Non-Newtonian Fluid Mechanics

(Part - V)

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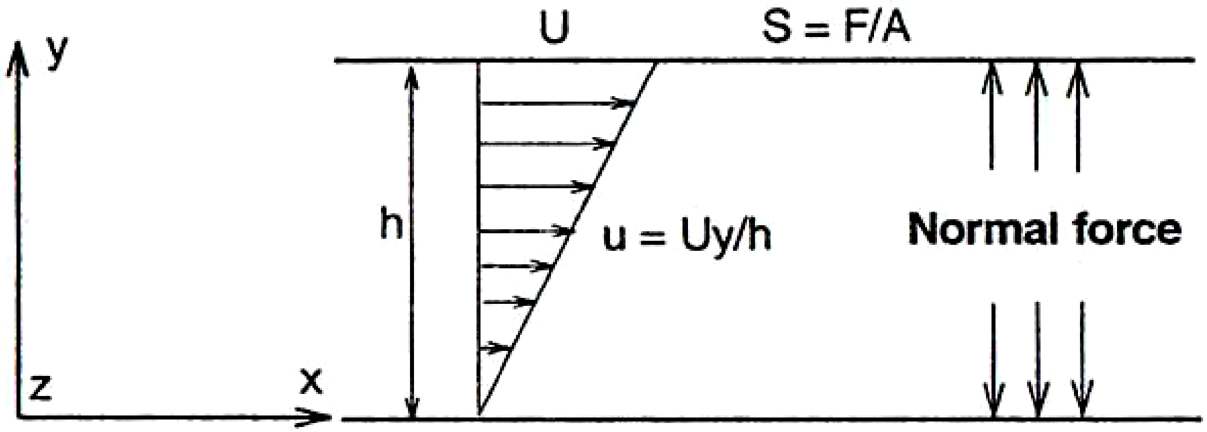
Visometric Flows & Viscometric Functions

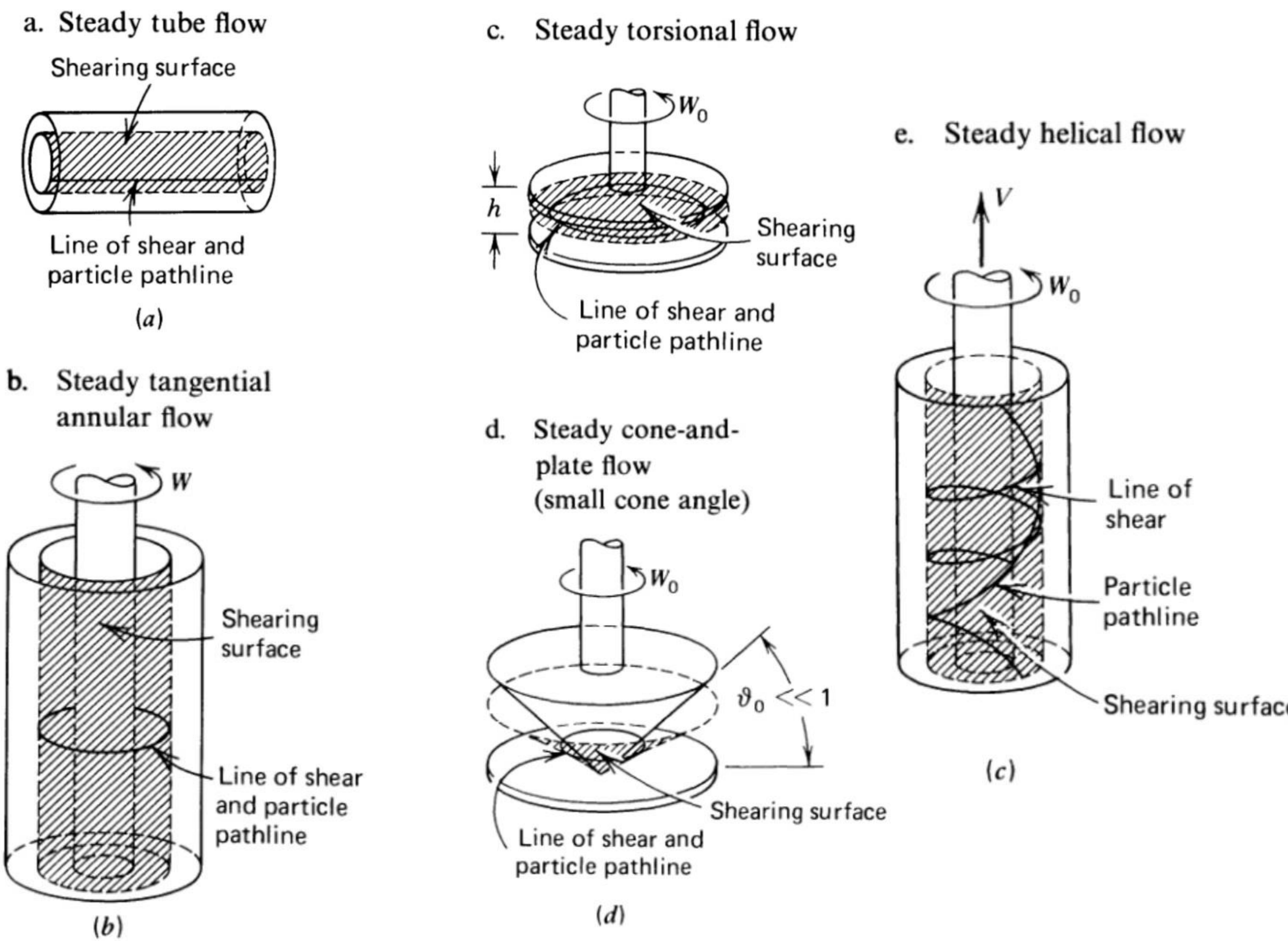
Viscometric flows are motions that are locally equivalent to steady simple shearing motion at every particle.

1. Shear flow of viscoelastic fluids is anisotropic.
2. This anisotropic behavior objects normal stress differences.
3. Unlike the shear stress, the normal stresses differences are even functions of shear rate.
4. The viscometric functions (viscosity and coefficients of normal stress differences are mostly nonlinear functions of shear rate.

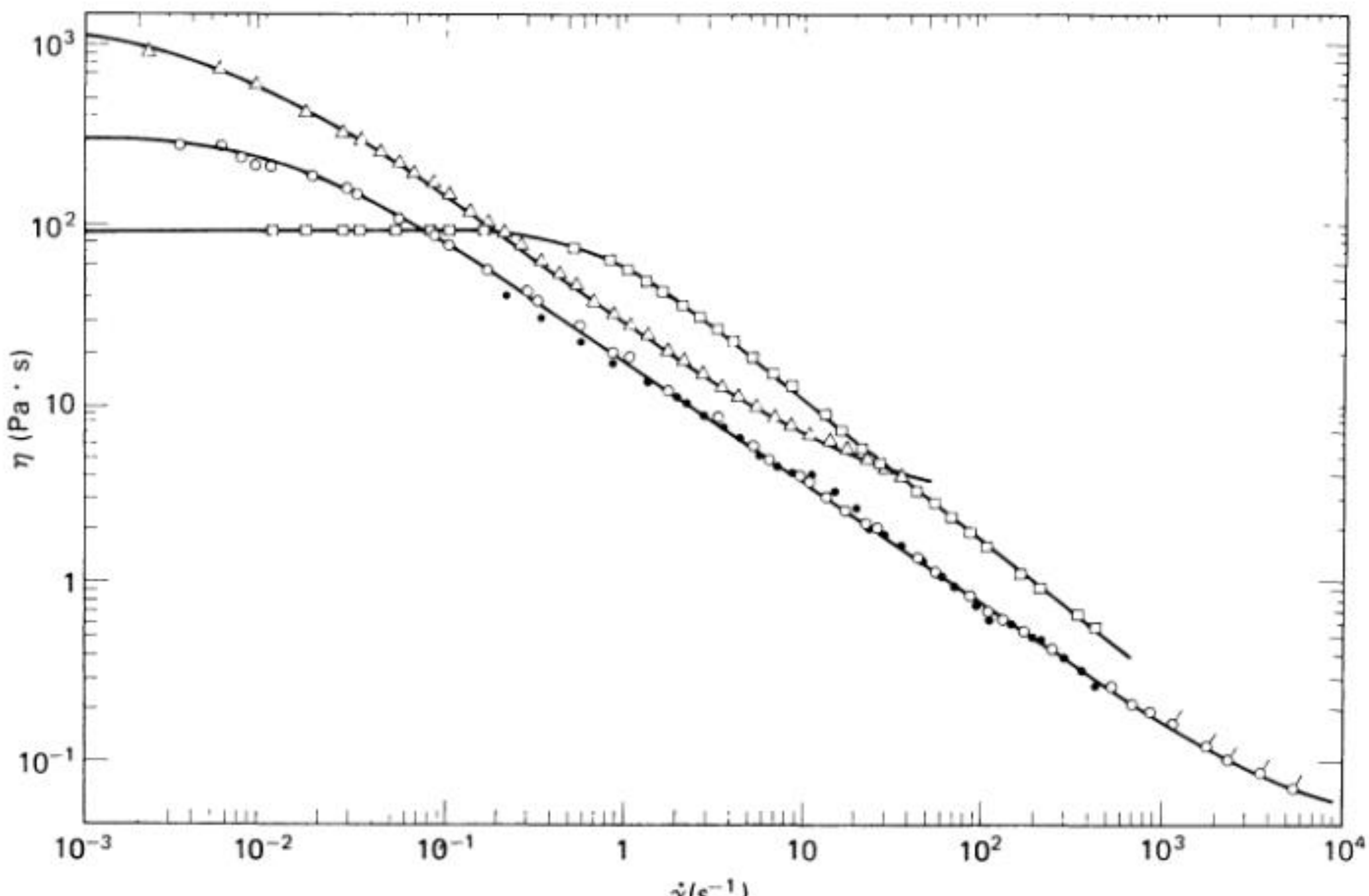
$$\tau_{12} = \eta\dot{\gamma}, \quad N_1 = \sigma_{11} - \sigma_{22} = \Psi_1 \dot{\gamma}^2 \quad \& \quad N_2 = \sigma_{22} - \sigma_{33} = \Psi_2 \dot{\gamma}^2$$



Examples of Viscometric Flow

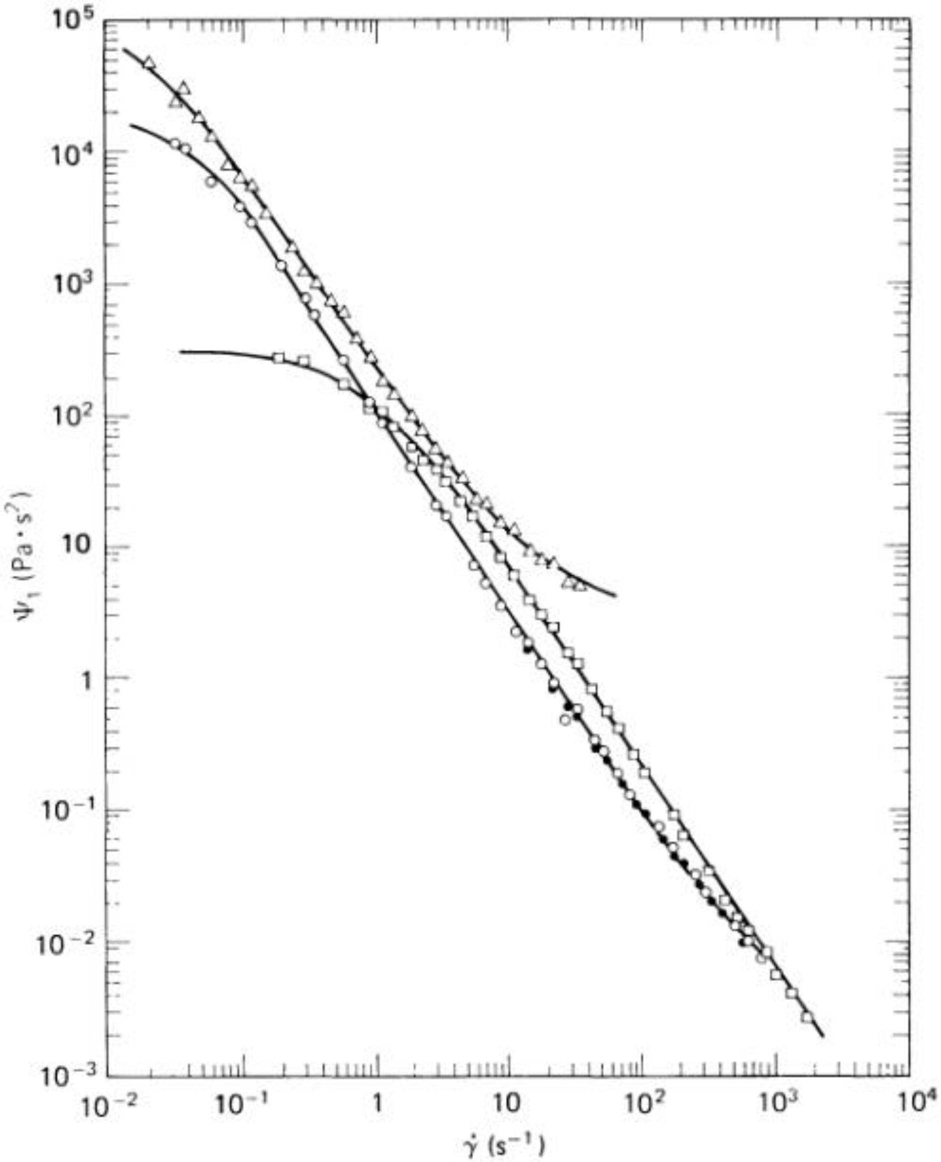


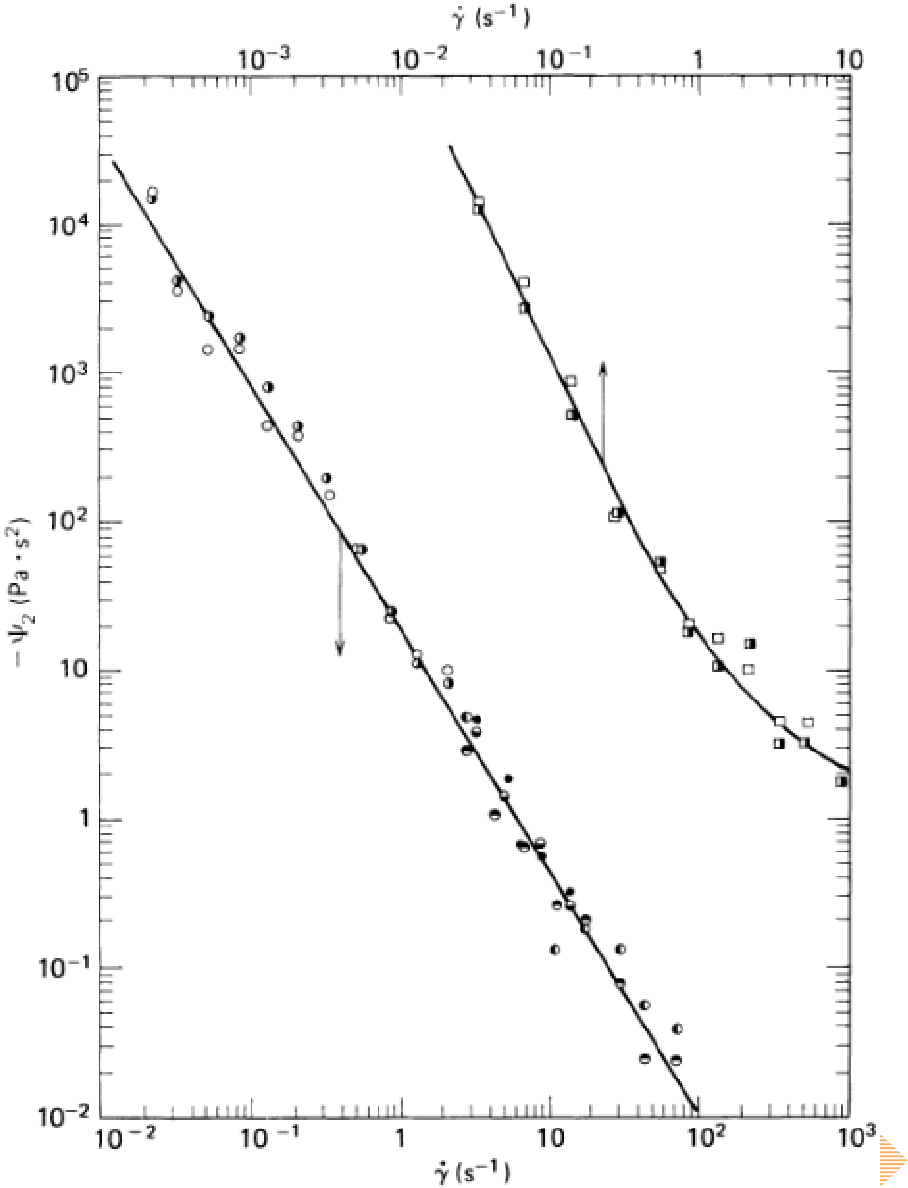
Viscosity



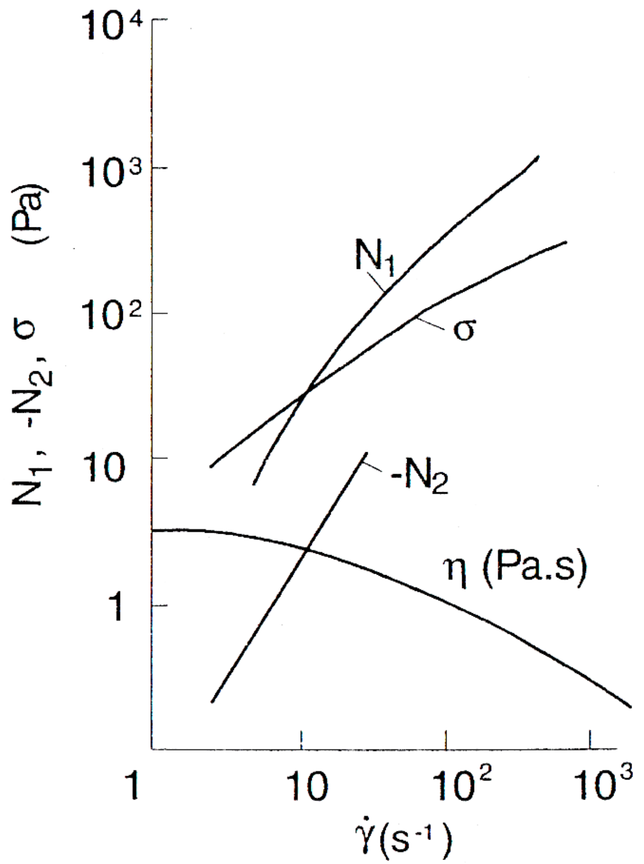
Dependence of viscosity of three viscoelastic fluids on shear rate.

Coefficients of Normal Stress Differences

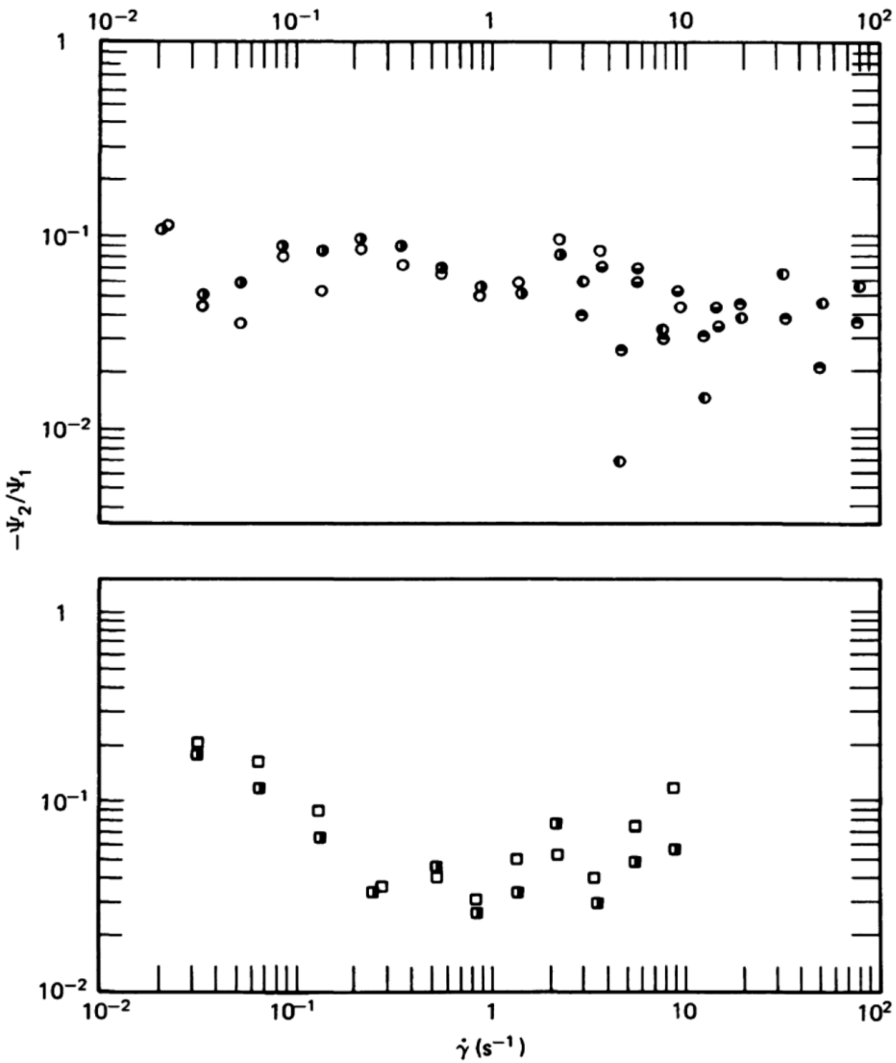




Normal Stress Differences



Viscometric functions of 6.8% polyisobutylene in cetane at 24C



Ratio of N2/N1 for two polymeric Solutions.

Modeling of Viscometric Functions

Modeling the Viscometric Functions: In nonlinear constitutive equations of viscoelastic fluids, the nonlinear viscometric functions are usually modeled implicitly. However, some explicit correlations for viscometric functions has been proposed in literature:

- **Power-law Equation:**

$$\eta = m\dot{\gamma}^{n-1}, \quad \Psi_1 = m'\dot{\gamma}^{n'-2}, \quad \Psi_2 = m''\dot{\gamma}^{n''-2}$$

- **Cross Equation:**

$$\frac{\eta - \eta_\infty}{\eta_0 - \eta_\infty} = \frac{1}{1+\lambda\dot{\gamma}^{n}}, \quad \frac{\Psi_1}{2\lambda'(\eta_0 - \eta_\infty)} = \frac{1}{1+\lambda\dot{\gamma}^{n'}}, \quad \frac{\Psi_2}{2\lambda''(\eta_0 - \eta_\infty)} = \frac{1}{1+\lambda\dot{\gamma}^{n''}}$$

Similar to Cross equation, it is possible to derive some correlations for coefficients of normal stress differences via Carreau-Yasuda model. It is important to mention that the coefficient of the first normal stress difference is positive while the coefficient of the second normal stress difference is negative. The absolute value of $|\Psi_2|$ is less than 20% of Ψ_1 and it is usually around 2 to 10% of Ψ_1. Due to the difficulty of measurement of the second normal stress difference, it could be simply determined via the first normal stress difference:

$$\Psi_2 = -X\Psi_1$$

where X is a positive constant and it is usually in the range of $0 \le X \le 0.1$.

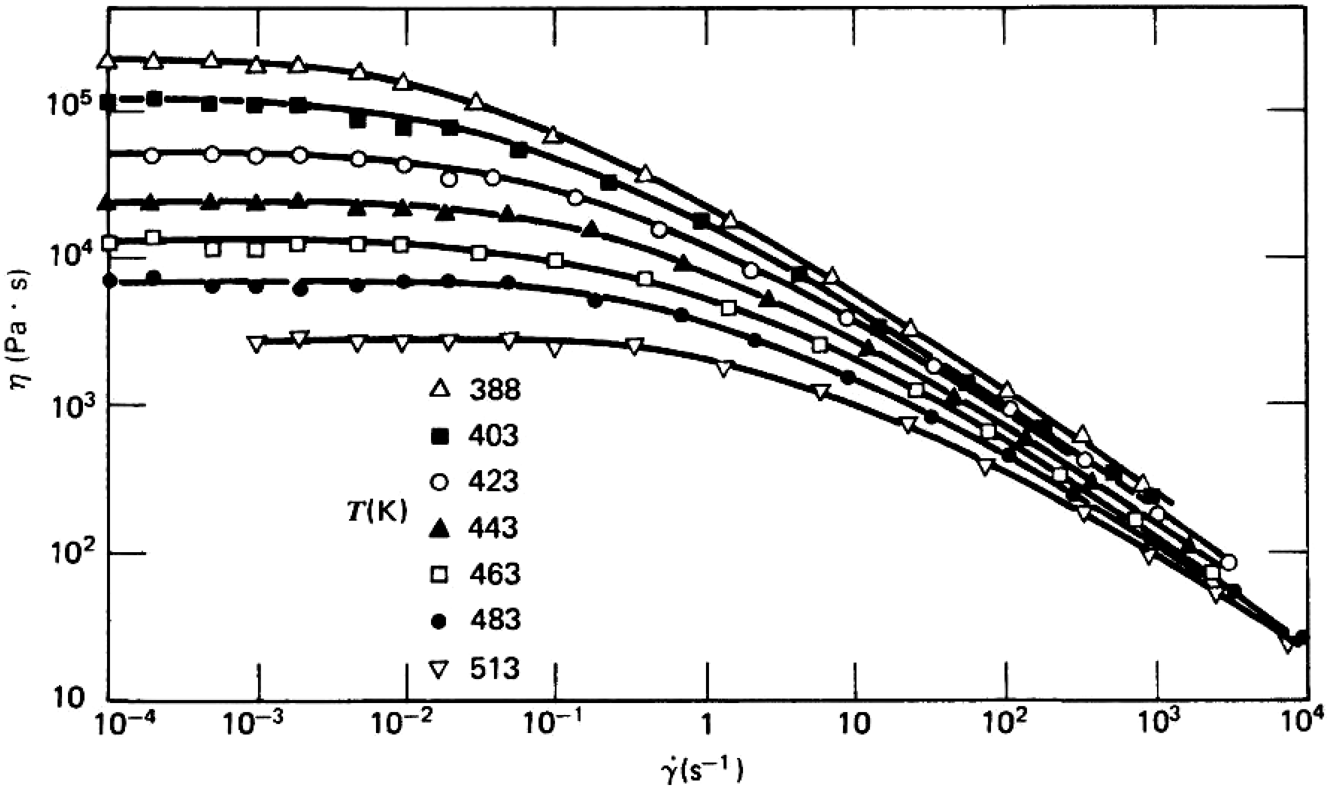
Modeling of Viscometric Functions

Power-Law Function Parameters for Describing the High Shear Rate Asymptotes of the Viscosity and First Normal Stress Coefficient

	$\eta = m\dot{\gamma}^{n-1}$		$\Psi_1 = m'\dot{\gamma}^{n'-2}$	
Fluid	m $\mathrm{Pa \cdot s}^n$	n	m' $\mathrm{Pa \cdot s}^{n'}$	n'
Solutions:[a]				
Hydroxyethylcellulose	21	0.400	30	0.567
Separan	25	0.333	410	0.830
Polyisobutylene	140	0.350	1700	0.677
Melts:[b]				
High-density polyethylene	1.89×10^4	0.40	4.28×10^4	0.41
Low-density polyethylene	0.48×10^4	0.45	1.61×10^4	0.51
Polystyrene	1.56×10^4	0.32	3.31×10^4	0.31
Polypropylene	0.44×10^4	0.50	1.02×10^4	0.51

Effect of Temperature

The viscosity of shear-thinning liquids has a strong dependency on temperature.



Viscosity of a low density polyethylene melt versus the shear rate.

Modeling the Effect of Temperature

In order to obtain a master curve for the viscosity function at an arbitrary reference temperature T_0 from plots of $\log \eta$ versus $\log \dot{\gamma}$ for a variety of temperatures T. We follow a two-step procedure: (1) the curve at temperature T is first shifted vertically upward by an amount $\log[\eta_0(T_0)/\eta_0(T)]$ and (2) the resulting curve is then shifted horizontally in such a way that any overlapping regions of the T_0-curve and shifted T-curve superpose. The amount by which $\eta(\dot{\gamma}, t)$ must be translated to the right in order to achieve superposition is defined as $\log aT$. Often it is found that the shift factor is given by

$$a_T = \frac{\eta_0(T)T_0\rho_0}{\eta_0(T_0)T\rho} \qquad (17)$$

where ρ is the density at temperature T, and ρ_0, the density at T_0. Thus, the method of reduced variables predicts that a single master curve can be obtained by plotting reduced viscosity η_r versus reduced shear rate $\dot{\gamma}_r$ where these are defined by:

$$\eta_r = \eta(\dot{\gamma}, T)\frac{\eta_0(T_0)}{\eta_0(T)} \doteq \frac{\eta(\dot{\gamma}, T)T_0}{a_T T} \qquad (18)$$

$$\dot{\gamma}_r = a_T\dot{\gamma} \qquad (19)$$

The First Normal Stress Difference

In the last terms of Eq. (18), we have neglected the temperature dependence of density. Moreover, the ratio ρ_0 / ρ is about unity and changes very little over ordinary temperature ranges. For example, this ratio is 0.92 for low-density polyethylene at 150°C relative to 200°C, whereas the complete shift factor a_T has a value of 0.32 for the same temperature change. If zero-shear-rate viscosity data are not available, we should use the other technique to estimate the shift factor (refer to book of B. Bird). Master curves for other material functions are obtained similarly to η. To decide on a proper form for the reduced material functions we assume that all components of the stress are reduced in the same manner as the shear stress. For example, we assume the first normal stress difference $N_1 = \Psi_1 \dot{\gamma}^2$ shifts in the same way as τ_{xy}, so that the reduced first normal stress difference is given by

$$N_{1,r}(\dot{\gamma}, T_0) = N_1(\dot{\gamma}, T) \frac{T_0}{T} \frac{\rho_0}{\rho} \qquad \Psi_{1,r}(\dot{\gamma}, T_0) = \Psi_1 \frac{T_0}{a_T^2 T} \tag{20}$$

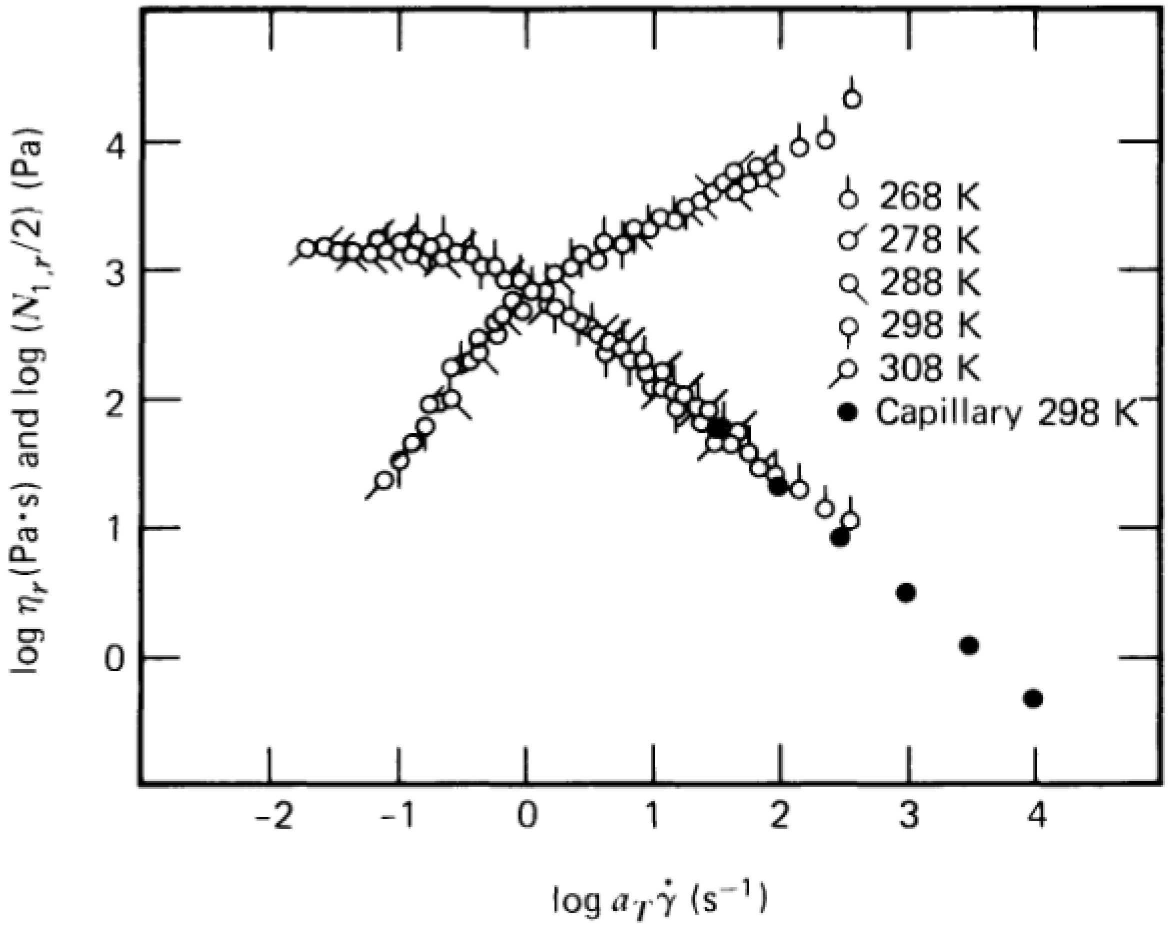
where we have taken $\rho_0 / \rho = 1$. For the linear viscoelastic properties (for which the method of reduced variables was originally developed), the reduced moduli are defined by:

$$G'_r(\omega, T_0) = G'(\omega, T) \frac{T_0 \rho_0}{T \rho} \qquad G''_r(\omega, T_0) = G''(\omega, T) \frac{T_0 \rho_0}{T \rho} \tag{21}$$

Note that the moduli are reduced in exactly the same way as stress. Corresponding to Eq. (21), we have:

$$\eta'_r(\omega, T_0) = \eta'(\omega, T) \frac{T_0}{a_T T} \qquad \eta''_r(\omega, T_0) = \eta''(\omega, T) \frac{T_0}{a_T T} \tag{22}$$

The First Normal Stress Difference



Master curves for viscosity and first normal stress difference for a solution of 2×10^6 molecular weight, monodisperse, linear polystyrene in 1-chloronaphthalene.

Shearfree Flow

Simple shearfree flows are given by the velocity field

$$v_x = -\frac{1}{2}\dot{\varepsilon}(1+b)x$$

$$v_y = -\frac{1}{2}\dot{\varepsilon}(1-b)y \qquad (23)$$

$$v_z = +\dot{\varepsilon}z$$

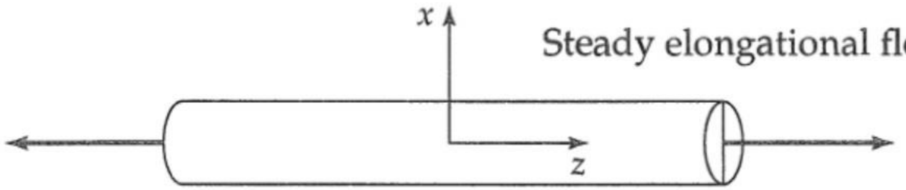
The effects of the three kinds of shearfree flow on a cube of material are illustrated for steady state in the figure of next slide, where these deformations are compared with steady shearing. By steady shearfree flow we mean that $\dot{\varepsilon}$ is independent of time; it is presumed that the elongation rate has been constant for such a long time that all the stresses in the fluid are time-independent. For steady simple shearfree flows, we define two viscosity functions $\bar{\eta}_1$ and $\bar{\eta}_2$ to describe the two normal stress differences:

$$\tau_{zz} - \tau_{xx} = \bar{\eta}_1(\dot{\varepsilon}, b)\dot{\varepsilon} \quad \& \quad \tau_{yy} - \tau_{xx} = \bar{\eta}_2(\dot{\varepsilon}, b)\dot{\varepsilon} \qquad (24)$$

For the special steady-state shearfree flow where $b = 0$, $\bar{\eta}_2 = 0$ and $\bar{\eta}_1$ is equal to the elongational viscosity $\bar{\eta}$:

$$\bar{\eta}(\dot{\varepsilon}) = \bar{\eta}_1(\dot{\varepsilon}, 0) \quad \& \quad \bar{\eta}_2(\dot{\varepsilon}, 0) = 0 \qquad (25)$$

Shearfree Flow



Steady elongational flow with elongation rate $\dot{\varepsilon} = dv_z/dz$.

$$v_z = \dot{\epsilon} z, \quad v_x = -\frac{1}{2}\dot{\epsilon} x, \quad v_y = -\frac{1}{2}\dot{\epsilon} y$$

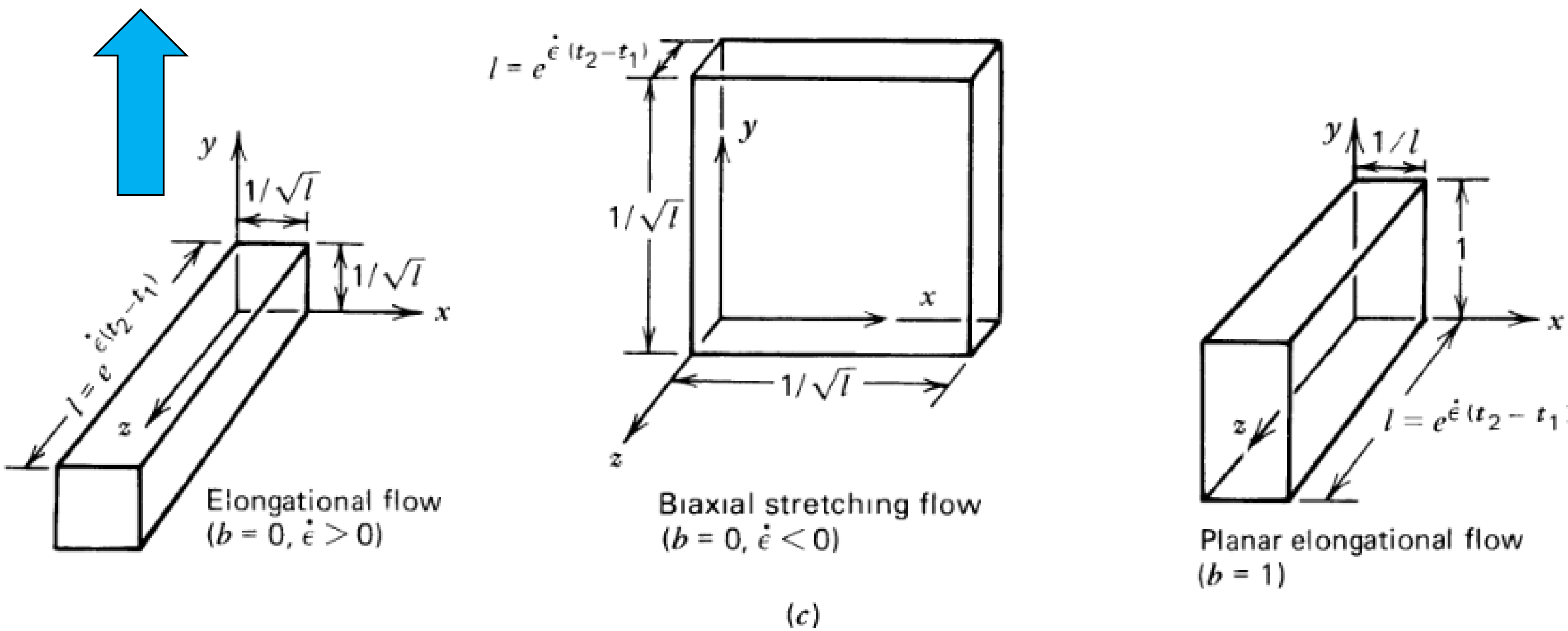


FIGURE 3.1-3. Deformation of (*a*) unit cube of material from time t_1 to t_2 ($t_2 > t_1$) in (*b*) steady simple shear flow and (*c*) three kinds of shearfree flow. The volume of material is preserved in all of these flows.

Some Tips on Elongational Viscosity

Some tips on elongational viscosity

- Positive and negative $\bar{\eta}$ referred as uniaxial and biaxial stretching.
- In uniaxial stretching of Newtonian fluids, we have: $\bar{\eta} = 3\mu$

Proof:

$$\boldsymbol{\tau} = \mu\dot{\boldsymbol{\gamma}} \xrightarrow{\text{Uniaxial Stretching}} \boldsymbol{\tau} = \mu\begin{bmatrix} 2\dfrac{\partial v_x}{\partial x} & 0 & 0 \\ 0 & 2\dfrac{\partial v_y}{\partial y} & 0 \\ 0 & 0 & 2\dfrac{\partial v_z}{\partial z} \end{bmatrix} = \mu\begin{bmatrix} 2\dfrac{-1}{2}\dot{\varepsilon} & 0 & 0 \\ 0 & 2\dfrac{-1}{2}\dot{\varepsilon} & 0 \\ 0 & 0 & 2\dot{\varepsilon} \end{bmatrix} = \mu\begin{bmatrix} -\dot{\varepsilon} & 0 & 0 \\ 0 & -\dot{\varepsilon} & 0 \\ 0 & 0 & 2\dot{\varepsilon} \end{bmatrix}$$

$$\bar{\eta} = \frac{\tau_{zz} - \tau_{xx}}{\dot{\varepsilon}} = \frac{2\mu\dot{\varepsilon} - (-\mu\dot{\varepsilon})}{\dot{\varepsilon}} = \frac{3\mu\dot{\varepsilon}}{\dot{\varepsilon}} \longrightarrow \bar{\eta} = 3\mu$$

- At low elongation rates, the elongational viscosity approaches a constant value known as the zero-elongation-rate elongational viscosity $\bar{\eta}_0 = 3\eta_0$, which is three times the zero-shear-rate viscosity.
- Because of the difficulty in reaching steady state in many shearfree flows of polymeric liquids, transient shearfree flows are very important. For start-up of steady elongational flow the transient stress is described by the elongational stress growth function $\bar{\eta}^+$.
- The ratio of extensional viscosity to viscosity at zero shear rate is known as the Trouton's ratio: $TR = \bar{\eta}^+/3\eta_0$

Elongational Viscosity

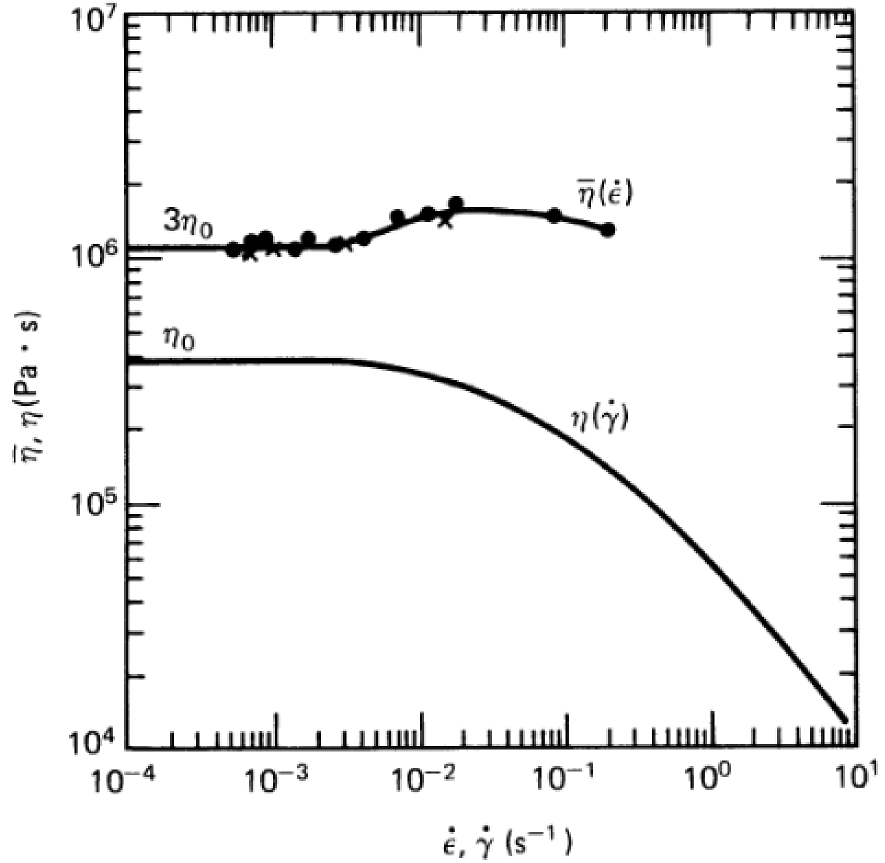
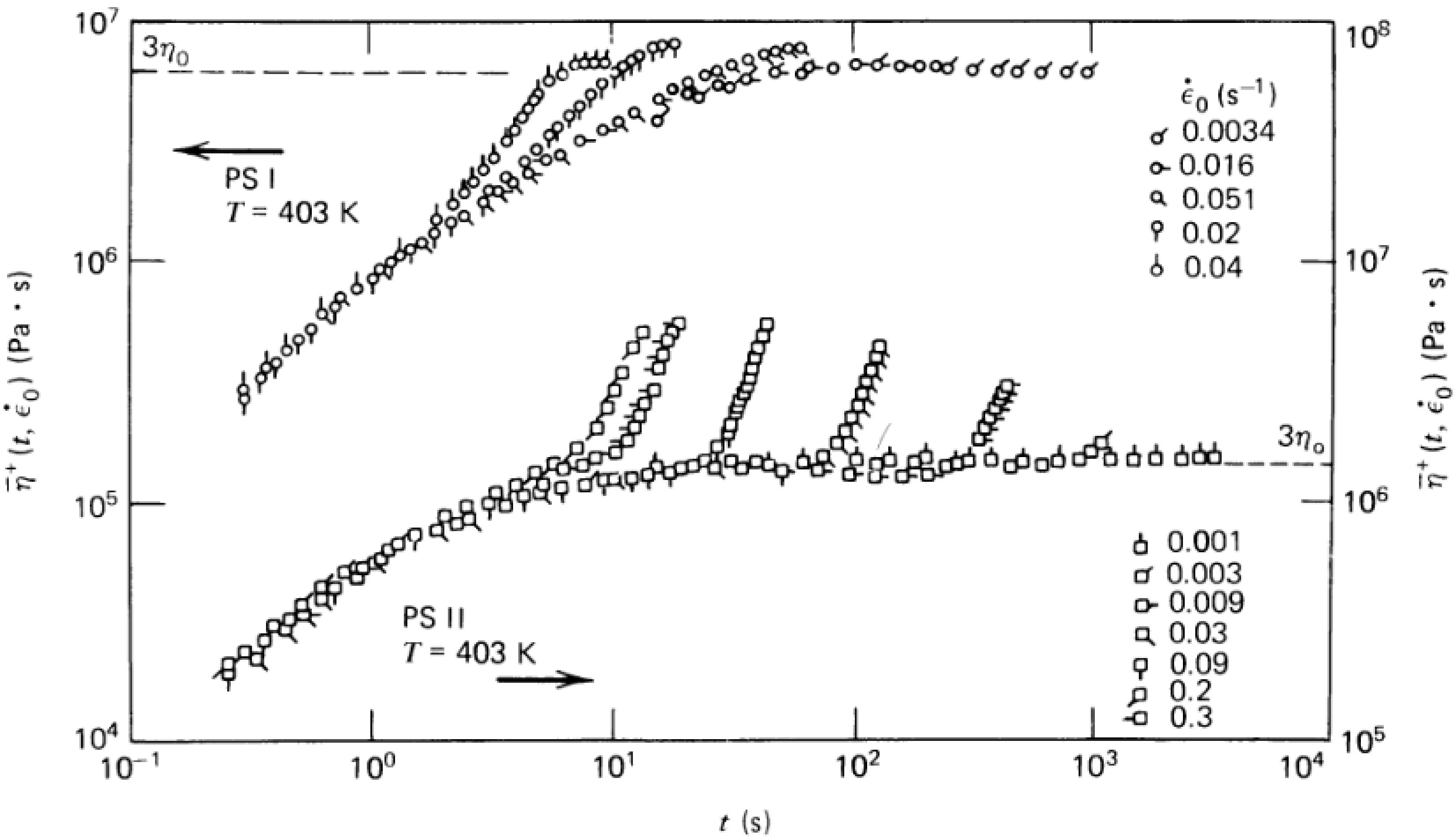


FIGURE 3.5-1. Elongational viscosity $\bar{\eta}$ and viscosity η for a polystyrene melt ($\bar{M}_w = 2.19 \times 10^5$, $\bar{M}_w/\bar{M}_n = 2.3$; sample denoted as PS IV in Fig. 3.5-2) as functions of elongation rate and shear rate, respectively. The experimentally determined zero-strain-rate values of $\bar{\eta}_0 = 1.1 \times 10^6$ Pa·s and $\eta_0 = 3.7 \times 10^5$ Pa·s agree closely with the relation $\bar{\eta}_0 = 3\eta_0$, as they must. The temperature is $T = 433$ K. [Data replotted from H. Münstedt, *J. Rheol.*, **24**, 847–867 (1980).]

Elongational Viscosity



Elongational Viscosity

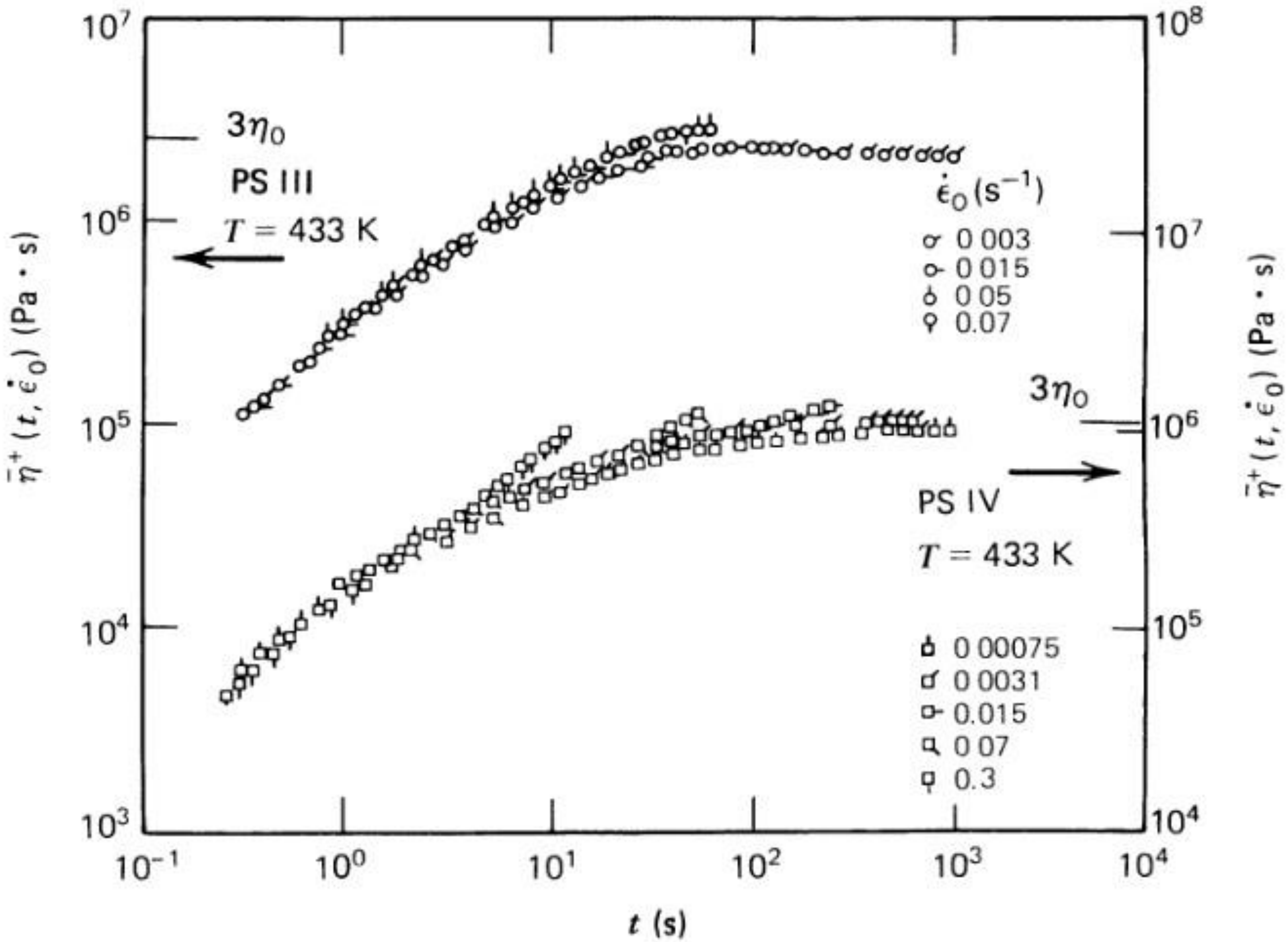


FIGURE 3.5-2. Time dependence of the elongational stress growth viscosity $\bar{\eta}^+$ for four polystyrene melts. The constant elongation rate $\dot{\epsilon}_0$ is applied to the samples for $t \geq 0$.


It will be all worth it in the END