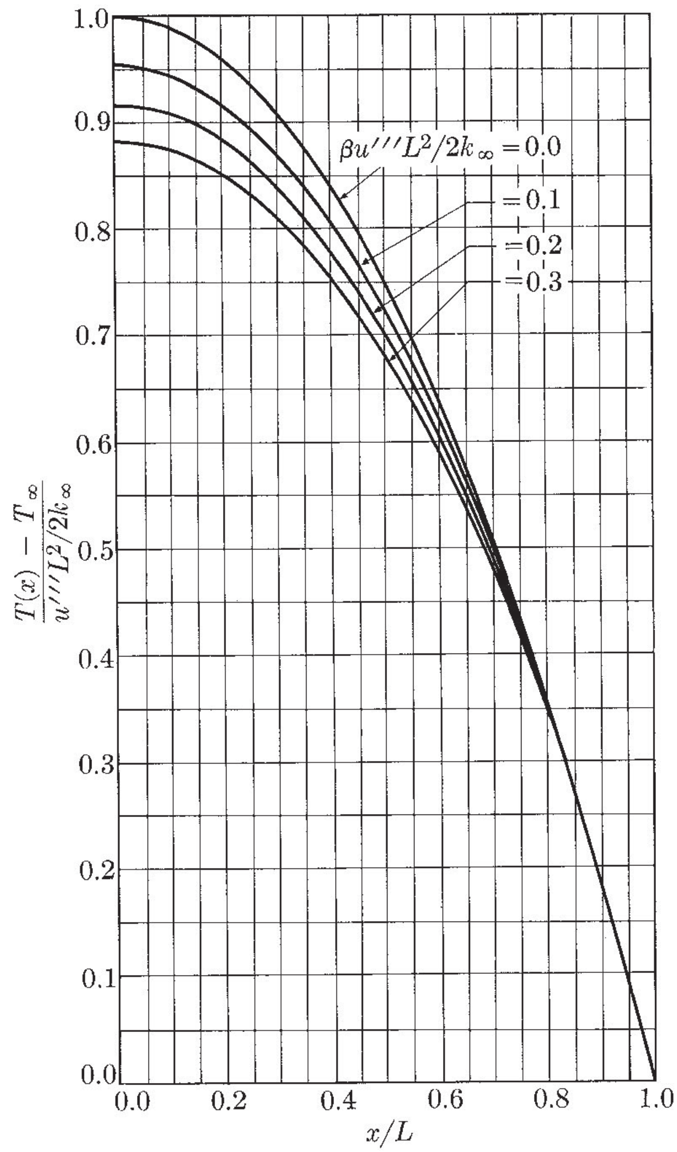

FIG. 3-23

obtained by substituting $\beta T_\infty = 0$ into Eq. (3-101) and noting that $k_R = k_\infty$ for this case. In Fig. 3-23 the effect of linear conductivity on the temperature of the plate, calculated from Eq. (3-102) for various values of $\beta u'''L^2/2k_\infty$, is shown as a function of x/L.

3-6. Power Series Solutions. Bessel Functions

In Section 3-7 we shall discuss a class of one-dimensional problems associated with extended surfaces (fins, pins, or spines). When the cross section of an extended surface is variable, the formulation of the problem results in a second-order linear differential equation with variable coefficients. This differential equation is a form of *Bessel's equation,* except in a special case which leads to the so-called *equidimensional equation.* The solution methods suitable to second-order linear differential equations with constant coefficients are not suitable to those with variable coefficients. We may, however, recall that equations with variable coefficients possess solutions expressible, over an appropriate interval, in terms of power series. This section is therefore devoted to a brief review of the power series solution of Bessel's equation and the properties of *Bessel functions*. Before this review, let us first recall the definition of power series.

An infinite series in the form

$$y(x) = a_0 + a_1(x - x_0) + a_2(x - x_0)^2 + \cdots = \sum_{k=0}^{\infty} a_k(x - x_0)^k$$

is called a *power series* expansion of the function $y(x)$ in the neighborhood of $x = x_0$, and is defined as

$$y(x) = \lim_{K\to\infty} \sum_{k=0}^{K} a_k(x - x_0)^k.$$

For an interval of x in which the foregoing limit exists, the series is said to *converge* in this interval. The reader may refer to any text on differential equations for the convergence criteria of power series.

We may now consider the method of power series solutions. Since the method is applicable to linear differential equations with constant as well as variable coefficients, it may be illustrated in terms of the following simple differential equation with constant coefficients:

$$\frac{d^2y}{dx^2} + y = 0. \tag{3-103}$$

Let us assume a power series in the form

$$y(x) = a_0 + a_1x + a_2x^2 + \cdots = \sum_{k=0}^{\infty} a_kx^k, \tag{3-104}$$

which converges in an interval including $x = 0$. Inserting Eq. (3-104) into Eq. (3-103) and rearranging gives

$$(a_0 + 2a_2) + (a_1 + 6a_3)x + (a_2 + 12a_4)x^2 + \cdots = 0. \tag{3-105}$$

Equation (3-105) is valid over an interval of x provided the coefficients of all powers of x vanish independently in this interval. It then follows that

$$\begin{aligned} a_2 &= -\tfrac{1}{2}a_0, \\ a_3 &= -\tfrac{1}{6}a_1, \\ a_4 &= -\tfrac{1}{12}a_2 = \tfrac{1}{24}a_0, \\ a_5 &= -\tfrac{1}{20}a_3 = \tfrac{1}{120}a_1, \\ &\vdots \end{aligned}$$

Introducing these values into Eq. (3-104), we obtain the solution of Eq. (3-103) in the form

$$y(x) = a_0(1 - x^2/2 + x^4/24 - \cdots) + a_1(x - x^3/6 + x^5/120 - \cdots),$$

which may also be expressed as

$$y(x) = a_0 \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k}}{(2k)!} + a_1 \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{(2k+1)!}. \tag{3-106}$$

The two series appearing in this equation are the Maclaurin expansions of $\cos x$ and $\sin x$, respectively. Hence Eq. (3–106) is equivalent to

$$y(x) = a_0 \cos x + a_1 \sin x. \tag{3-107}$$

Clearly, Eq. (3–107) may also be obtained by the classical method which suggests solutions in the form $y = e^{rx}$, where r is to be determined from the characteristic equation that is obtained by the introduction of $y = e^{rx}$ into the differential equation (3–103).

We next consider the second-order linear differential equation with variable coefficients, Bessel's equation,

$$x \frac{d}{dx}\left(x \frac{dy}{dx}\right) + (m^2x^2 - \nu^2)y = 0, \tag{3-108}$$

where m is a parameter, and ν may be zero, a fractional number, or an integer.

The solution of Eq. (3–108) may be obtained by the use of power series in a manner analogous to, but somewhat more involved than, the solution of Eq. (3–103). The result is

$$y(x) = a_0 J_\nu(mx) + a_1 Y_\nu(mx). \tag{3-109}$$

In Eq. (3–109),

$$J_\nu(mx) = \sum_{k=0}^{\infty} (-1)^k \frac{(mx/2)^{2k+\nu}}{k!\,\Gamma(k+\nu+1)} \tag{3-110}$$

and

$$Y_\nu(mx) = \frac{(\cos \nu\pi) J_\nu(mx) - J_{-\nu}(mx)}{\sin \nu\pi}, \tag{3-111}$$

where

$$J_{-\nu}(mx) = \sum_{k=0}^{\infty} (-1)^k \frac{(mx/2)^{2k-\nu}}{k!\,\Gamma(k-\nu+1)}. \tag{3-112}$$

The function appearing in the denominator of Eqs. (3–110) and (3–112), known as the *Gamma function*, is defined in the form

$$\Gamma(n+1) = n\Gamma(n) = n!, \qquad \Gamma(1) = 0! = 1$$

for integers, and in the form

$$\Gamma(\nu)\Gamma(\nu-1) = \pi/\sin \pi\nu, \qquad \Gamma(\tfrac{1}{2}) = \pi^{1/2}$$

for fractional numbers.

If ν is not an integer, $J_\nu(mx)$ and $J_{-\nu}(mx)$ are independent solutions of Eq. (3–108), but if ν is an integer, say n, then

$$J_n(mx) = (-1)^n J_{-n}(mx). \tag{3–113}$$

To obtain a second solution of Eq. (3–108) which is valid for all values of ν, Eq. (3–111) is so defined that

$$\lim_{\nu \to n} Y_\nu(mx) \to Y_n(mx),$$

where

$$\pi Y_n(mx) = 2\left(\ln \frac{mx}{2} + \gamma\right) J_n(mx) - \sum_{k=0}^{n-1} \frac{(n-k-1)!}{k!}\left(\frac{mx}{2}\right)^{2k-n}$$
$$+ \sum_{k=0}^{\infty} (-1)^{k+1}[\varphi(k) + \varphi(k+n)] \frac{(mx/2)^{2k+n}}{k!(n+k)!}, \tag{3–114}$$

in which

$$\varphi(k) = \sum_{m=1}^{k} \frac{1}{m}, \qquad \varphi(0) = 0, \qquad \text{and} \qquad \gamma = 0.5772\ldots$$

The function $J_\nu(mx)$ is known as the *Bessel function of the first kind, of order* ν, and the function $Y_\nu(mx)$ as the *Bessel function of the second kind, of order* ν.

An equation related to Eq. (3–108) is the *modified Bessel equation*

$$x\frac{d}{dx}\left(x\frac{dy}{dx}\right) - (m^2x^2 + \nu^2)y = 0. \tag{3–115}$$

Inspection reveals that the replacement of x by ix reduces Eq. (3–115) to Eq. (3–108). Hence the solution of Eq. (3–115) may readily be obtained by replacing x by ix in Eq. (3–109). We have then

$$y(x) = a_0 J_\nu(imx) + a_1 Y_\nu(imx). \tag{3–116}$$

It follows from Eq. (3–110) that

$$J_\nu(imx) = \sum_{k=0}^{\infty} (-1)^k i^{2k+\nu} \frac{(mx/2)^{2k+\nu}}{k!\Gamma(k+\nu+1)} = i^\nu \sum_{k=0}^{\infty} \frac{(mx/2)^{2k+\nu}}{k!\Gamma(k+\nu+1)}. \tag{3–117}$$

However, rather than using Eq. (3–116) as the general solution of Eq. (3–115), it is customary to replace $J_\nu(imx)$ by $I_\nu(mx)$ as a first particular solution defined in the form

$$I_\nu(mx) = \sum_{k=0}^{\infty} \frac{(mx/2)^{2k+\nu}}{k!\Gamma(k+\nu+1)}. \tag{3–118}$$

The comparison of Eqs. (3–117) and (3–118) gives

$$J_\nu(imx) = i^\nu I_\nu(mx). \tag{3–119}$$

If ν is not an integer, $I_{-\nu}(mx)$ is independent of $I_\nu(mx)$ and is therefore another particular solution of Eq. (3-115); the complete solution may then be written as a linear combination of $I_\nu(mx)$ and $I_{-\nu}(mx)$. However, to obtain a second particular solution suitable to all values of ν, we define*

$$K_\nu(mx) = \frac{\pi}{2}\,\frac{I_{-\nu}(mx) - I_\nu(mx)}{\sin \nu\pi}, \tag{3-120}$$

so that

$$\lim_{\nu \to n} K_\nu(mx) \to K_n(mx),$$

where n is an integer. With this definition

$$\begin{aligned} K_n(mx) &= (-1)^{n+1}\left(\ln \frac{mx}{2} + \gamma\right) I_n(mx) \\ &\quad + \frac{1}{2}\sum_{k=0}^{n-1} (-1)^k \frac{(n-k-1)!}{k!}\left(\frac{mx}{2}\right)^{2k-n} \\ &\quad + \tfrac{1}{2}(-1)^n \sum_{k=0}^{\infty} [\varphi(k) + \varphi(k+n)]\,\frac{(mx/2)^{2k+n}}{k!(n+k)!}, \end{aligned} \tag{3-121}$$

where the definitions of $\varphi(k)$ and γ are identical to those given above in conjunction with Eq. (3-114).

Now we may write the general solution of Eq. (3-115) in the alternative form

$$y(x) = a_0 I_\nu(mx) + a_1 K_\nu(mx). \tag{3-122}$$

The function $I_\nu(mx)$ is known as the *modified Bessel function of the first kind, of order* ν, and the function $K_\nu(mx)$ as the *modified Bessel function of the second kind, of order* ν.

Many tables of the Bessel functions and the modified Bessel functions have been compiled. The reader is referred to such tables for numerical computations. (Because of the size of the literature no specific reference is cited here.)

We next demonstrate that the solution of differential equations in the general form

$$x^2 \frac{d^2y}{dx^2} + ax\frac{dy}{dx} + b^2x^2y = 0 \tag{3-123}$$

can be expressed in terms of the Bessel functions. First, we introduce the dependent variable change $y = x^\nu z$ and rearrange Eq. (3-123) to give

$$x^\nu \frac{d^2z}{dx^2} + (a + 2\nu)x^{\nu-1}\frac{dz}{dx} + \{b^2x^\nu + [(a-1)\nu + \nu^2]x^{\nu-2}\}z = 0.$$

Now, adjusting ν so that $a + 2\nu = 1$ and dividing each term by $x^{\nu-2}$, we get

$$x\frac{d}{dx}\left(x\frac{dz}{dx}\right) + (b^2x^2 - \nu^2)z = 0, \tag{3-124}$$

* Note that $K_\nu(mx)$ is not defined through $Y_\nu(mx)$.

which is identical to Eq. (3–108). Therefore, if the solution of Eq. (3–124) is $Z_\nu(bx)$, then that of Eq. (3–123) is

$$y(x) = x^\nu Z_\nu(bx), \tag{3–125}$$

where Z_ν is used for the general representation of the Bessel functions of order ν, and $\nu = (1 - a)/2$.

Finally, we consider differential equations in the general form

$$\frac{d}{dx}\left(x^\alpha \frac{dy}{dx}\right) + \gamma^2 x^\beta y = 0, \tag{3–126}$$

where α and β are positive, and γ may be real or imaginary. The formulation of a problem related to an extended surface with variable cross section frequently leads to the general form given in Eq. (3–126). We now show, by means of a variable change, that Eq. (3–126) is another form of Bessel's equation. Introducing the independent variable change $x = t^\mu$, we rearrange Eq. (3–126) to give

$$t^2 \frac{d^2y}{dt^2} + t[\mu(\alpha - 1) + 1]\frac{dy}{dt} + \gamma^2\mu^2 t^{\mu(\beta-\alpha+2)} y = 0.$$

Adjusting μ so that $\mu(\beta - \alpha + 2) = 2$, we have

$$t^2 \frac{d^2y}{dt^2} + t[\mu(\alpha - 1) + 1]\frac{dy}{dt} + \gamma^2\mu^2 t^2 y = 0, \tag{3–127}$$

which has the form of Eq. (3–123). Thus the solution of Eq. (3–126) may be written from Eq. (3–125) by appropriately relating the parameters involved in Eq. (3–127) to those in Eq. (3–123). We have then

$$y(t) = t^\nu Z_\nu(\gamma\mu t), \tag{3–128}$$

where $\nu = \mu(1 - \alpha)/2 = (1 - \alpha)/(\beta - \alpha + 2)$. If we return to the original independent variable and insert $x^{1/\mu}$ in place of t, Eq. (3–128) becomes

$$y(x) = x^{\nu/\mu} Z_\nu(\gamma\mu x^{1/\mu}) \tag{3–129}$$

provided $\beta - \alpha + 2 \neq 0$. The parameters involved in Eq. (3–129) are related to those in Eq. (3–126) by $\nu = (1 - \alpha)/(\beta - \alpha + 2)$, $1/\mu = (\beta - \alpha + 2)/2$, and $\nu/\mu = (1 - \alpha)/2$.

If the sign of the second term of Eq. (3–126) is negative, then by replacing γ by $i\gamma$ in Eq. (3–129), we may express the solution in terms of the Bessel functions of the first and second kinds with imaginary argument, or, equivalently, in terms of the modified Bessel functions of the first and second kinds with real argument.

The special case of Eq. (3–126) for $\beta - \alpha + 2 = 0$ may be found by expanding the first term of this equation and dividing the result by $x^{\alpha-2}$.

TABLE 3–1

SOLUTION OF $\dfrac{d}{dx}\left(x^\alpha \dfrac{dy}{dx}\right) + \gamma^2 x^\beta y = 0$

Case (i): $\beta - \alpha + 2 \neq 0$. The general solution is

$$y(x) = x^{\nu/\mu} Z_\nu(|\gamma|\mu x^{1/\mu}),$$

where

$$\nu = (1-\alpha)/(\beta-\alpha+2), \qquad \mu = 2/(\beta-\alpha+2), \qquad \nu/\mu = (1-\alpha)/2;$$

two particular solutions, corresponding to Z_ν and to be selected according to γ and ν, are shown below.

γ	ν	Particular solutions	
Real	Fractional	J_ν	$J_{-\nu}$ (or Y_ν)
	Zero or integer	J_n	Y_n
Imaginary	Fractional	I_ν	$I_{-\nu}$ (or K_ν)
	Zero or integer	I_n	K_n

Case (ii): $\beta - \alpha + 2 = 0$. The general solution is

$$y(x) = x^r;$$

two particular solutions, to be determined according to the roots of

$$r^2 + (\alpha - 1)r + \gamma^2 = 0,$$

are shown below.

$(\alpha-1)^2 - 4\gamma^2$	Particular solutions	
Positive	x^{r_1}	x^{r_2}
Zero	x^δ	$x^\delta \ln x$
Negative	$x^\delta \cos(\epsilon \ln x)$	$x^\delta \sin(\epsilon \ln x)$

Here

$$r_{1,2} = \tfrac{1}{2}\{(1-\alpha) \pm [(\alpha-1)^2 - 4\gamma^2]^{1/2}\},$$
$$\delta = \tfrac{1}{2}(1-\alpha), \qquad \epsilon = \tfrac{1}{2}[4\gamma^2 - (\alpha-1)^2]^{1/2}.$$

Thus we obtain

$$x^2 \frac{d^2y}{dx^2} + \alpha x \frac{dy}{dx} + \gamma^2 y = 0, \tag{3–130}$$

which is known as the *equidimensional equation* (also called *Euler's equation* or *Cauchy's equation*). It may easily be shown that Eq. (3–130) is reduced to an equation with constant coefficients by the transformation $x = e^u$. The result is

$$\frac{d^2y}{du^2} + \alpha \frac{dy}{du} + \gamma^2 y = 0. \tag{3–131}$$

The general solution of Eq. (3–131), $y = e^{ru}$, readily gives that of Eq. (3–130) in the form

$$y(x) = (e^u)^r = x^r. \tag{3–132}$$

Inserting Eq. (3–132) into Eq. (3–130), we obtain the characteristic equation

$$r^2 + (\alpha - 1)r + \gamma^2 = 0. \tag{3–133}$$

Introducing the roots of Eq. (3–133) into Eq. (3–132) yields two particular solutions of Eq. (3–130).

For convenience in the solution of problems related to extended surfaces with variable cross sections, the particular solutions of Eq. (3–126) are summarized in Table 3–1.

3–7. Properties of Bessel Functions

In the properties considered below, Z_ν denotes any Bessel function of order ν, and x a complex number unless otherwise specified.

1. *Bessel functions of the third kind, or Hankel functions of the first and second kinds, of order ν* are defined to be

$$H_\nu^{(1),(2)}(x) = J_\nu(x) \pm Y_\nu(x). \tag{3–134}$$

2. *Derivatives of Bessel functions:*

$$\frac{d}{dx}[x^\nu Z_\nu(mx)] = \begin{cases} mx^\nu Z_{\nu-1}(mx), & Z = J,\, Y,\, I,\, H^{(1)},\, H^{(2)} \\ -mx^\nu Z_{\nu-1}(mx), & Z = K \end{cases} \tag{3–135}$$

$$\frac{d}{dx}[x^{-\nu} Z_\nu(mx)] = \begin{cases} -mx^{-\nu} Z_{\nu+1}(mx), & Z = J,\, Y,\, K,\, H^{(1)},\, H^{(2)} \\ mx^{-\nu} Z_{\nu+1}(mx), & Z = I. \end{cases} \tag{3–136}$$

A special case of Eq. (3–136) corresponding to $\nu = 0$ is

$$\frac{d}{dx}[Z_0(mx)] = \begin{cases} -mZ_1(mx), & Z = J,\, Y,\, K,\, H^{(1)},\, H^{(2)} \\ mZ_1(mx), & Z = I \end{cases} \tag{3–137}$$

$$\frac{d}{dx}[Z_\nu(mx)] = \begin{cases} mZ_{\nu-1}(mx) - (\nu/x)Z_\nu(mx), & Z = J,\, Y,\, I,\, H^{(1)},\, H^{(2)} \\ -mZ_{\nu-1}(mx) - (\nu/x)Z_\nu(mx), & Z = K \end{cases} \tag{3–138}$$

$$\frac{d}{dx}[Z_\nu(mx)] = \begin{cases} -mZ_{\nu+1}(mx) + (\nu/x)Z_\nu(mx), & Z = J,\, Y,\, K,\, H^{(1)},\, H^{(2)} \\ mZ_{\nu+1}(mx) + (\nu/x)Z_\nu(mx), & Z = I. \end{cases} \tag{3–139}$$

3. *Relations between some Bessel and circular (trigonometric) or hyperbolic functions:*

$$
\begin{aligned}
J_{1/2}(x) &= (2/\pi x)^{1/2} \sin x, \\
J_{-1/2}(x) &= (2/\pi x)^{1/2} \cos x,
\end{aligned}
\tag{3–140}
$$

$$
\begin{aligned}
I_{1/2}(x) &= (2/\pi x)^{1/2} \sinh x, \\
I_{-1/2}(x) &= (2/\pi x)^{1/2} \cosh x.
\end{aligned}
\tag{3–141}
$$

4. *Relations between some Bessel functions:*

$$J_\nu(xe^{im\pi}) = e^{im\nu\pi} J_\nu(x), \tag{3–142}$$

$$Y_\nu(xe^{im\pi}) = e^{-im\nu\pi} Y_\nu(x) + 2i \sin m\nu\pi \cot \nu\pi J_\nu(x), \tag{3–143}$$

$$I_\nu(xe^{\pm i\pi/2}) = e^{\pm i\nu\pi/2} J_\nu(x), \tag{3–144}$$

$$
\begin{aligned}
K_\nu(xe^{\pm i\pi/2}) &= \pm i \frac{\pi}{2} e^{\mp i\nu\pi/2}[-J_\nu(x) \pm iY_\nu(x)] \\
&= \mp i \frac{\pi}{2} e^{\mp i\nu\pi/2} H_\nu^{(2),(1)}(x),
\end{aligned}
\tag{3–145}
$$

$$J_\nu(xe^{\pm i3\pi/4}) = \mathrm{ber}_\nu\, x \pm i\, \mathrm{bei}_\nu\, x, \tag{3–146}$$

where x is real, and ber and bei stand for Bessel-real and Bessel-imaginary, respectively;

$$K_\nu(xe^{\pm i\pi/4}) = e^{\pm i\nu\pi/2}(\mathrm{ker}_\nu\, x \pm i\, \mathrm{kei}_\nu\, x), \tag{3–147}$$

where x is real, and ker and kei stand for Kelvin-real and Kelvin-imaginary, respectively.

5. *Behavior of Bessel functions for small arguments.* It follows from the form of the power series solutions discussed in this section that the series representations of the Bessel functions considered so far converge rapidly for small arguments. Retaining the first few terms of these series, we may obtain the behavior of Bessel functions for small arguments. For example, from Eq. (3–110),

$$J_0(x) = 1 - \frac{(x/2)^2}{(1!)^2} + \frac{(x/2)^4}{(2!)^2} - \cdots, \tag{3–148}$$

$$J_1(x) = \frac{x}{2} - \frac{(x/2)^3}{1!2!} + \frac{(x/2)^5}{2!3!} - \cdots, \tag{3–149}$$

$$\vdots$$

$$J_\nu(x) = \frac{(x/2)^\nu}{\Gamma(\nu + 1)} \left\{1 - \frac{(x/2)^2}{1!(\nu + 1)} + \frac{(x/2)^4}{2!(\nu + 1)(\nu + 2)} - \cdots\right\}, \tag{3–150}$$

and from Eq. (3–118),

$$I_0(x) = 1 + \frac{(x/2)^2}{(1!)^2} + \frac{(x/2)^4}{(2!)^2} + \cdots, \tag{3–151}$$

$$I_1(x) = \frac{x}{2} - \frac{(x/2)^3}{1!2!} + \frac{(x/2)^5}{2!3!} + \cdots, \tag{3–152}$$

$$\vdots$$

$$I_\nu(x) = \frac{(x/2)^\nu}{\Gamma(\nu+1)}\left\{1 + \frac{(x/2)^2}{1!(\nu+1)} + \frac{(x/2)^4}{2!(\nu+1)(\nu+2)} + \cdots\right\}. \tag{3–153}$$

The small-argument expansion of other Bessel functions may be written in a similar way.

6. *Asymptotic (large-argument) behavior of Bessel functions.* Asymptotic expansions require a different treatment than that of power series. This will not be given here. Only the results corresponding to a number of frequently encountered cases are listed below:

$$J_\nu(x) \sim \left(\frac{2}{\pi x}\right)^{1/2}\left\{\left[1 - \frac{(4\nu^2-1^2)(4\nu^2-3^2)}{2!(8x)^2} + \cdots\right]\cos\left(x - \frac{\pi}{4} - \frac{\nu\pi}{2}\right)\right.$$
$$\left. - \left[\frac{4\nu^2-1^2}{1!8x} - \cdots\right]\sin\left(x - \frac{\pi}{4} - \frac{\nu\pi}{2}\right)\right\}, \tag{3–154}$$

$$Y_\nu(x) \sim \left(\frac{2}{\pi x}\right)^{1/2}\left\{\left[1 - \frac{(4\nu^2-1^2)(4\nu^2-3^2)}{2!(8x)^2} + \cdots\right]\sin\left(x - \frac{\pi}{4} - \frac{\nu\pi}{2}\right)\right.$$
$$\left. + \left[\frac{4\nu^2-1^2}{1!8x} - \cdots\right]\cos\left(x - \frac{\pi}{4} - \frac{\nu\pi}{2}\right)\right\}, \tag{3–155}$$

$$I_\nu(x) \sim \frac{e^x}{(2\pi x)^{1/2}}\left\{1 - \frac{4\nu^2-1^2}{1!8x} + \frac{(4\nu^2-1^2)(4\nu^2-3^2)}{2!(8x)^2} - \cdots\right\}, \tag{3–156}$$

$$K_\nu(x) \sim \left(\frac{\pi}{2x}\right)^{1/2} e^{-x}\left\{1 + \frac{4\nu^2-1^2}{1!8x} + \frac{(4\nu^2-1^2)(4\nu^2-3^2)}{2!(8x)^2} + \cdots\right\}. \tag{3–157}$$

7. *Graphical representation of the general behavior of Bessel functions.* Graphs of the general behavior of Bessel functions are shown in Fig. 3–24.

Having thus completed our review of Bessel functions we may now proceed to demonstrate the use of these functions in the solution of problems related to extended surfaces.

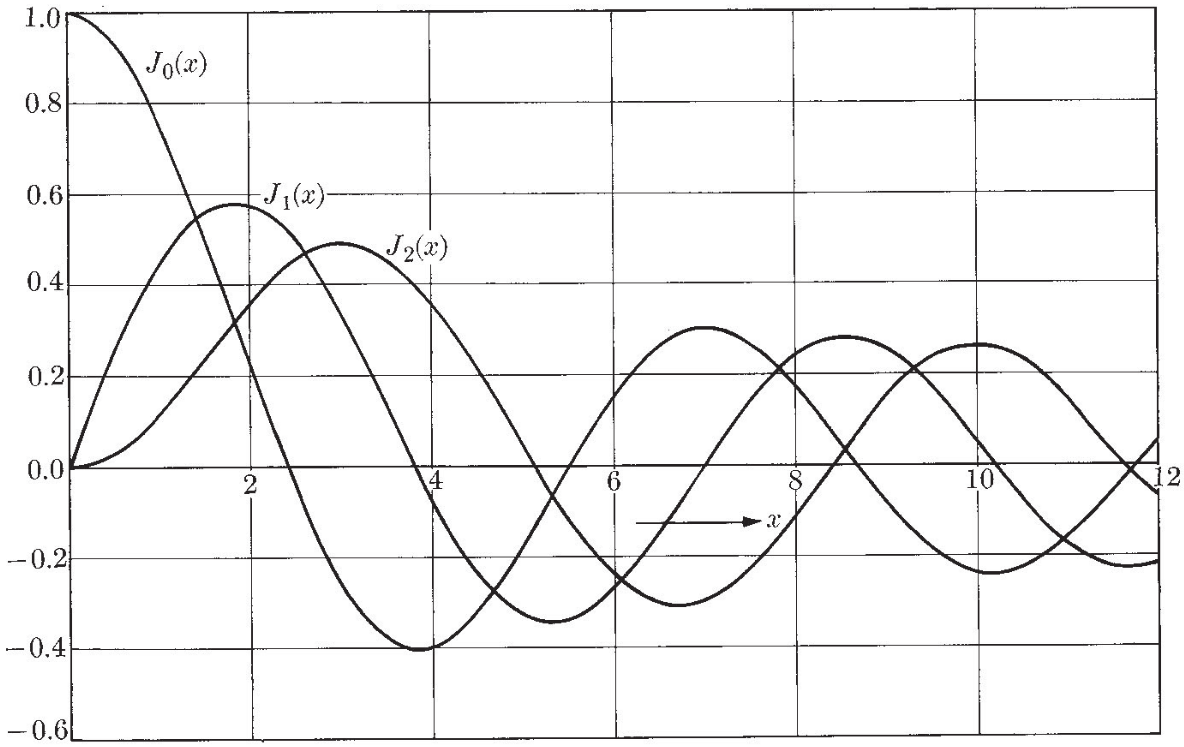

FIG. 3–24 (a)

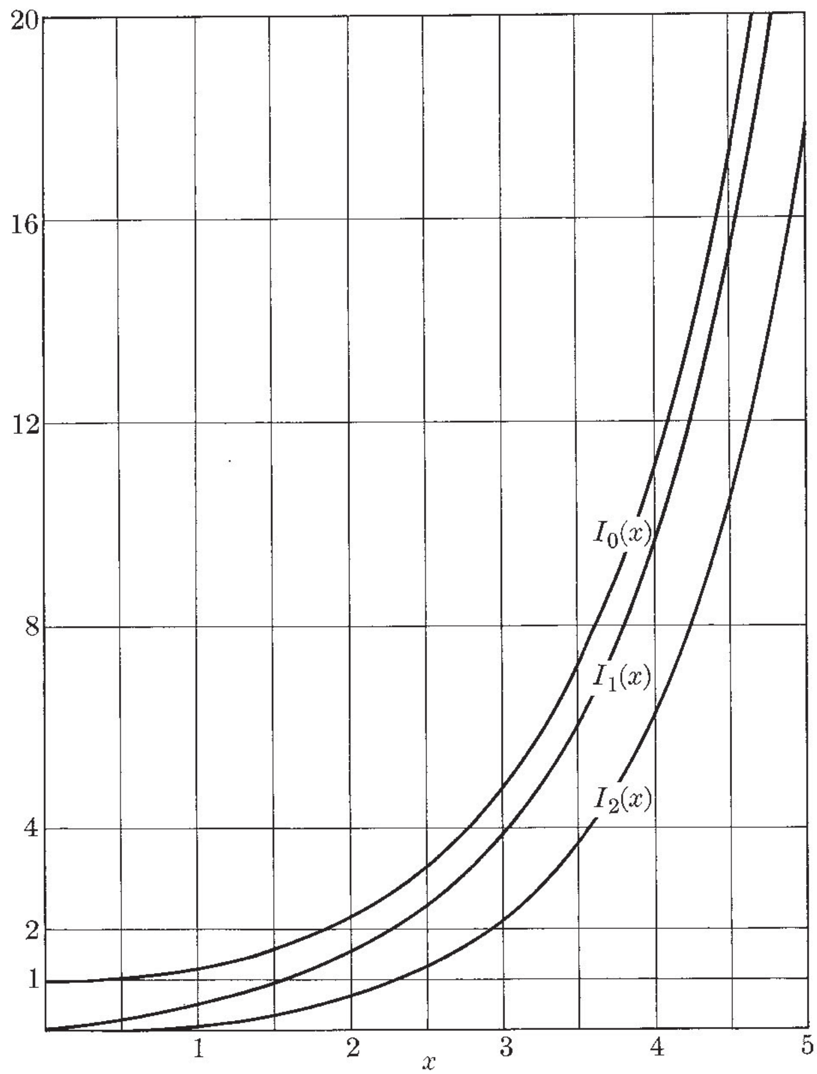

FIG. 3–24 (c)

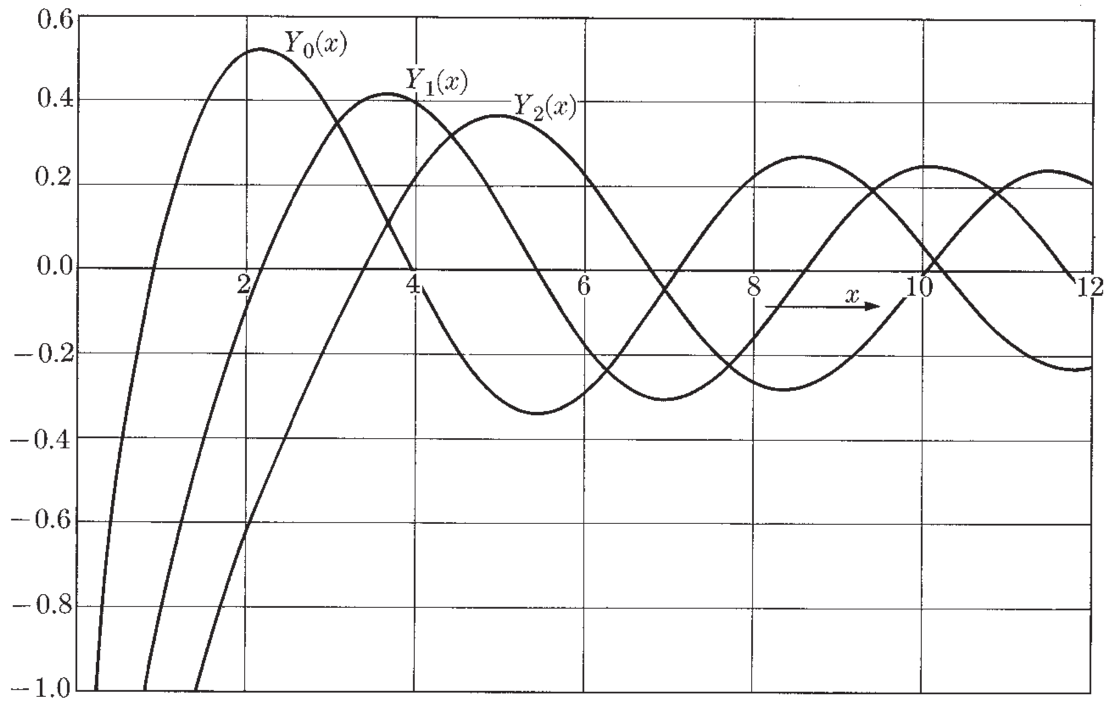

FIG. 3–24 (b)

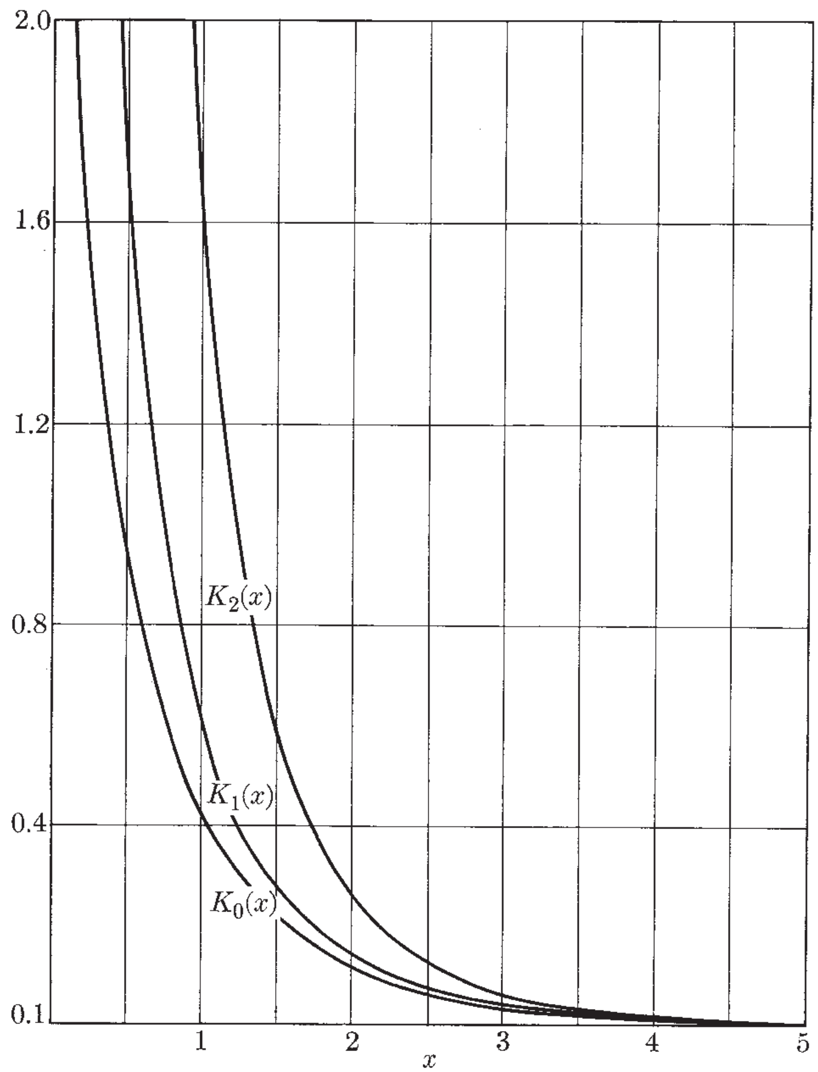

FIG. 3–24 (d)