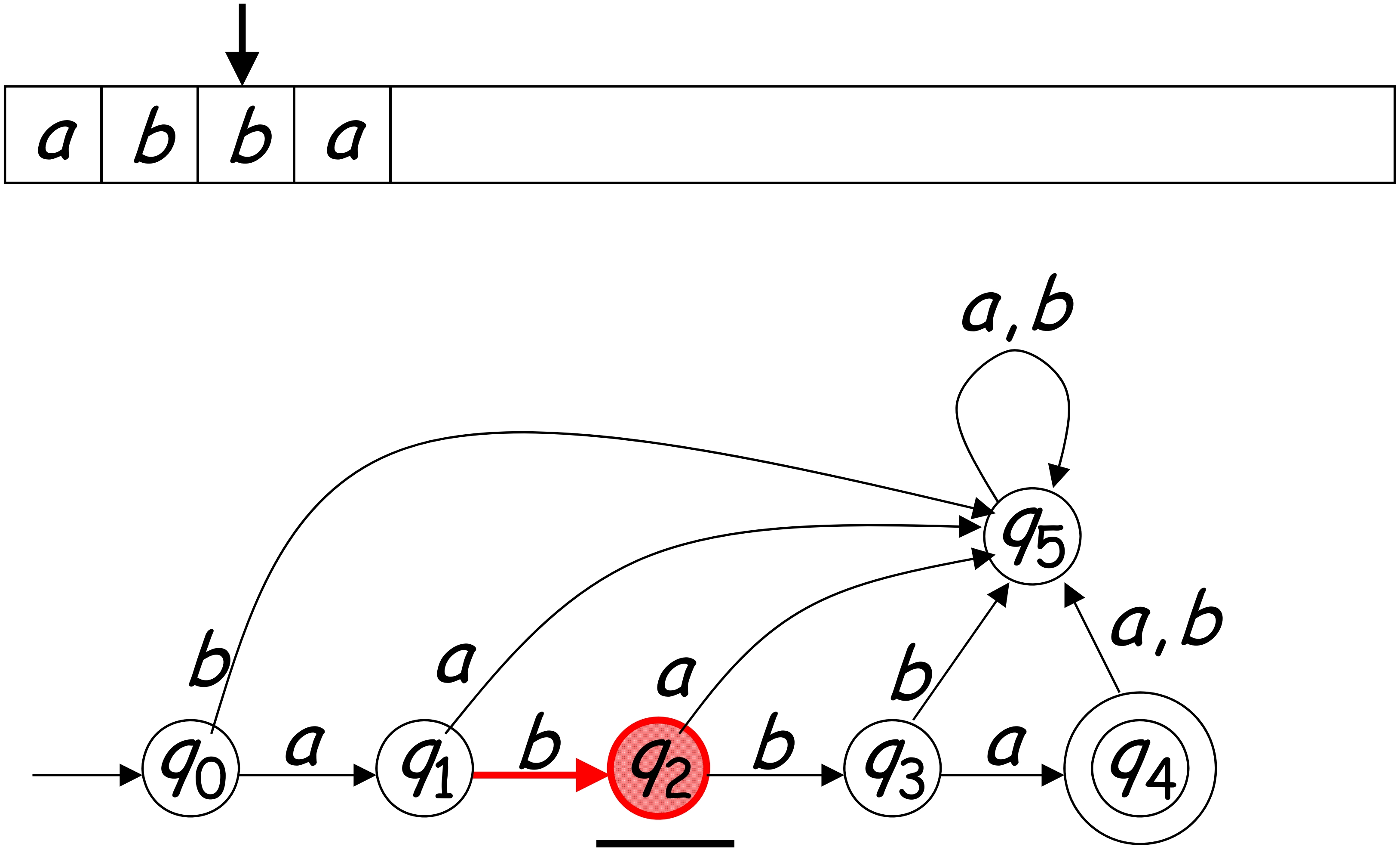


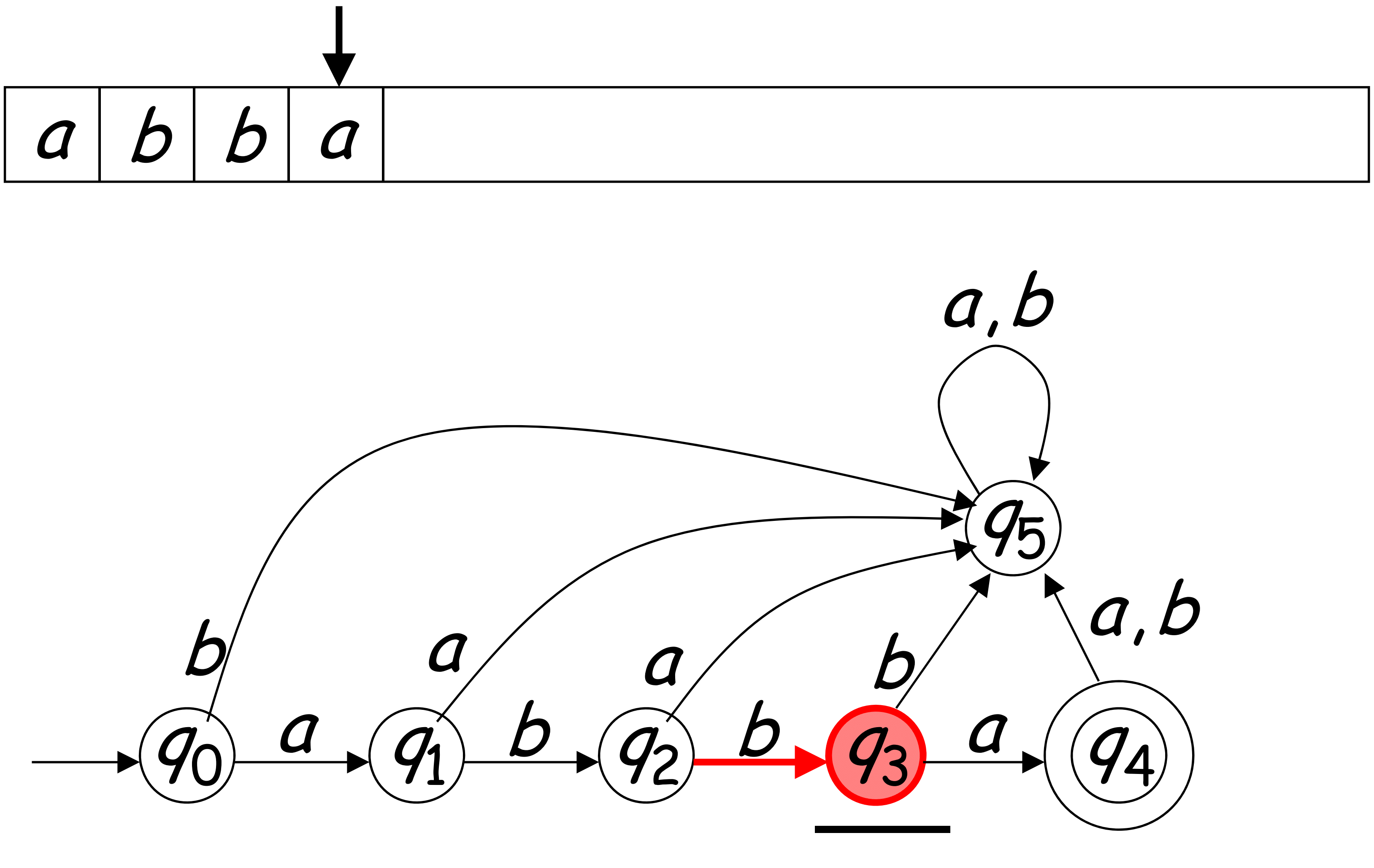
Initial Configuration

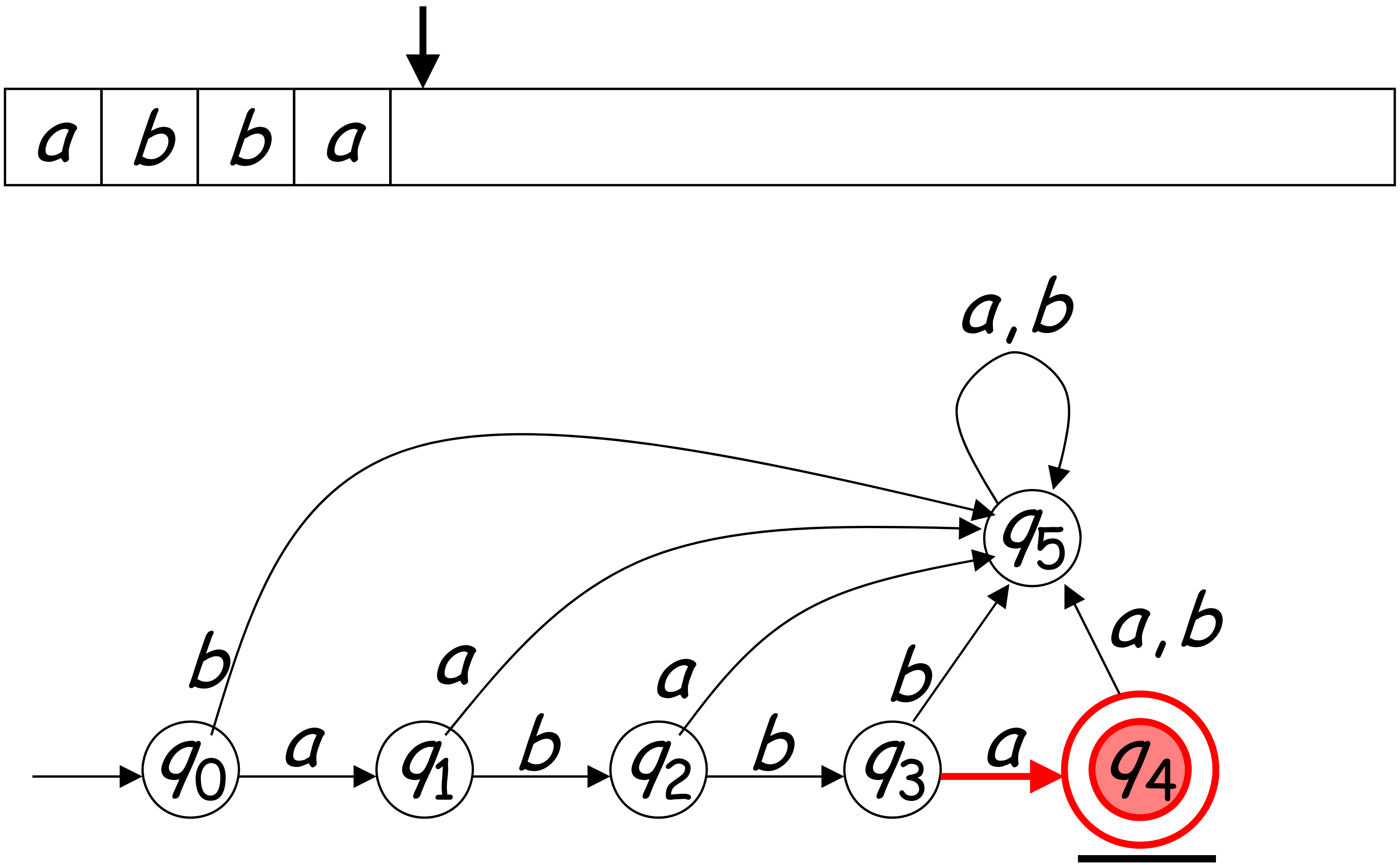


Reading the Input

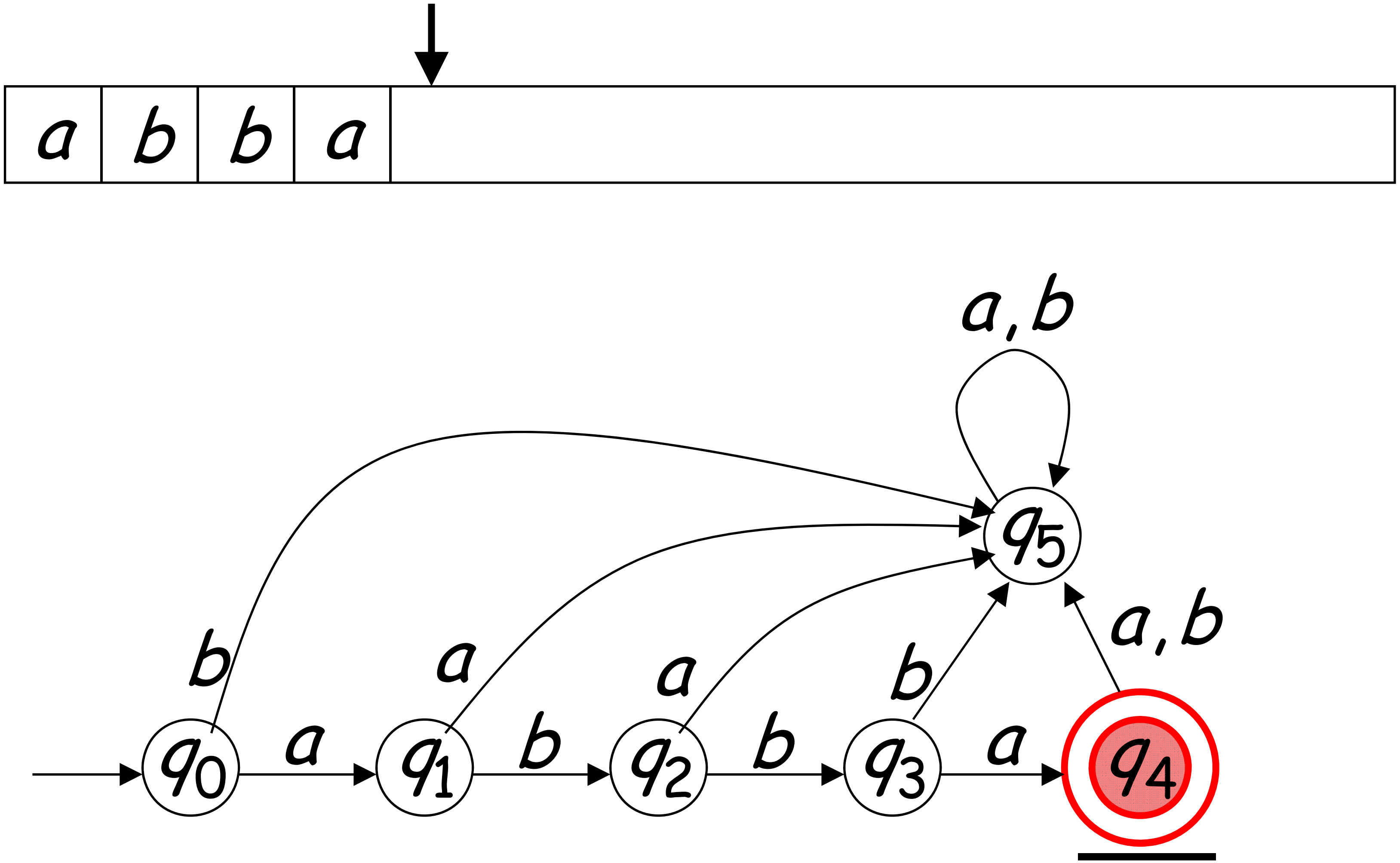



a
b
b
a
a,b
b
a
a
b
b
a,b
a
b
a
q_0
q_1
q_2
q_3
q_4
q_5


a
b
b
a
a,b
q_5
b
a
a
b
a,b
q_0
a
q_1
b
q_2
b
q_3
a
q_4

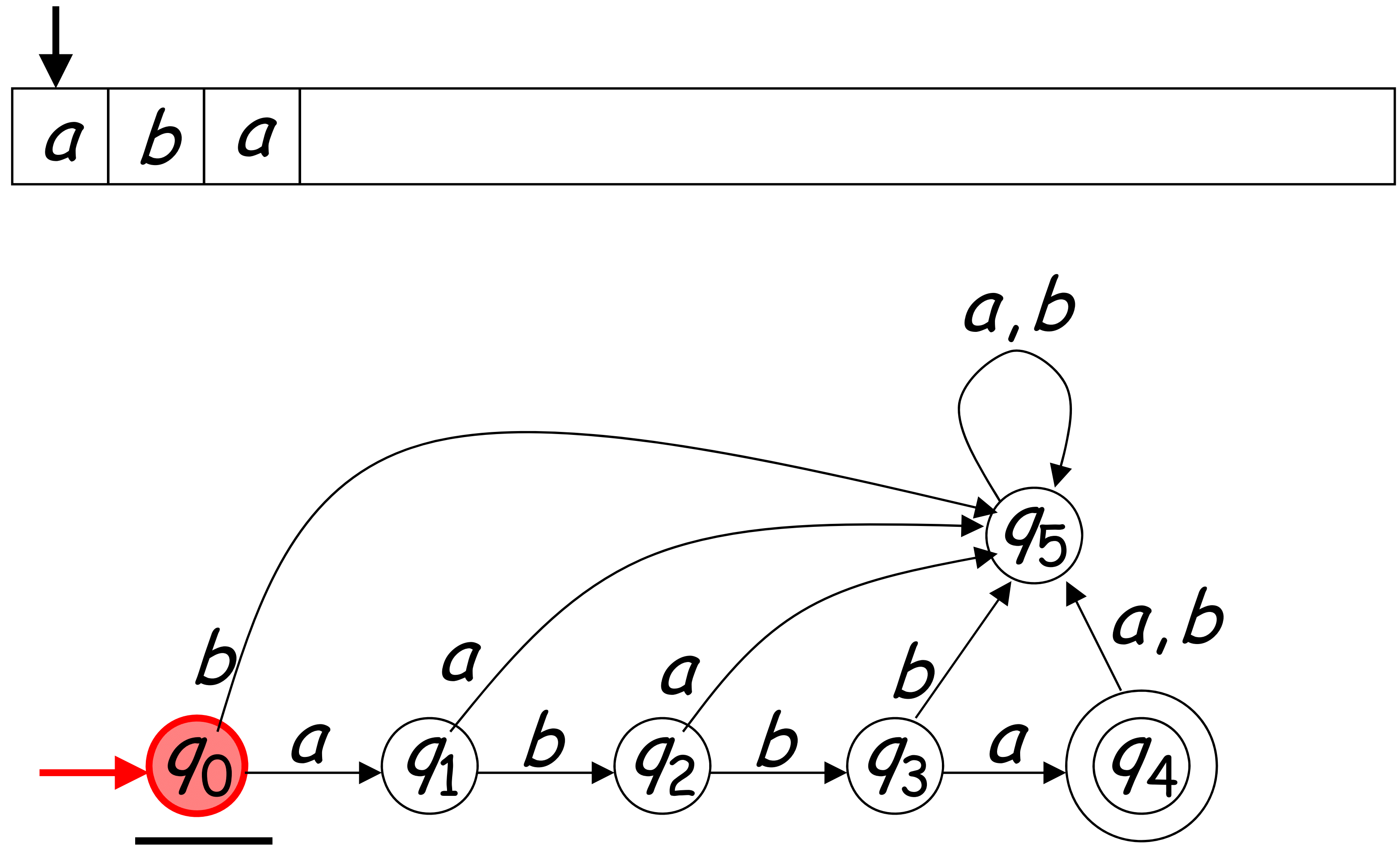


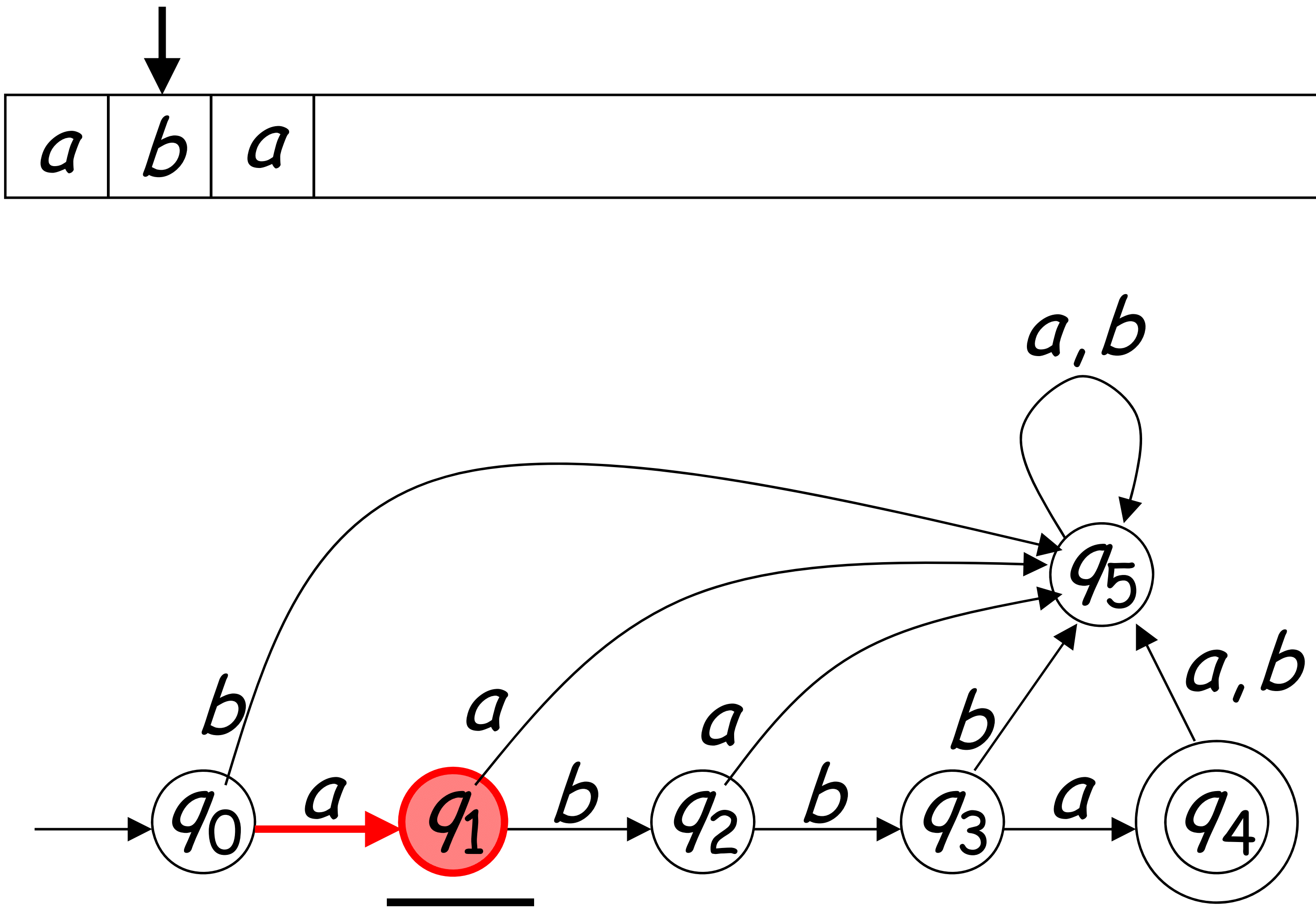
a
b
b
a
a,b
q_5
b
a
a
b
a,b
q_0
a
q_1
b
q_2
b
q_3
a
q_4

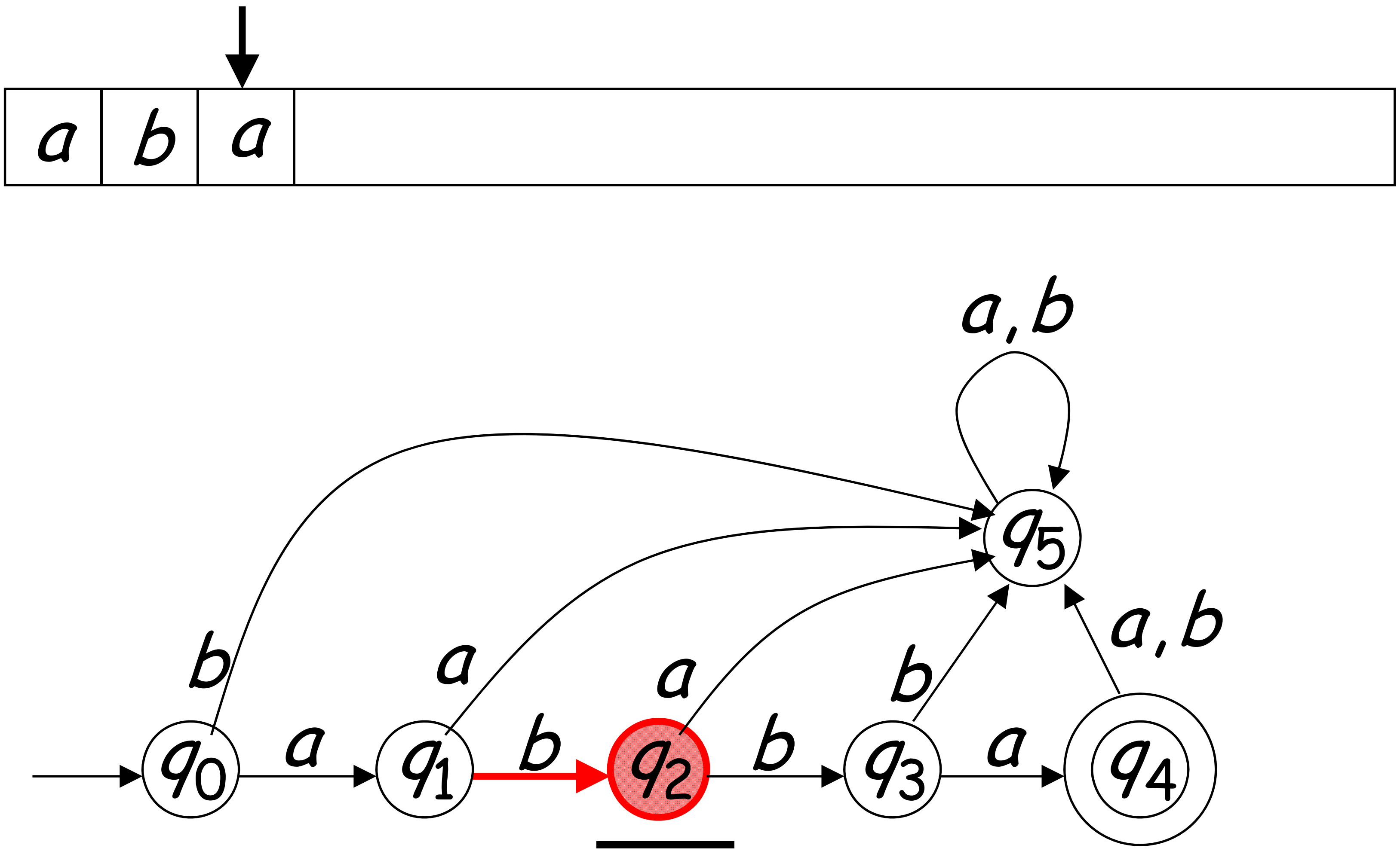


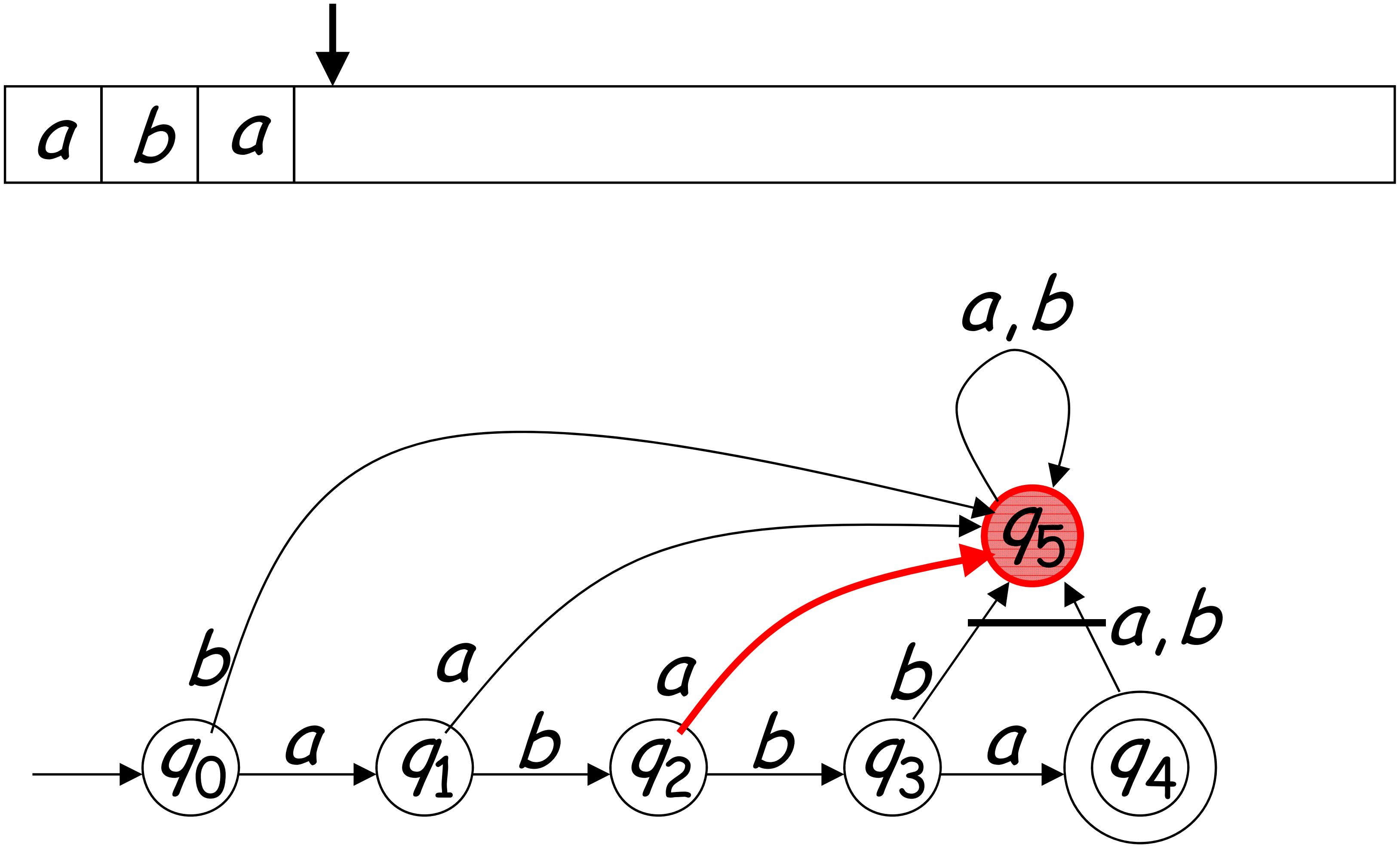
Output: “accept”

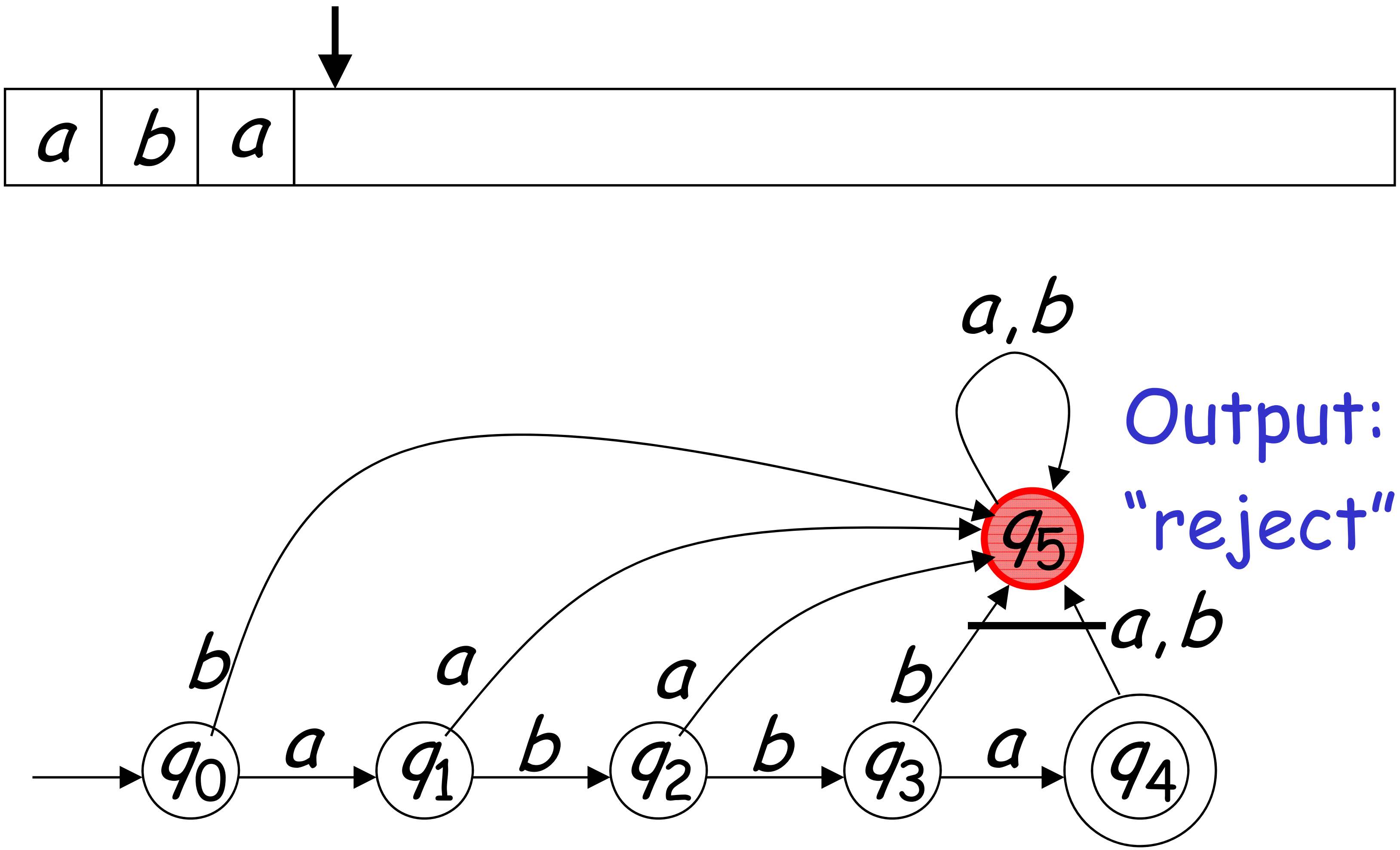
Rejection



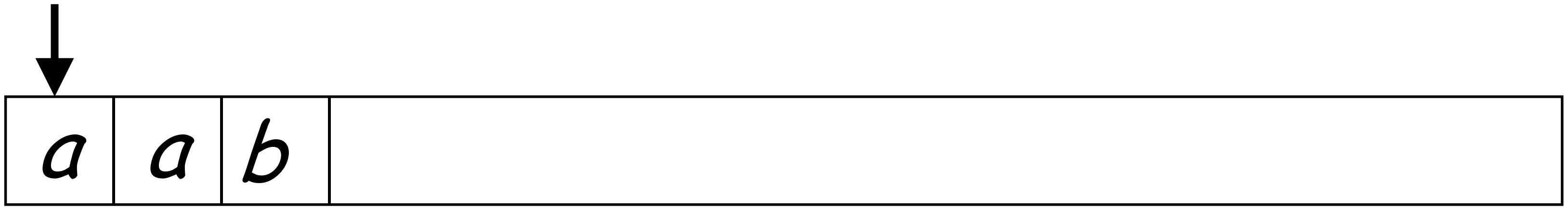

a b a
a,b
b
a
a
b
a
b
b
a
a,b
q0 q1 q2 q3 q4 q5

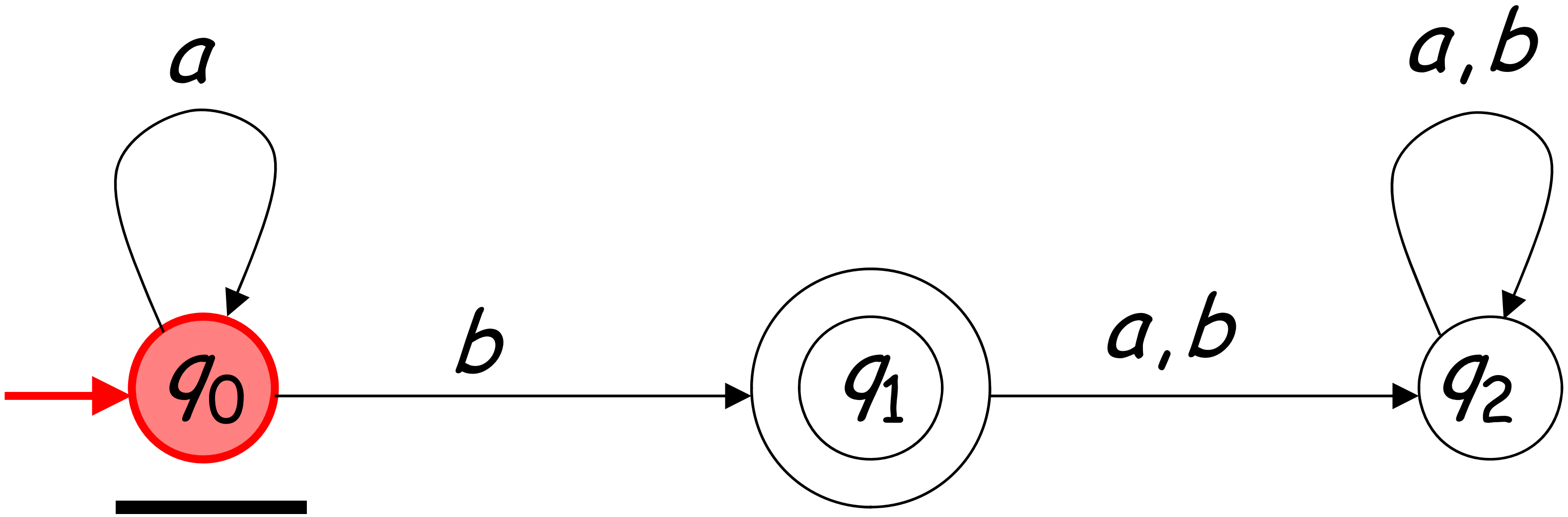

a b a
a,b
b
a
a
b
a,b
q0 a q1 b q2 b q3 a q4
q5

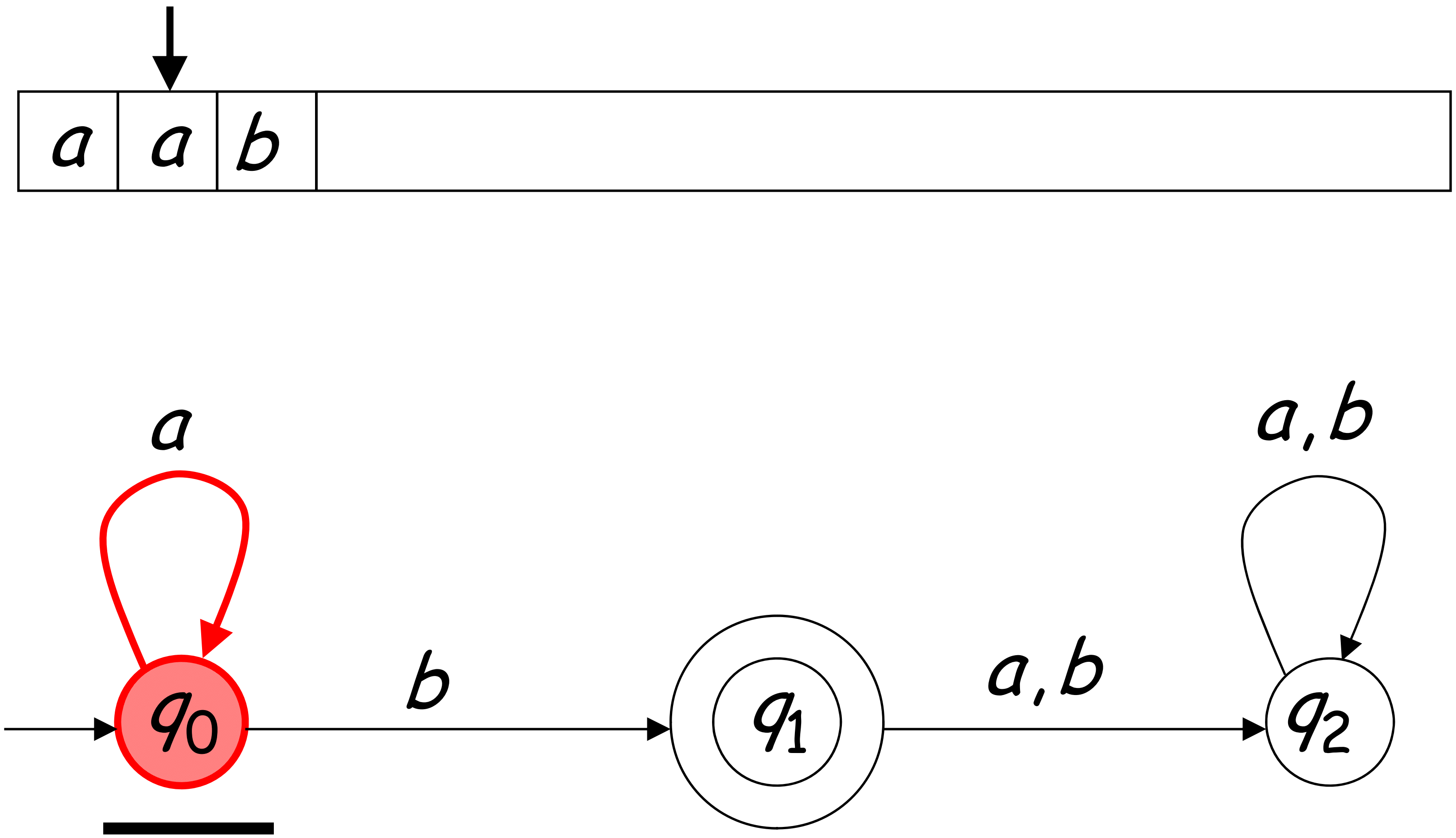

a
b
a
a,b
a,b
b
a
a
b
q0
a
q1
b
q2
b
q3
a
q4
q5

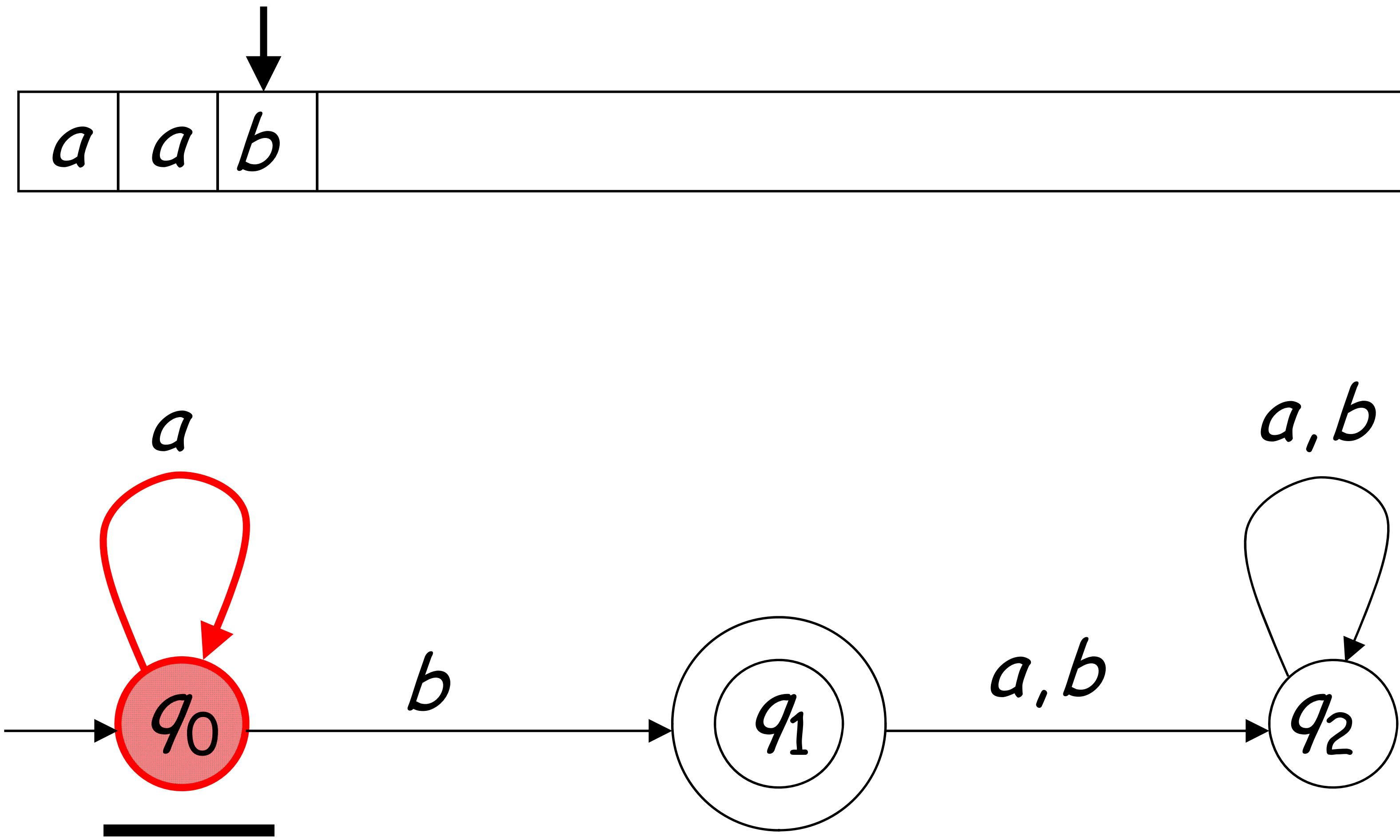

a
b
a
a,b
Output:
"reject"
a,b
b
a
a
b
q5
q0
a
q1
b
q2
b
q3
a
q4

Another Example

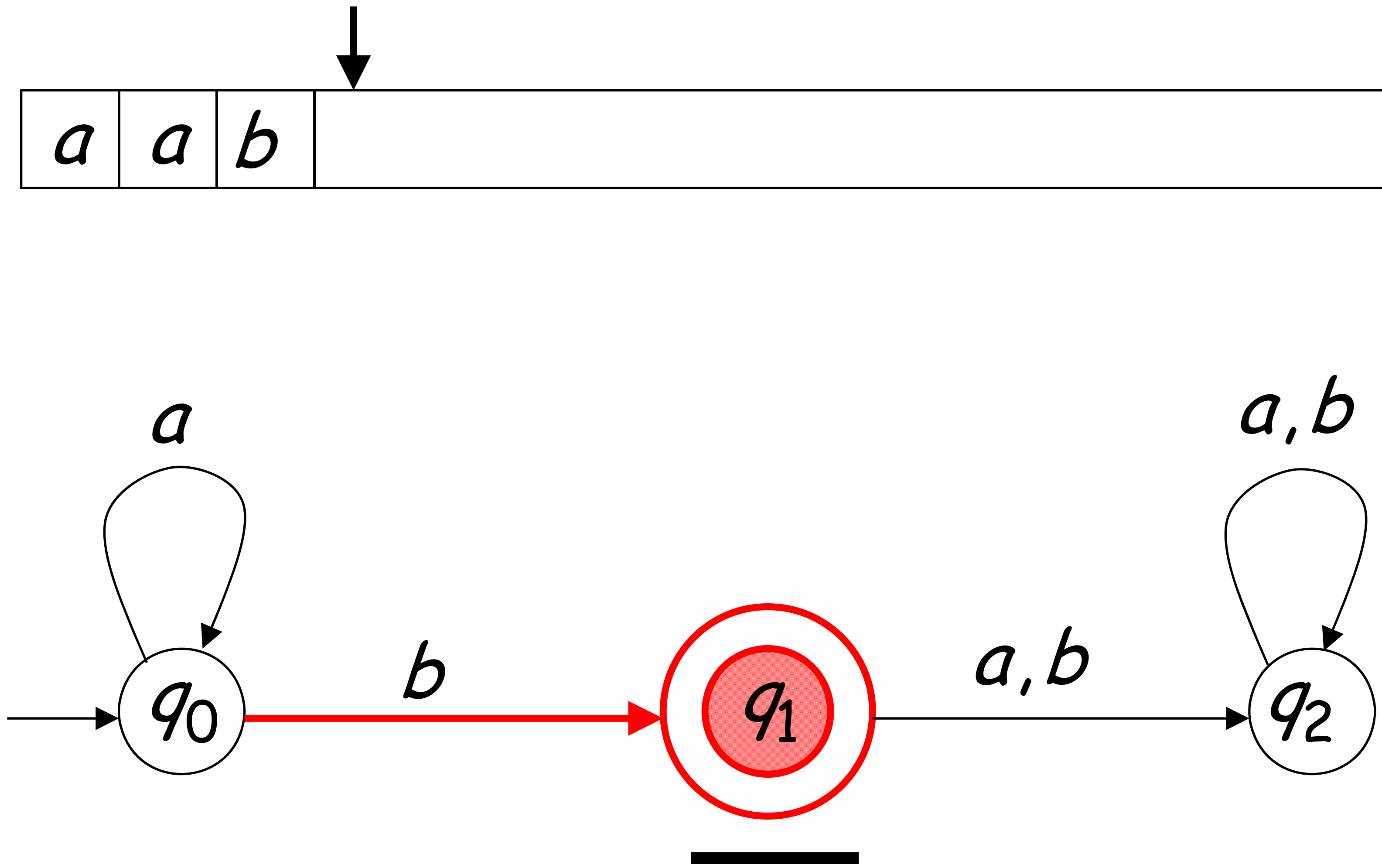


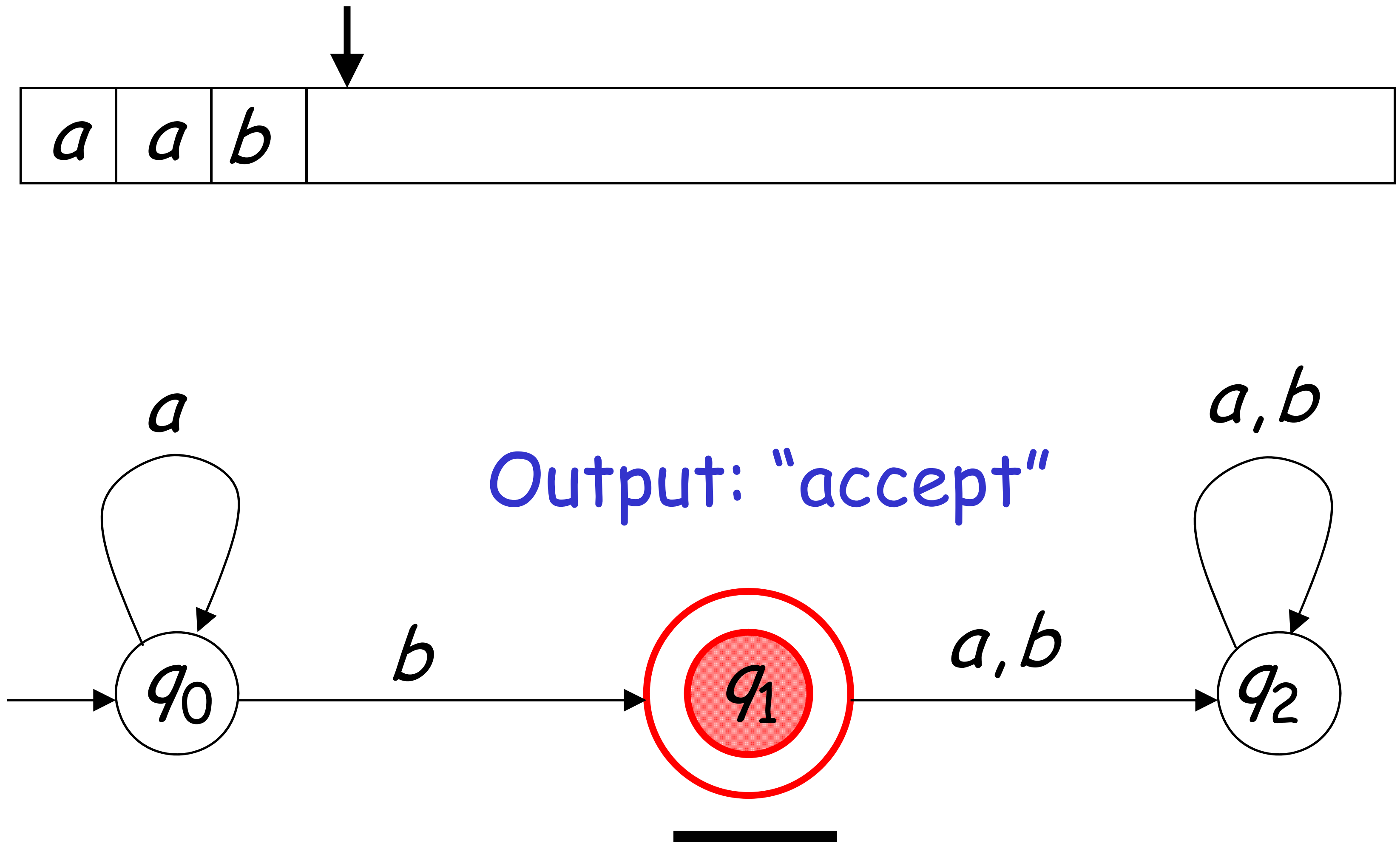



a
a
b
a
b
a,b
a,b
q0
q1
q2

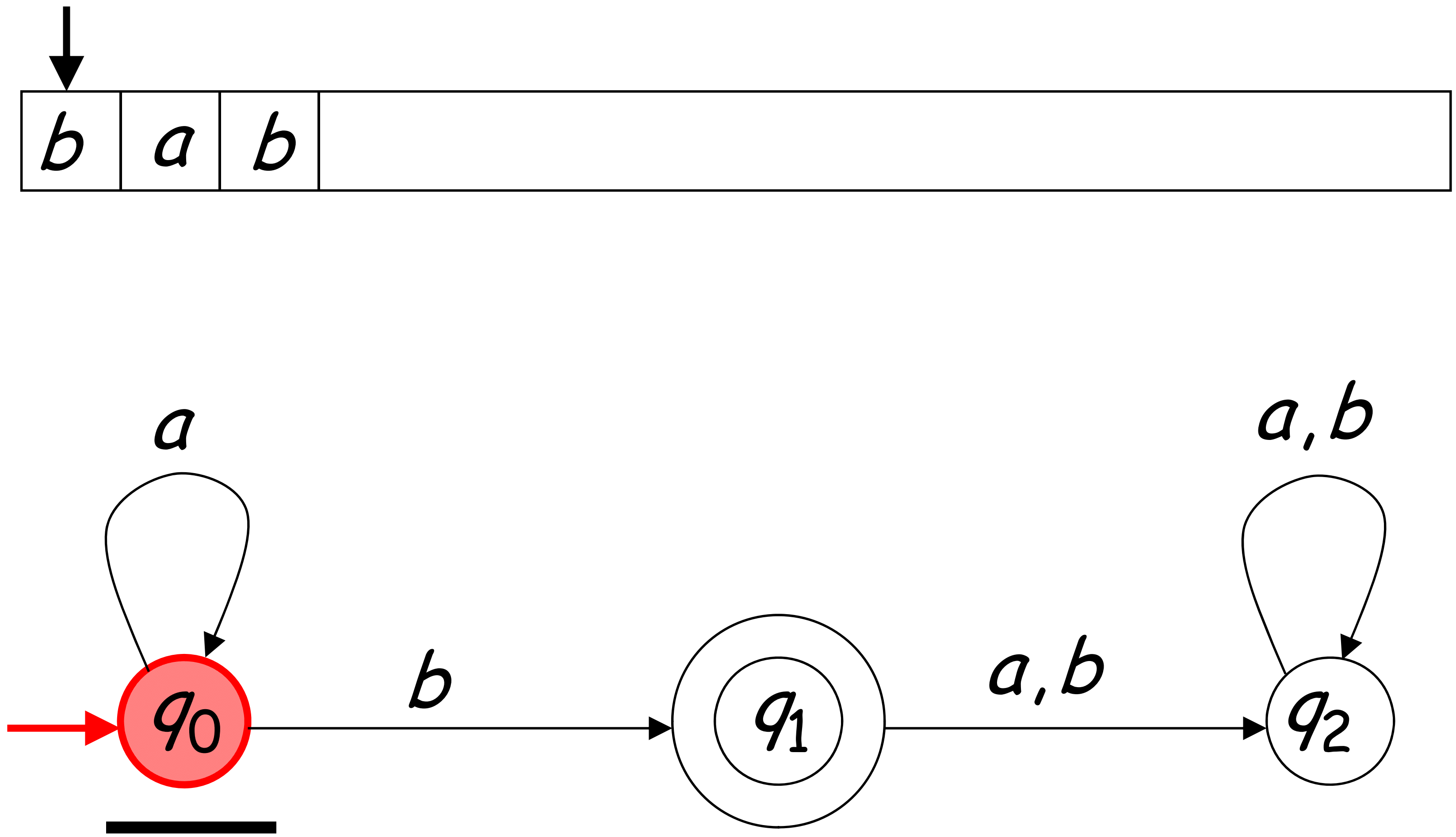


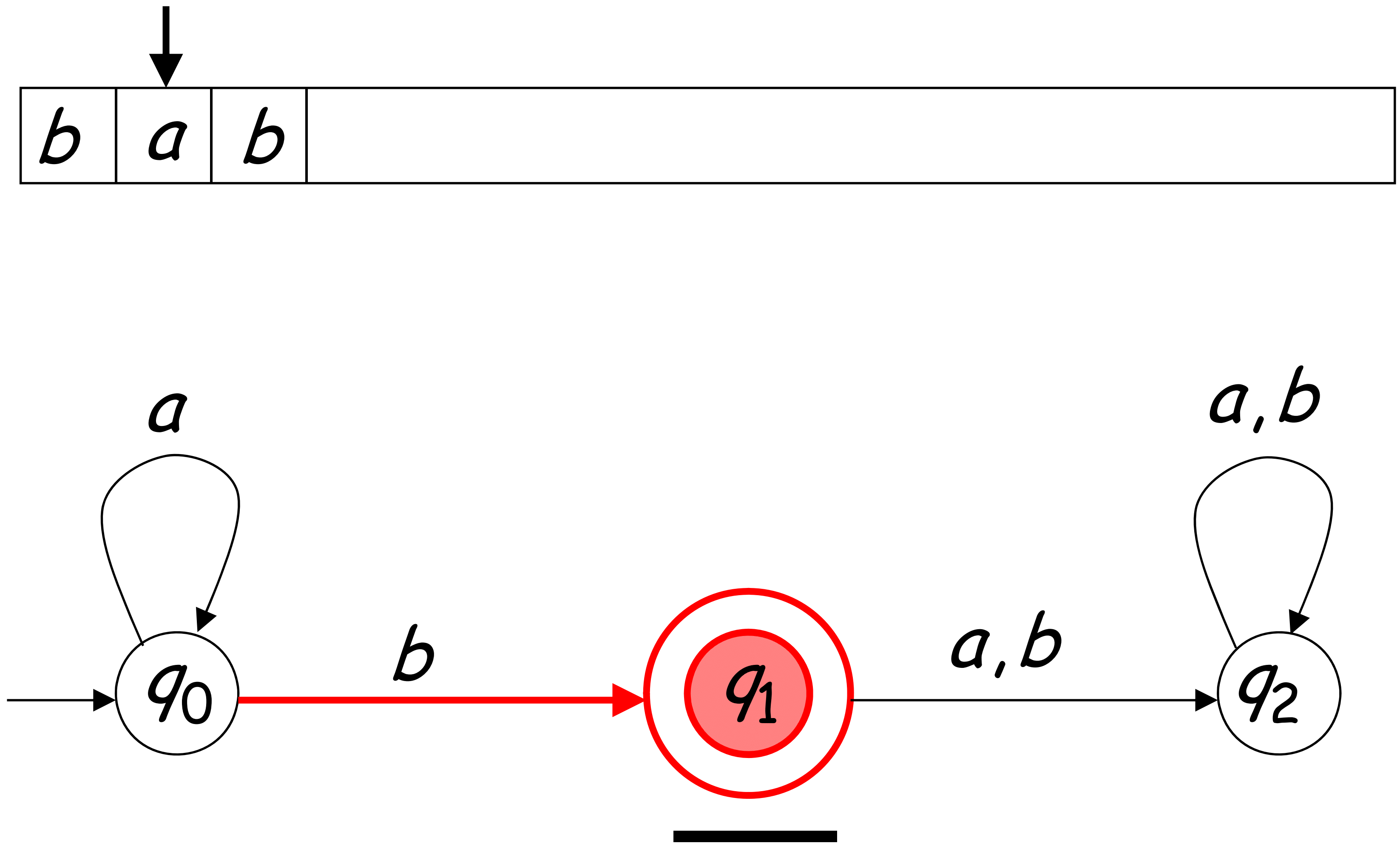
a
a
b
a
b
a, b
a, b
q_0
q_1
q_2

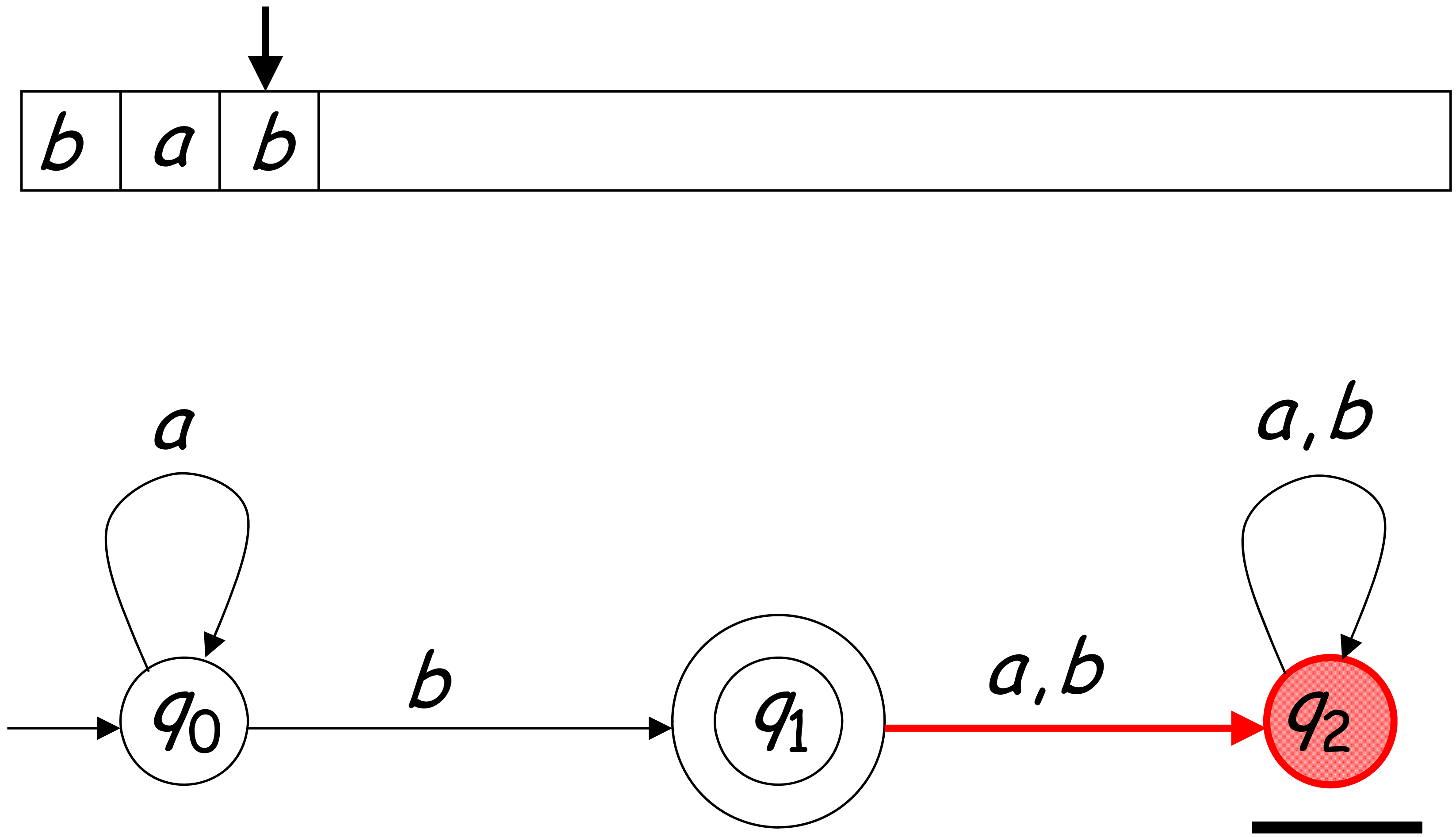

a a b
a
a,b
b
a,b
q0
q1
q2


a a b
a
a,b
Output: “accept”
b
a,b
q0
q1
q2

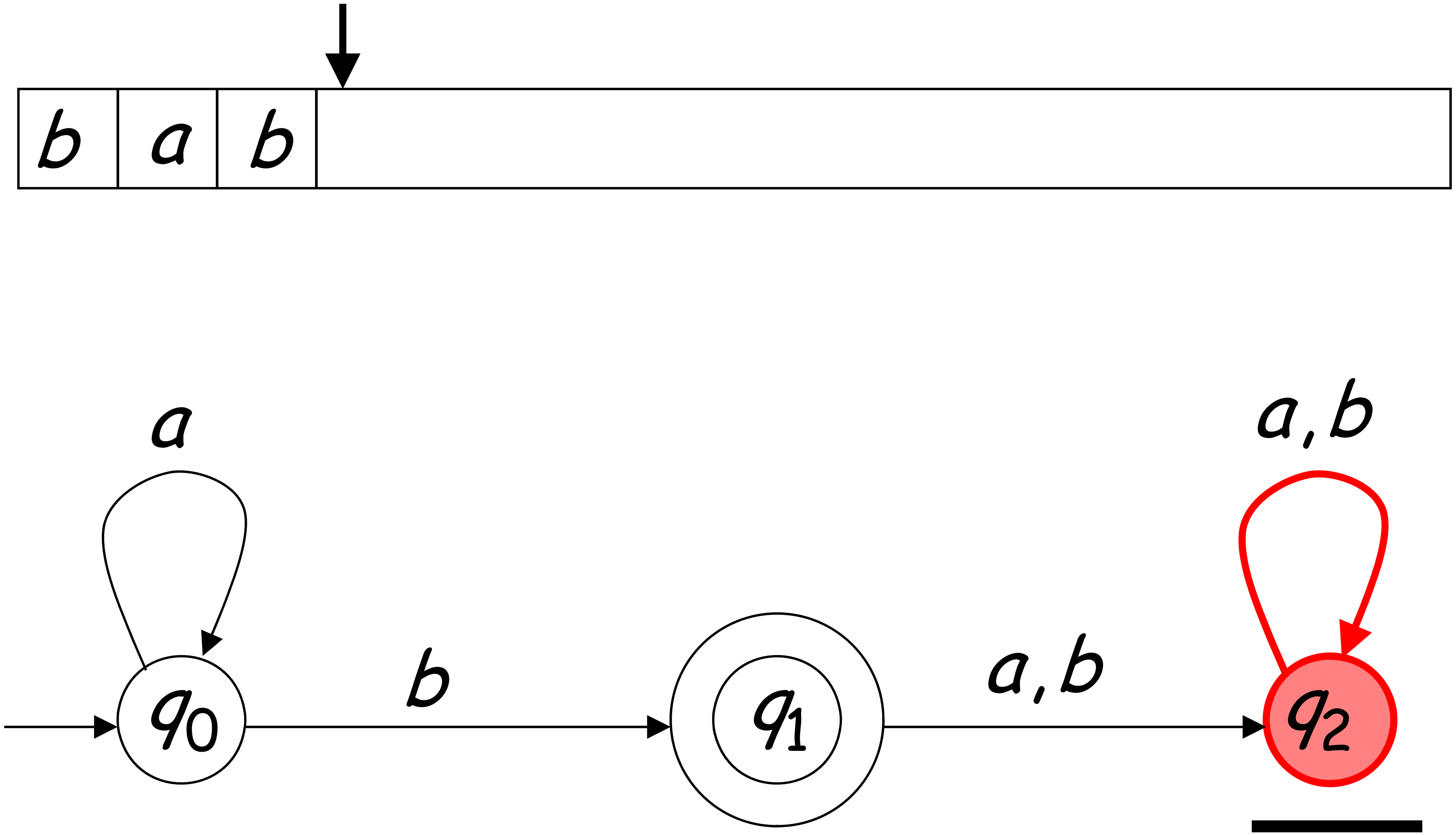
Rejection

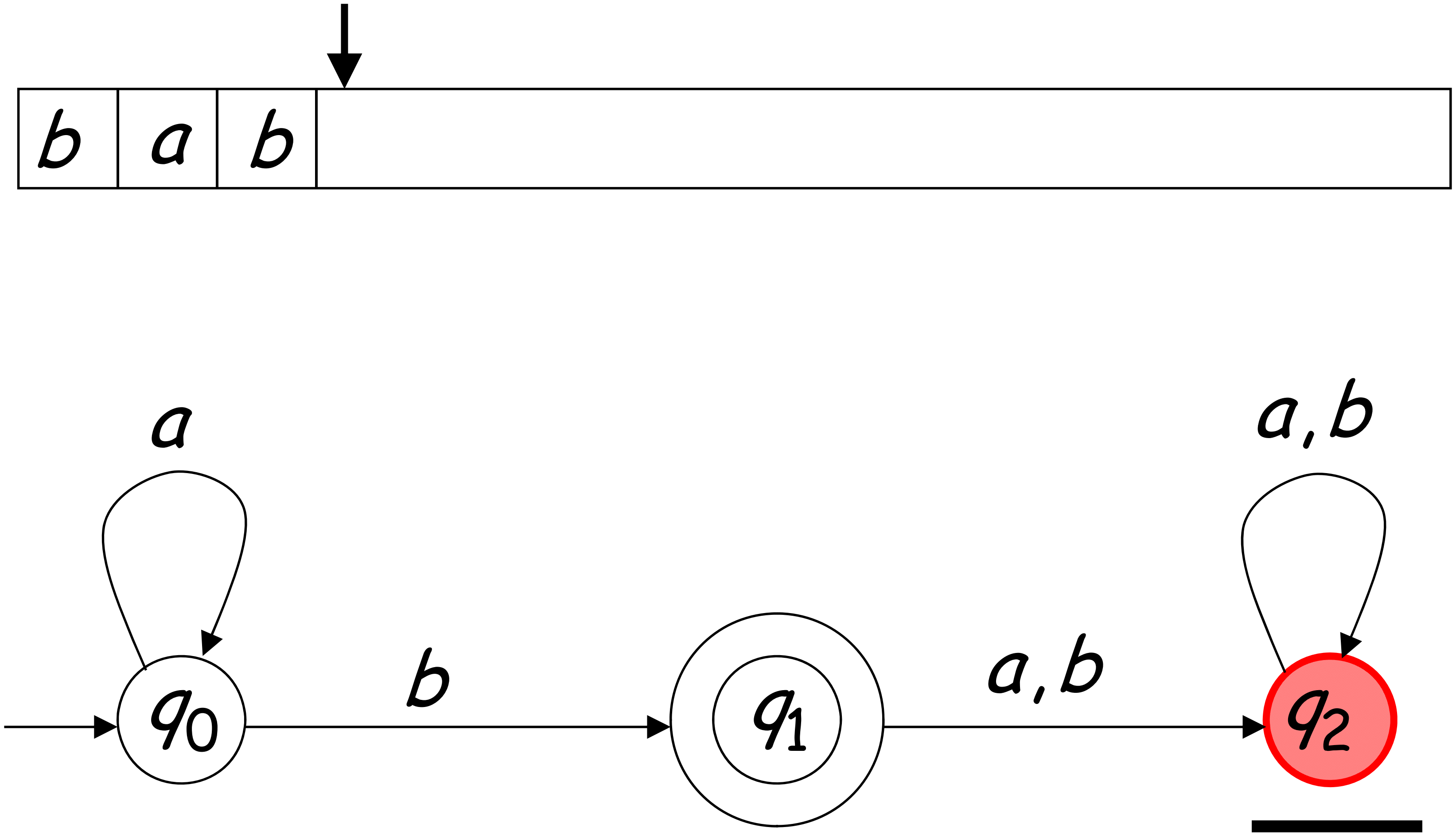



b a b
a
a,b
b
a,b
q_0
q_1
q_2



b
a
b
a
a,b
b
a,b
q_0
q_1
q_2


b a b
a
a,b
b
a,b
q0
q1
q2



Output: “reject”

Formalities

Deterministic Finite Accepter (DFA)

$$M = (Q, \Sigma, \delta, q_0, F)$$

Q : set of states

Σ : input alphabet

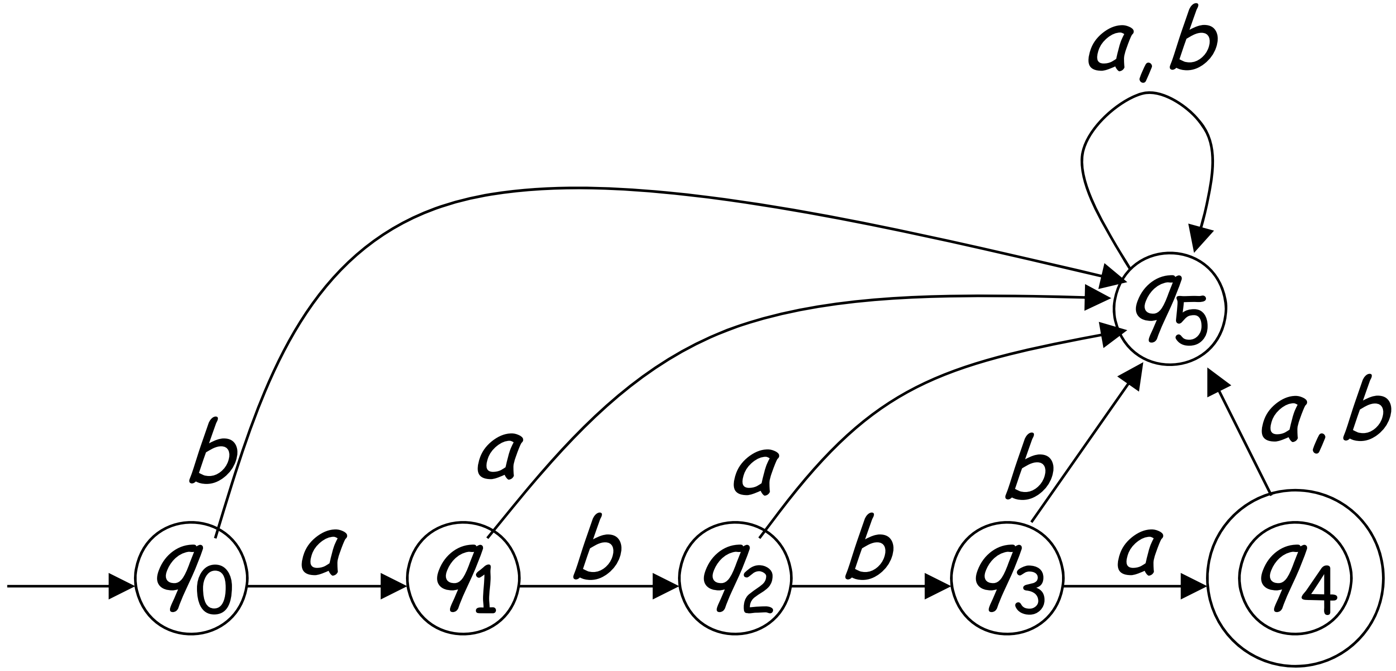
δ : transition function

q_0 : initial state

F : set of final states

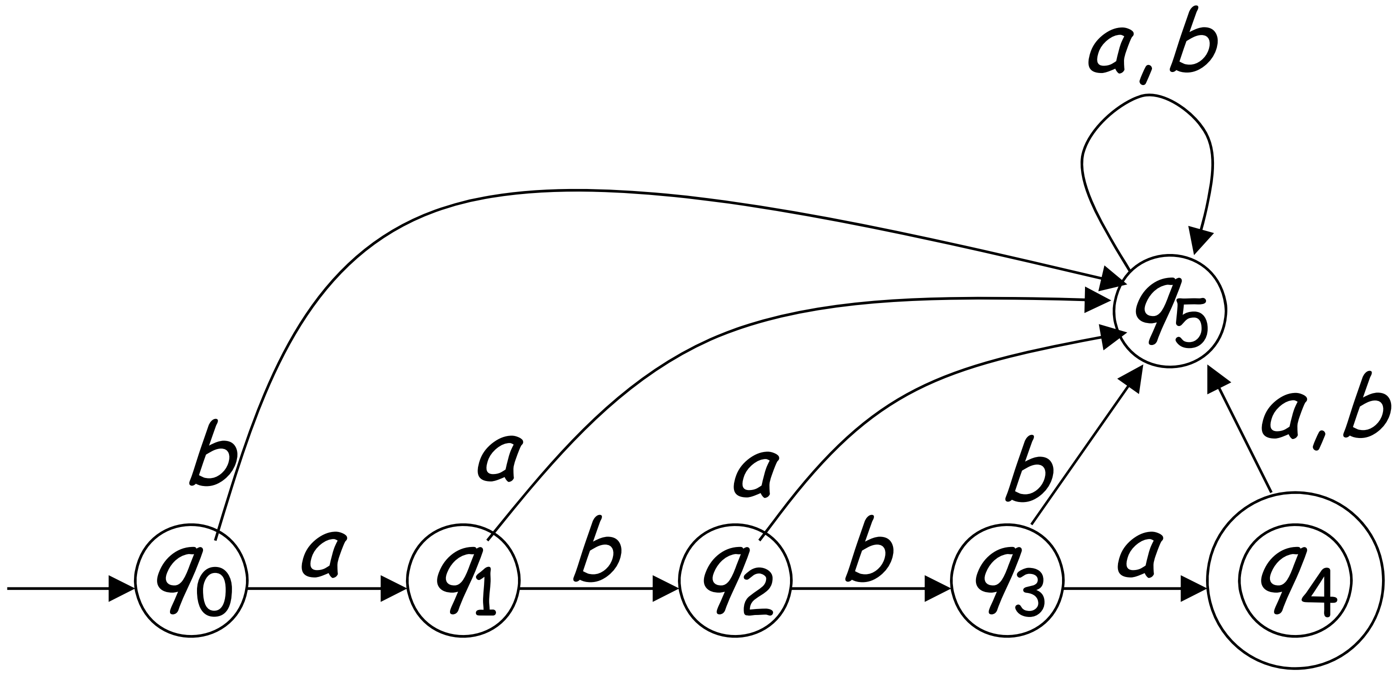
Input Aplhabet Σ

$$\Sigma = \{a, b\}$$

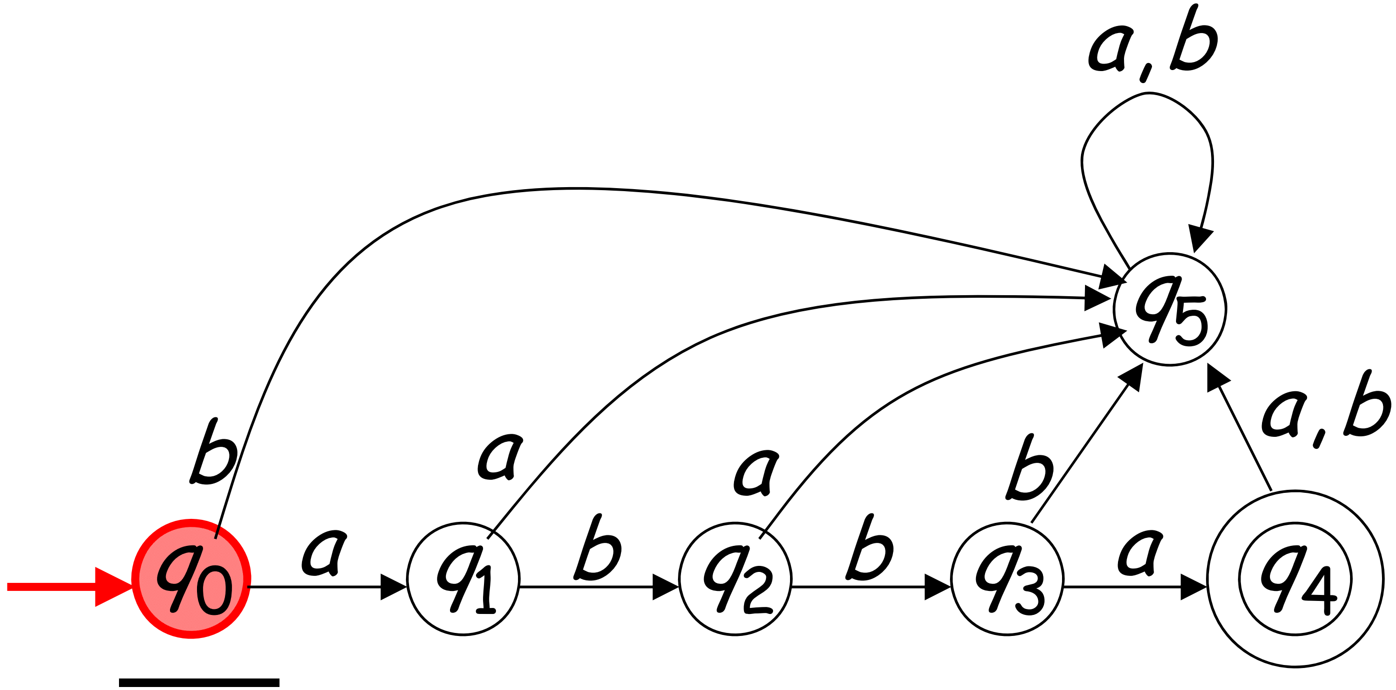


Set of States Q

$$Q = \{q_0, q_1, q_2, q_3, q_4, q_5\}$$

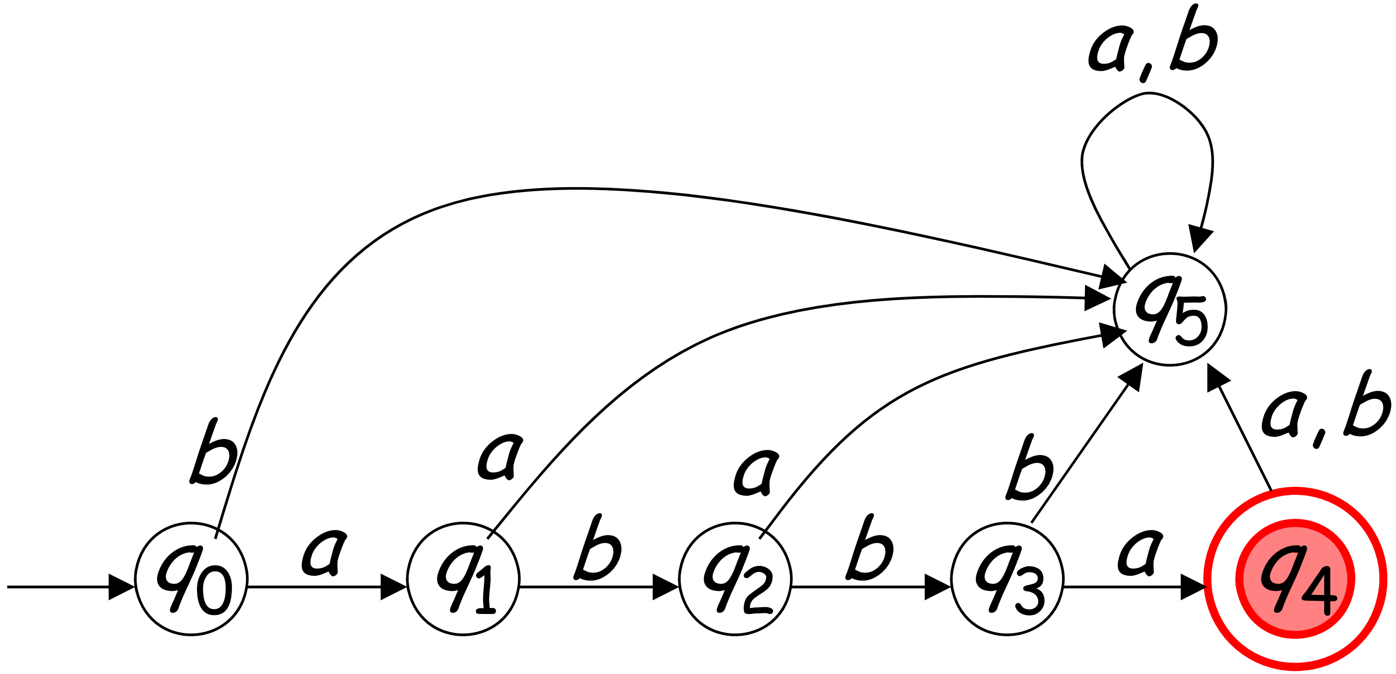


Initial State q_0



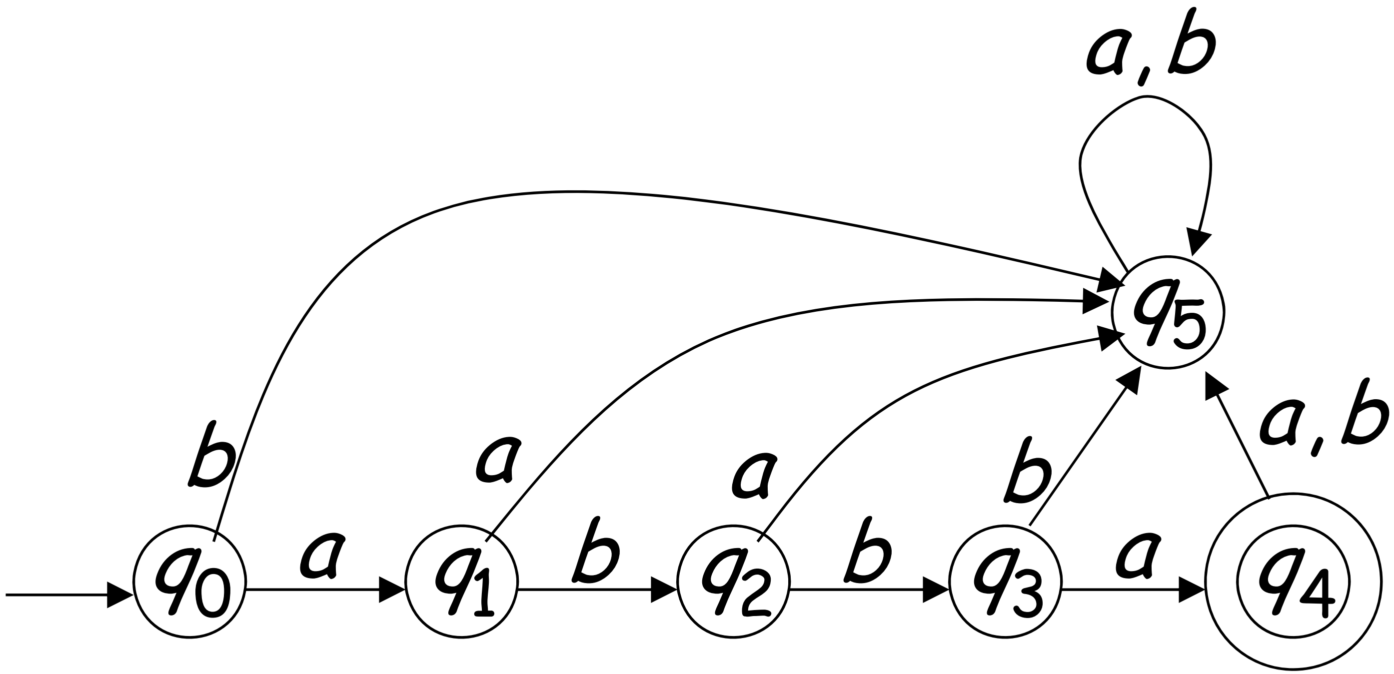
Set of Final States F

$$F = \{q_4\}$$

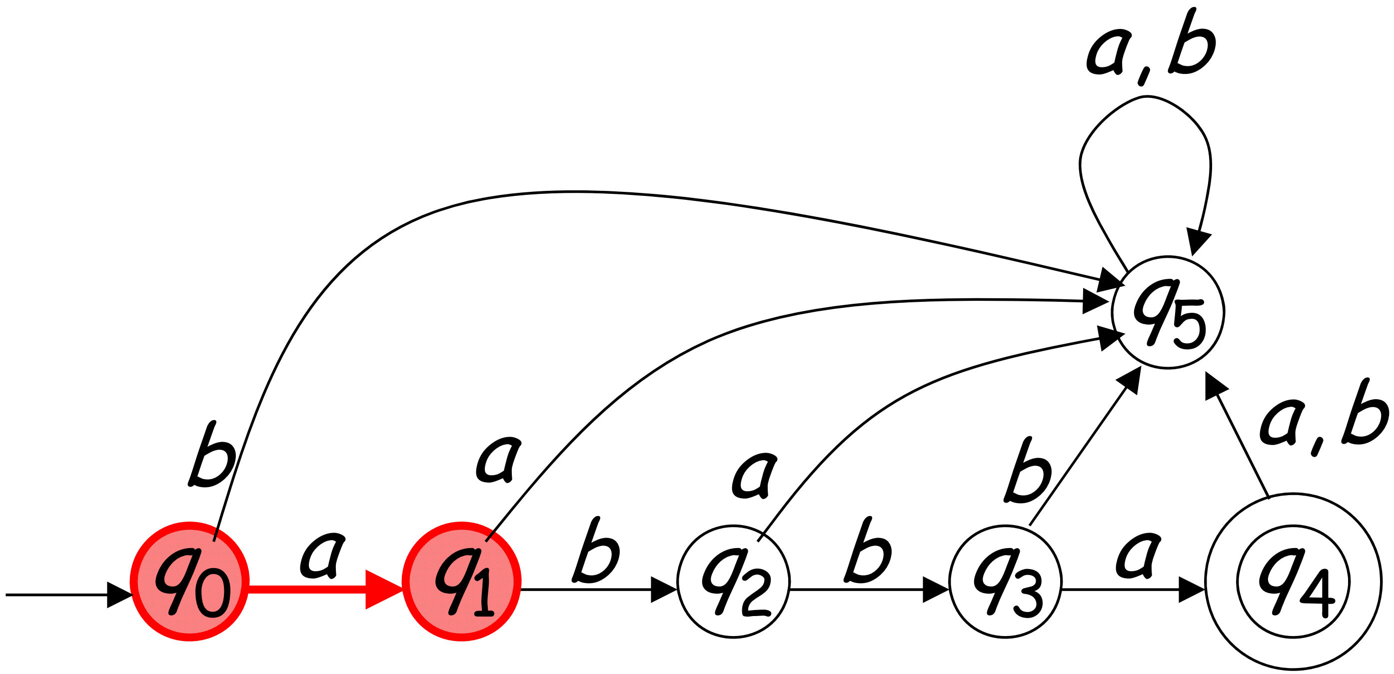


Transition Function δ

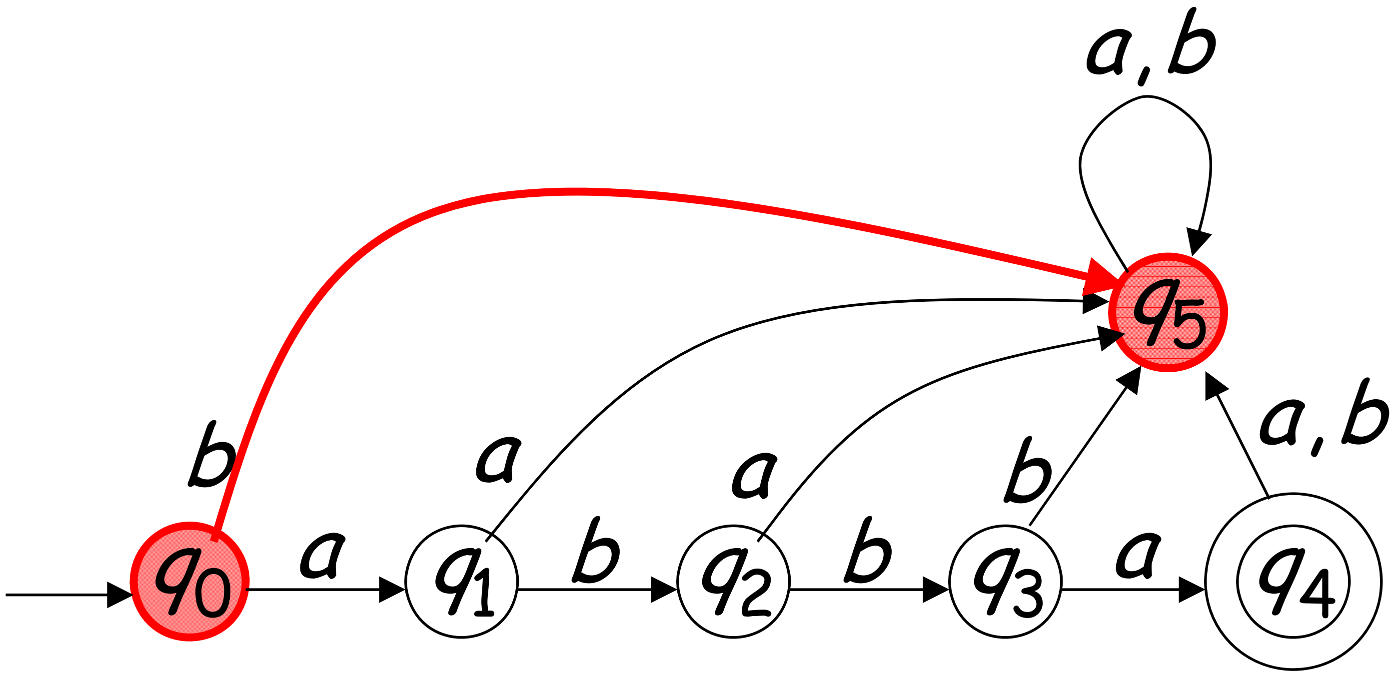
$$\delta : Q \times \Sigma \rightarrow Q$$



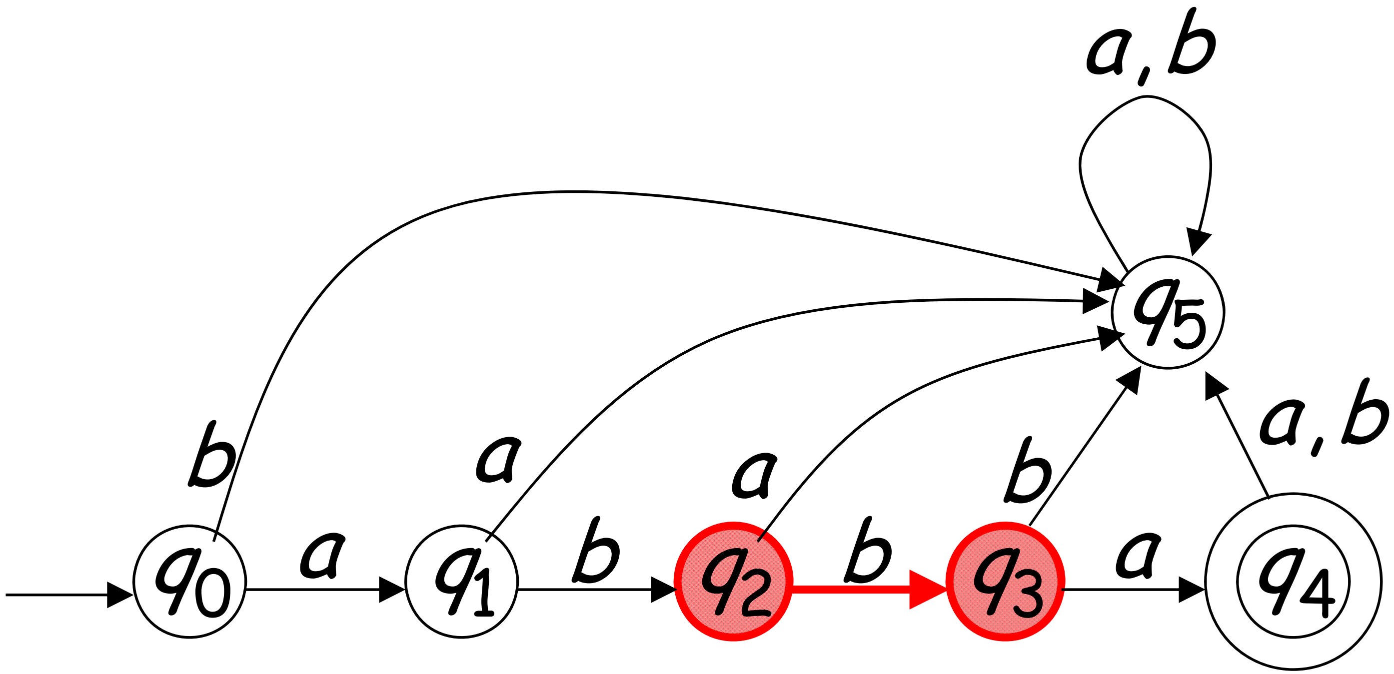
$$\delta(q_0, a) = q_1$$



$$\delta(q_0, b) = q_5$$

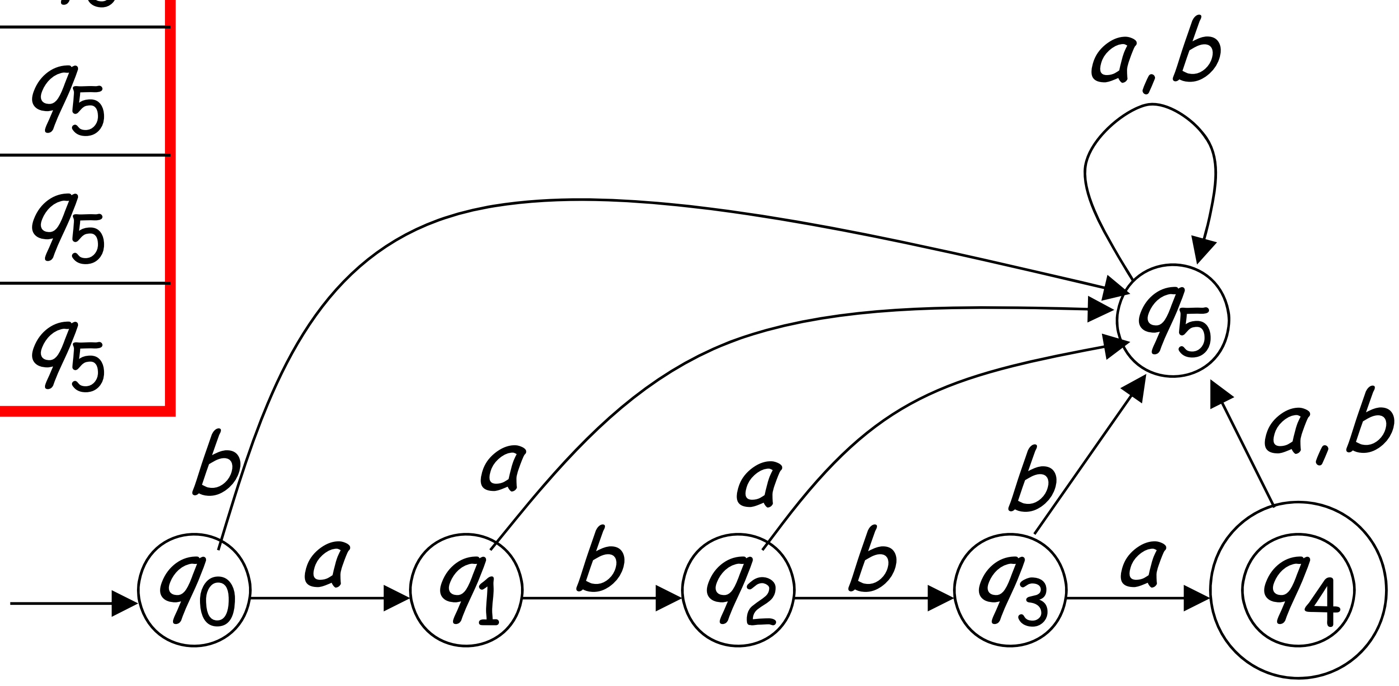


$$\delta(q_2, b) = q_3$$



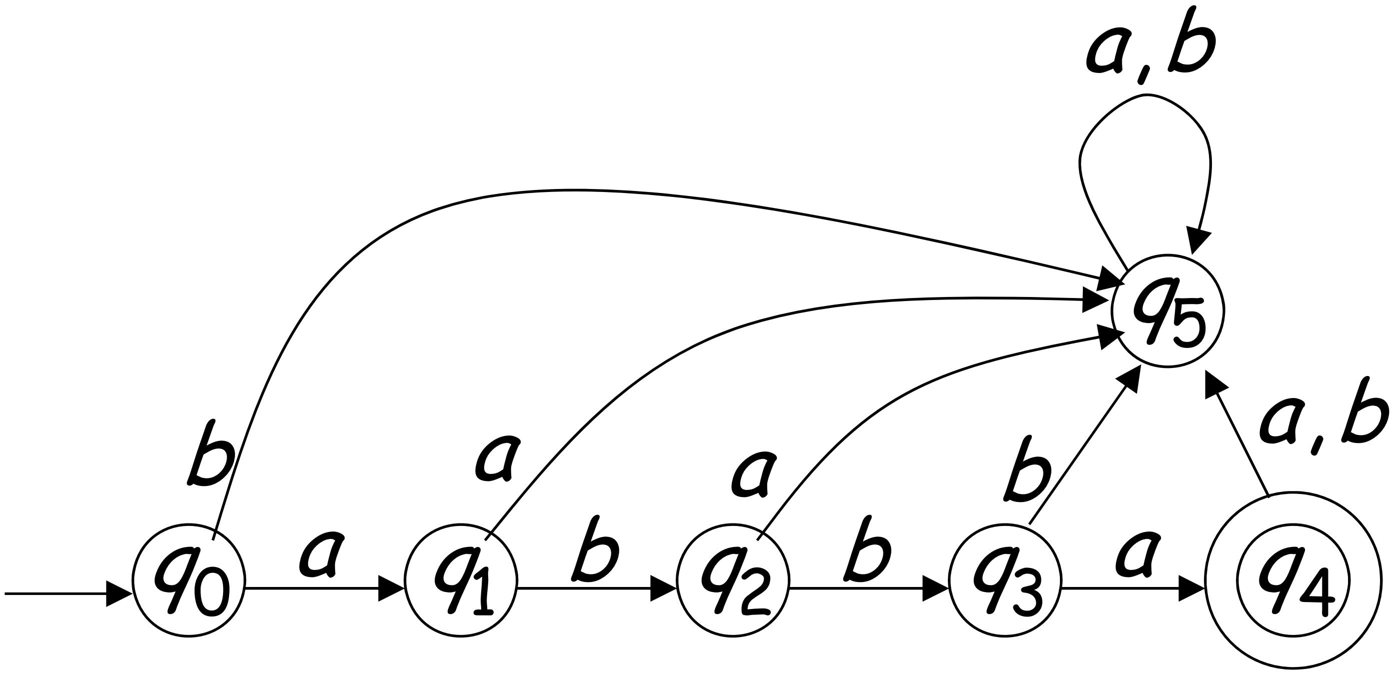
Transition Function δ

δ	a	b
q_0	q_1	q_5
q_1	q_5	q_2
q_2	q_5	q_3
q_3	q_4	q_5
q_4	q_5	q_5
q_5	q_5	q_5

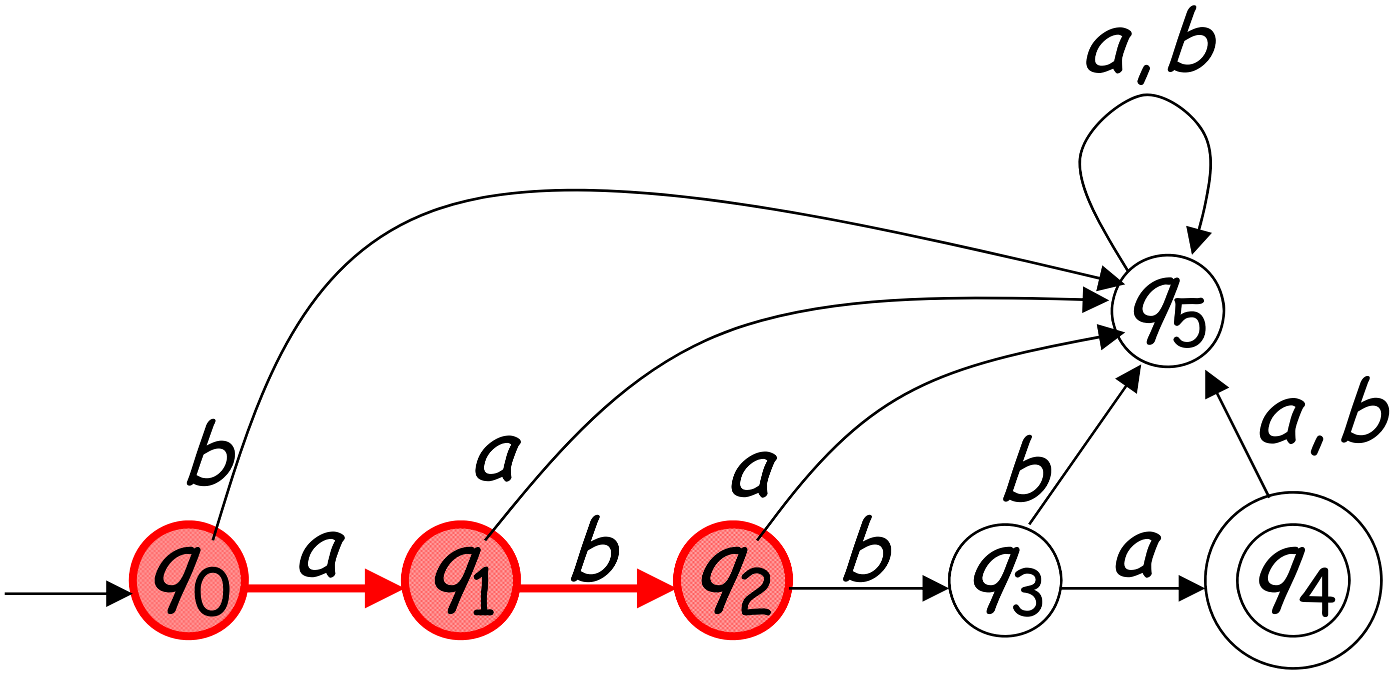


Extended Transition Function δ^*

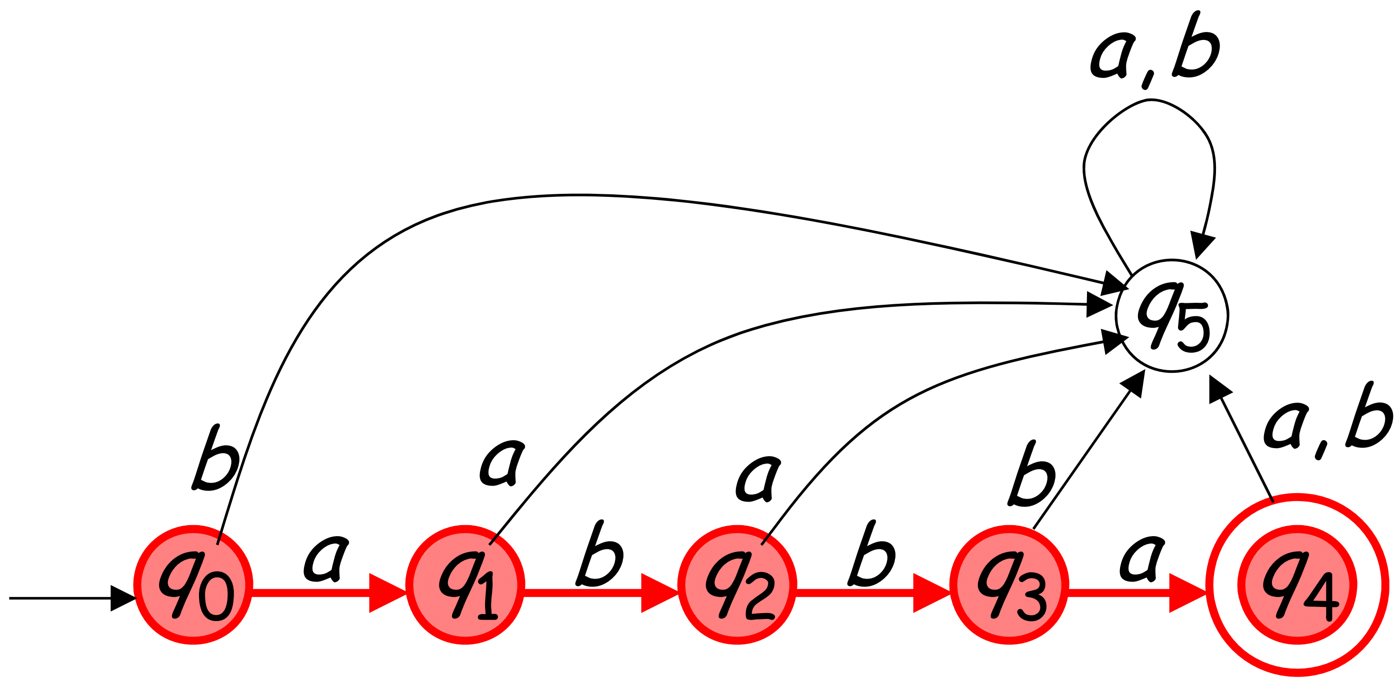
$$\delta^* : Q \times \Sigma^* \rightarrow Q$$



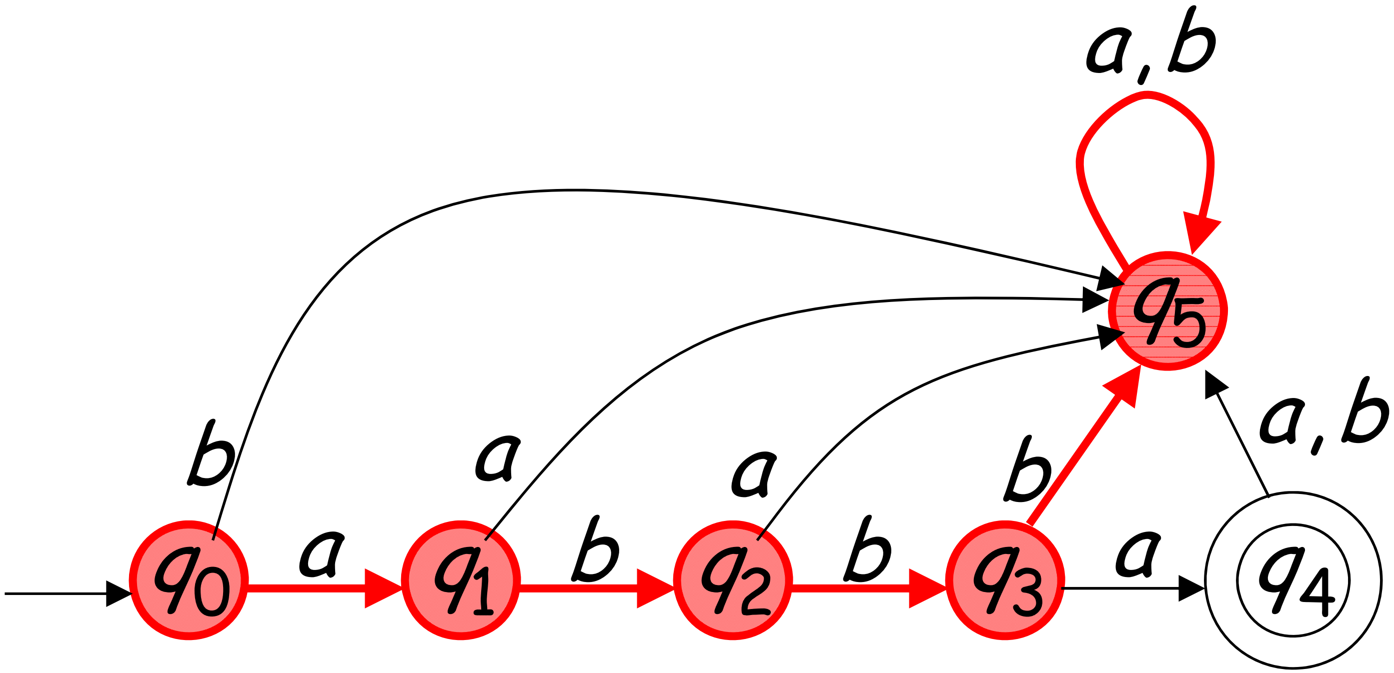
$$\delta^*(q_0, ab) = q_2$$



$$\delta^*(q_0, abba) = q_4$$

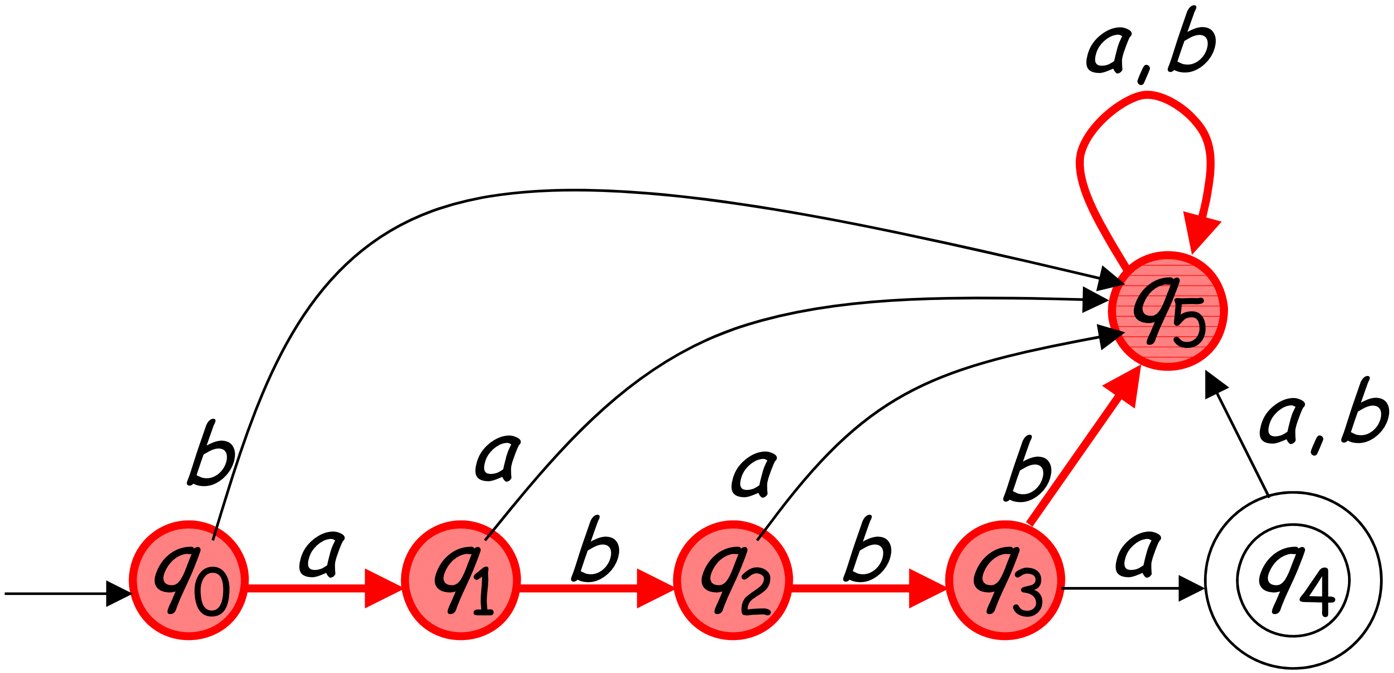


$$\delta^*(q_0, abbbaa) = q_5$$



Observation: There is a walk from q_0 to q_5 with label $abbbaa$

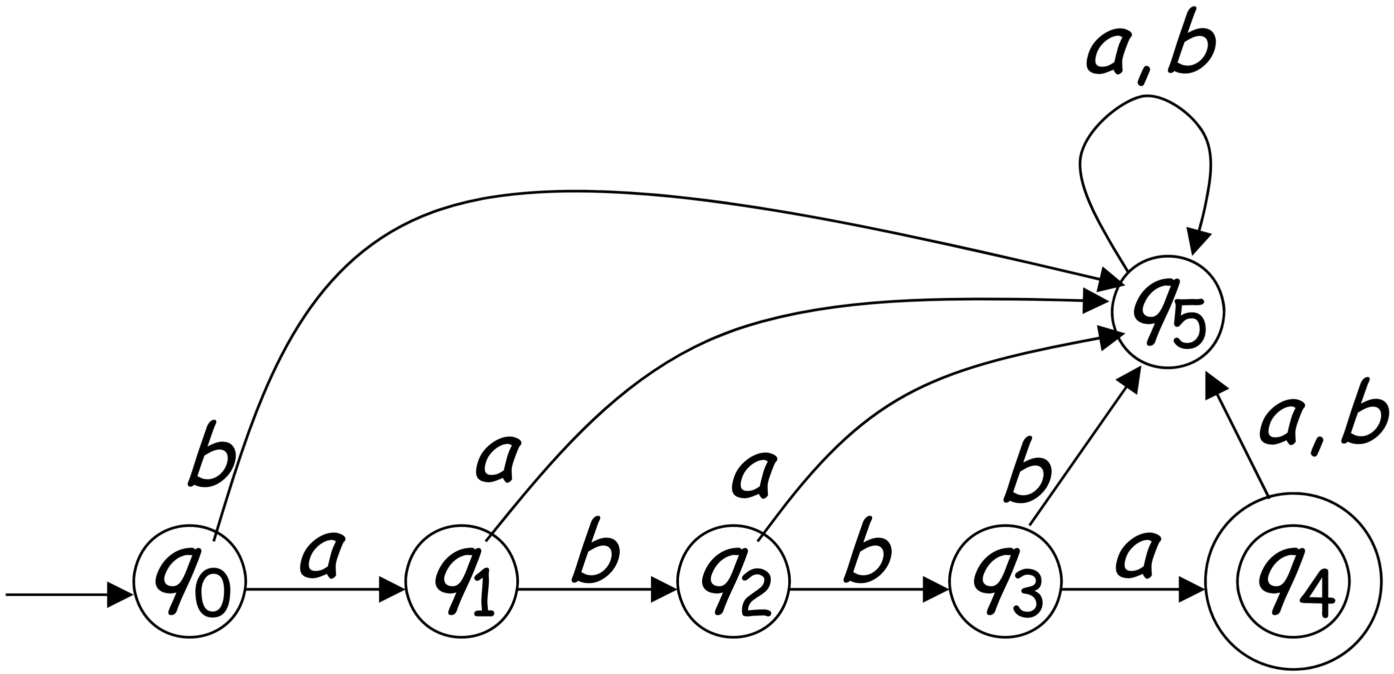
$$\delta^*(q_0, abbbaa) = q_5$$



Recursive Definition

$$\delta^*(q, \lambda) = q$$

$$\delta^*(q, wa) = \delta(\delta^*(q, w), a)$$



$$\delta^*(q_0, ab) =$$

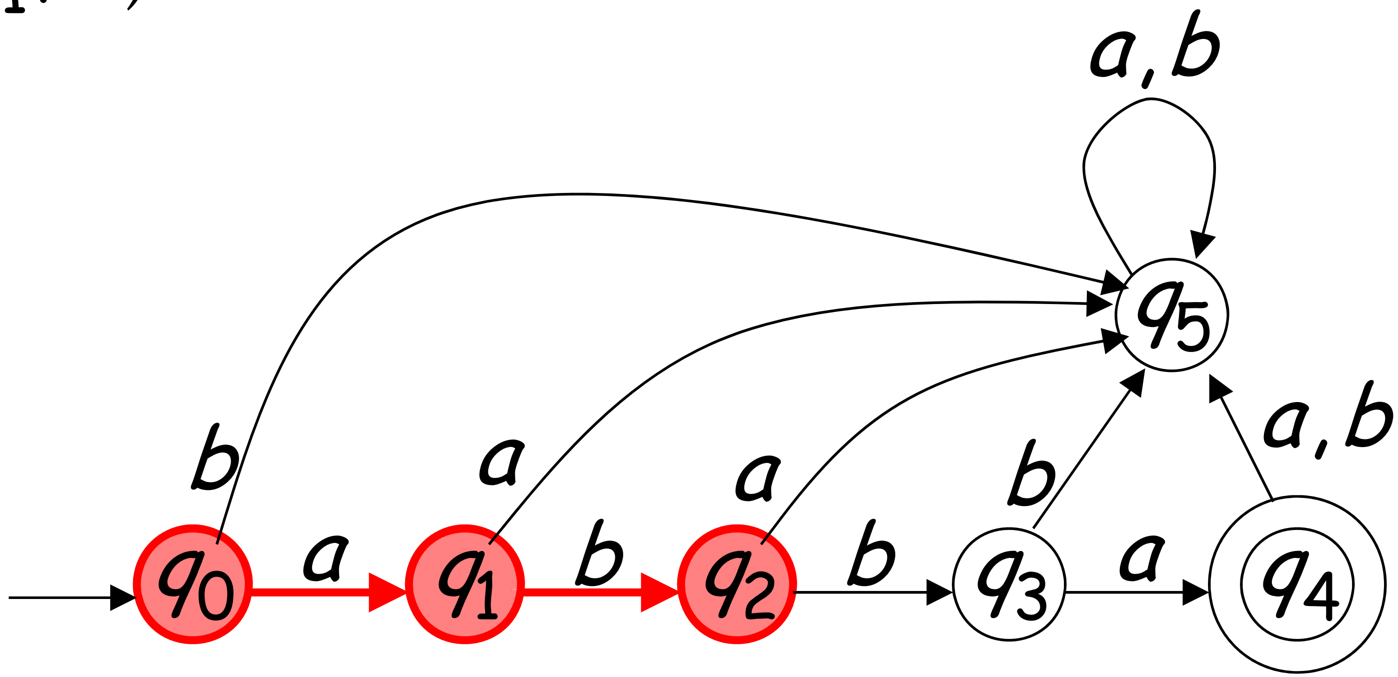
$$\delta(\delta^*(q_0, a), b) =$$

$$\delta(\delta(\delta^*(q_0, \lambda), a), b) =$$

$$\delta(\delta(q_0, a), b) =$$

$$\delta(q_1, b) =$$

$$q_2$$



Languages Accepted by DFAs

Take DFA M

Definition:

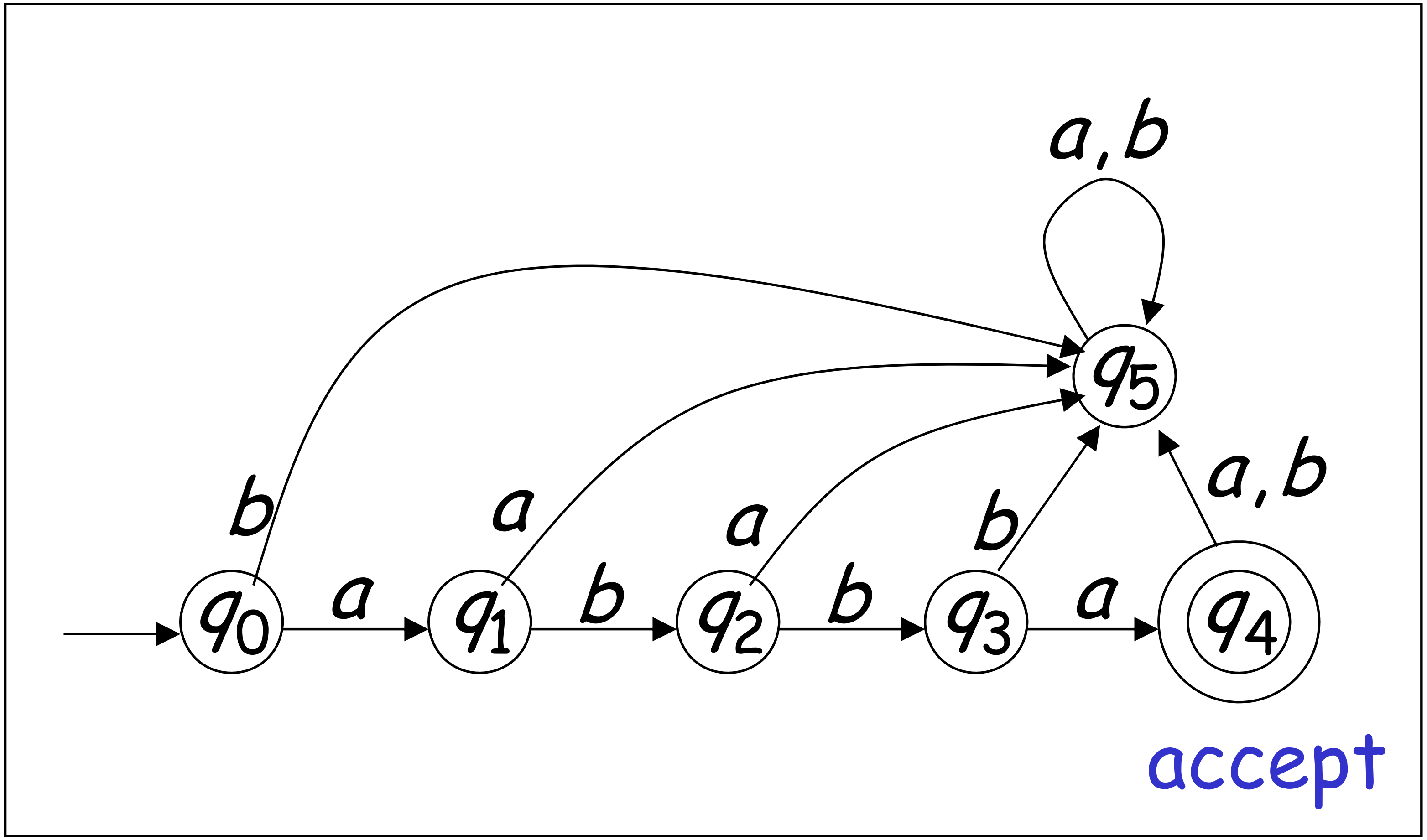
The language $L(M)$ contains all input strings accepted by M

$L(M)$ = { strings that drive M to a final state}

Example

$$L(M) = \{abba\}$$

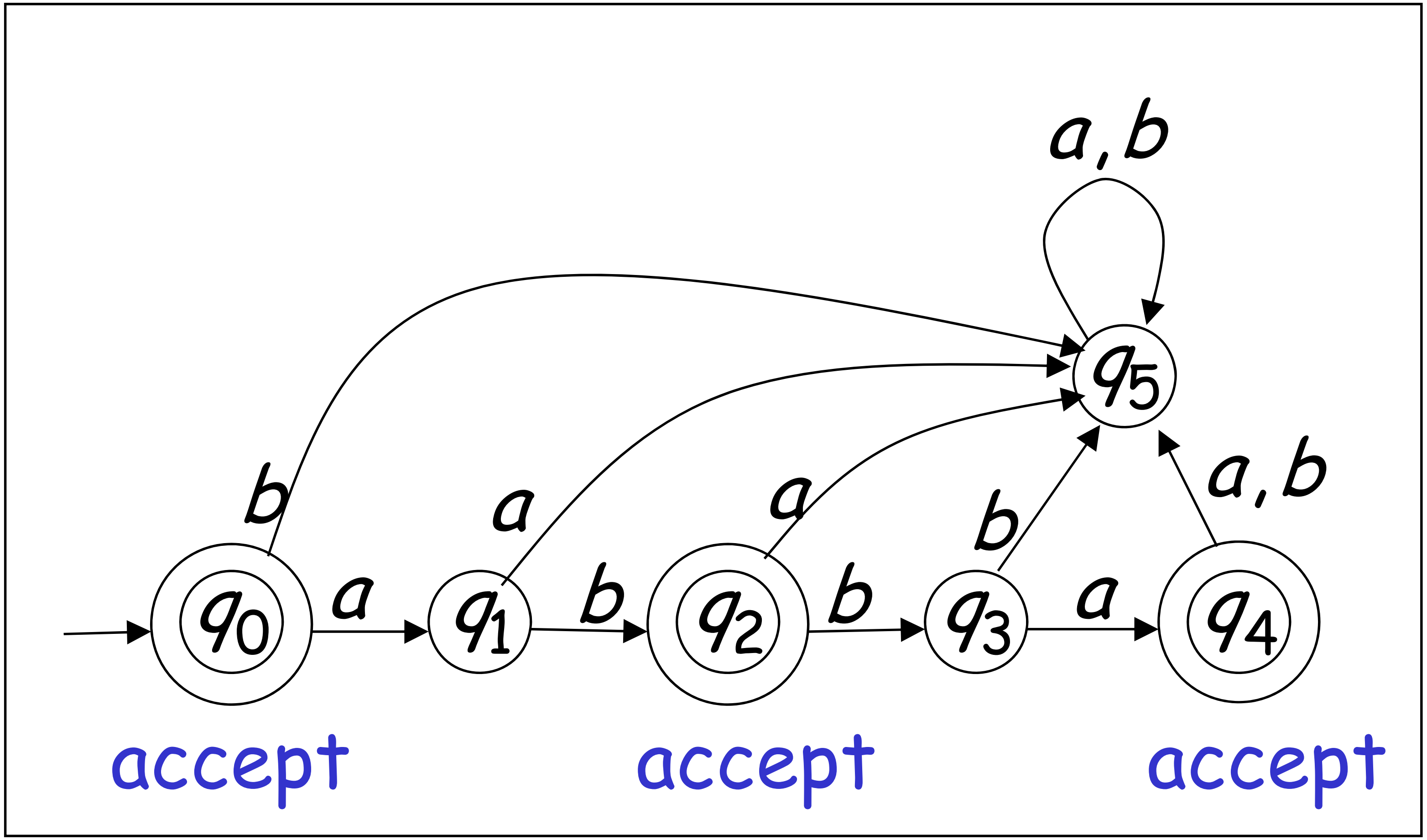
M



Another Example

$$L(M) = \{\lambda, ab, abba\}$$

M



Formally

For a DFA $M = (Q, \Sigma, \delta, q_0, F)$

Language accepted by M :

$$L(M) = \{ w \in \Sigma^* : \delta^*(q_0, w) \in F \}$$

alphabet

transition function

initial state

final states

Observation

Language accepted by M:

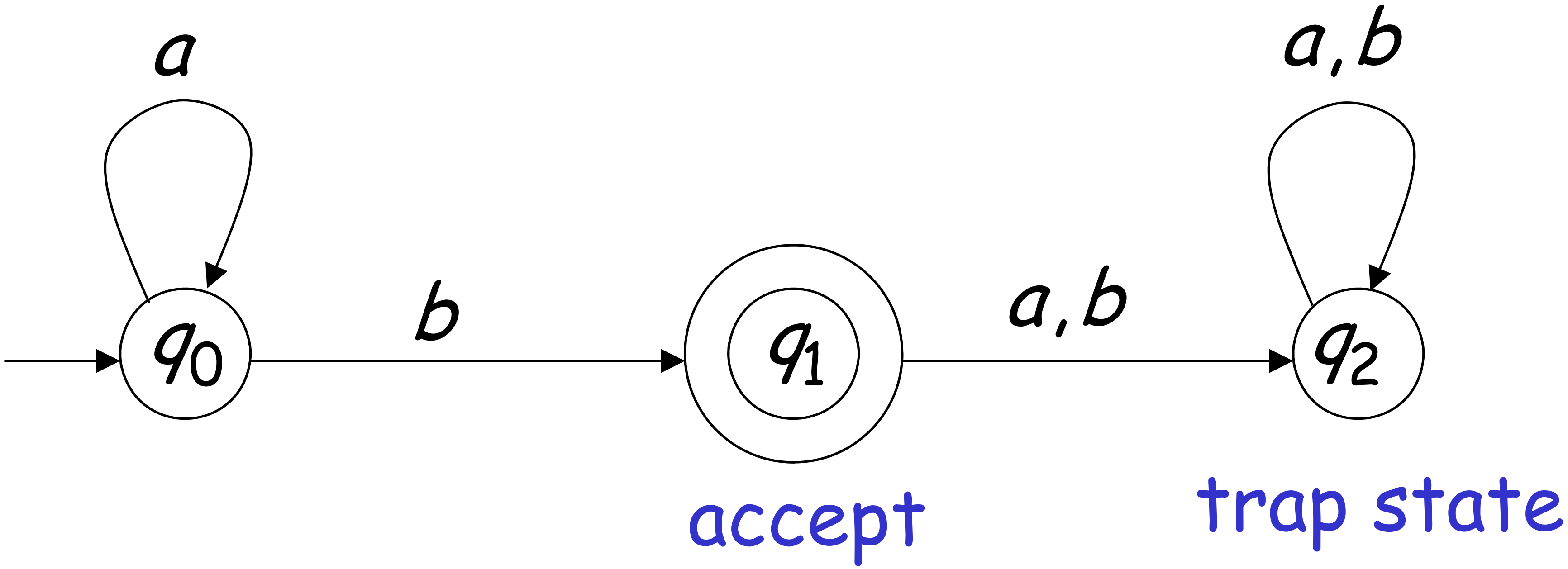
$$L(M) = \{w \in \Sigma^* : \delta^*(q_0, w) \in F\}$$

Language rejected by M:

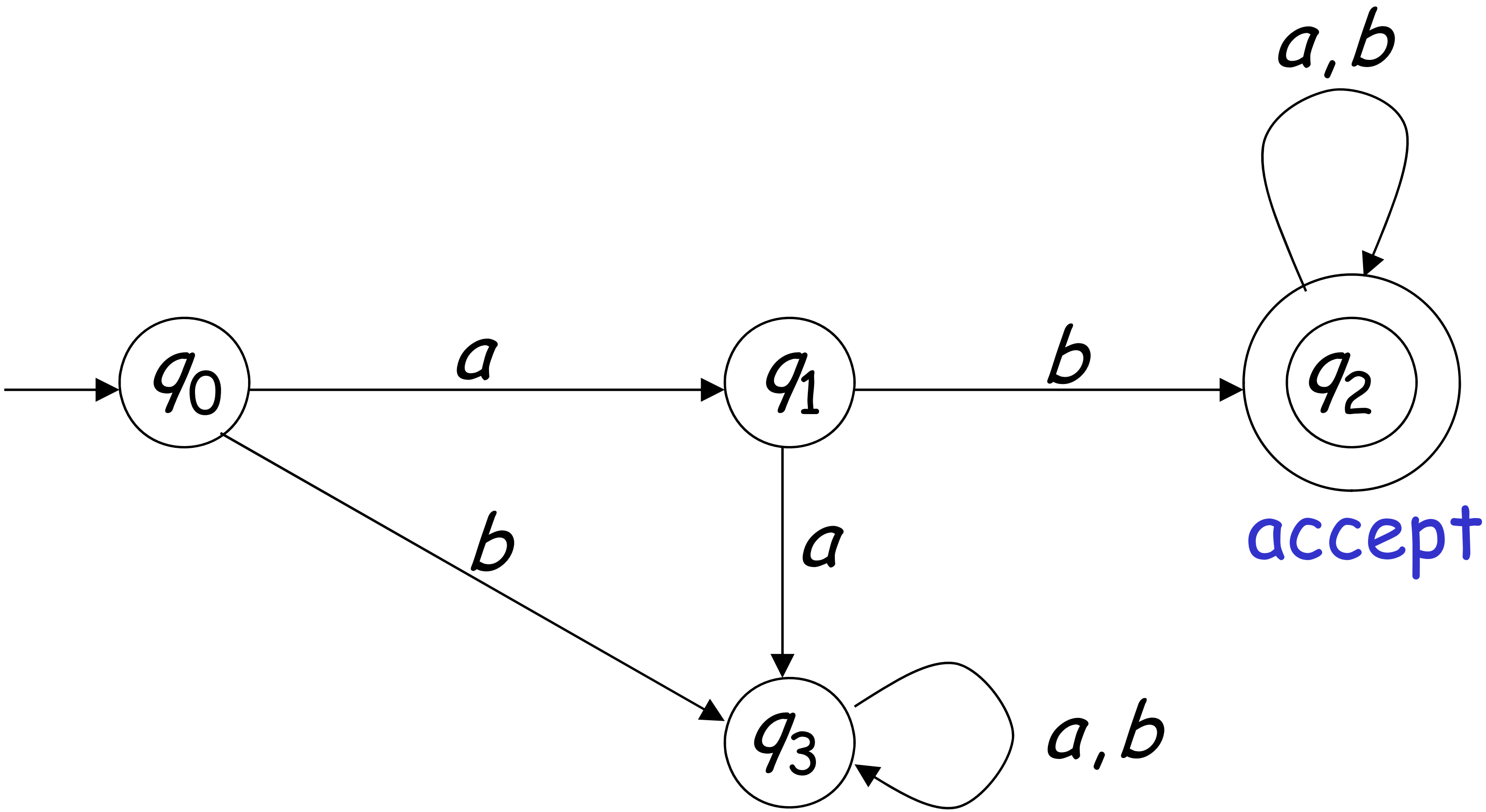
$$\overline{L(M)} = \{w \in \Sigma^* : \delta^*(q_0, w) \notin F\}$$

More Examples

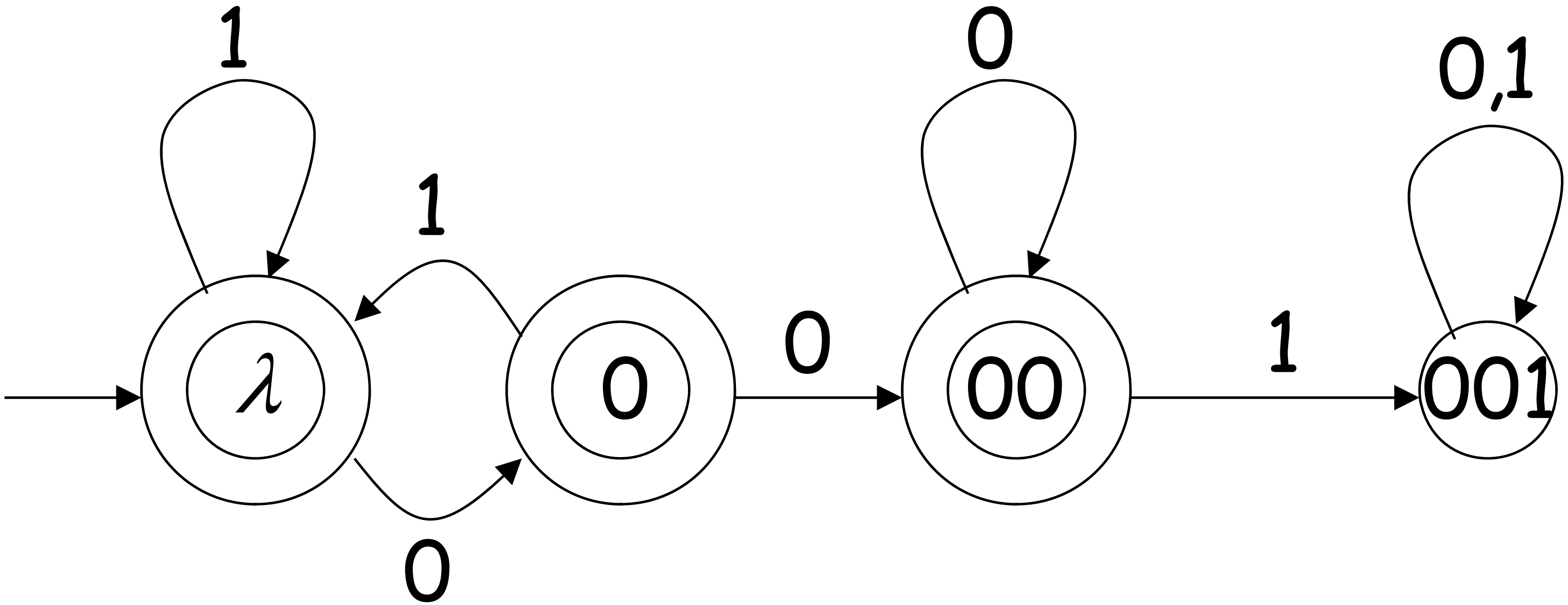
$$L(M) = \{a^n b : n \geq 0\}$$



$L(M)$ = { all substrings with prefix ab }



$L(M)$ = { all strings without
substring 001 }



Regular Languages

A language L is regular if there is
a DFA M such that $L = L(M)$

All regular languages form a language family

Example

The language $L = \{awa : w \in \{a,b\}^*\}$
is regular: