



Non-Newtonian Fluid Mechanics

(Part - I)

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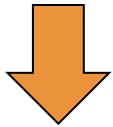
March 2021



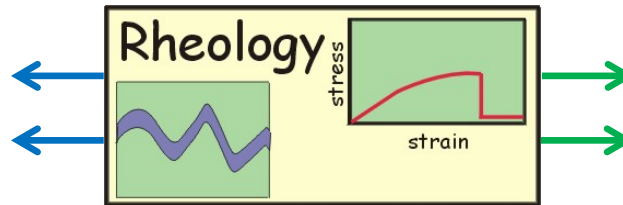
Preface



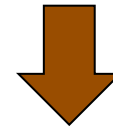
Newtonian Fluid



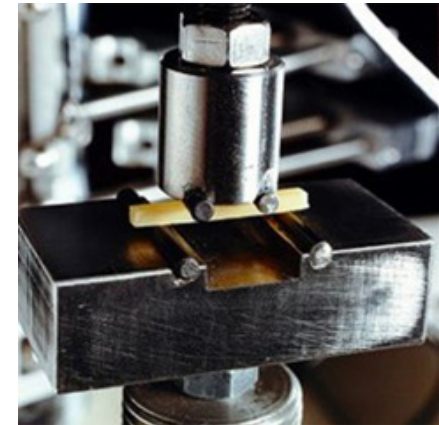
Fluid Mechanics



Hookean Solid



Solid Mechanics





Rheology



Prof. E. C. Bingham (1878-1945)

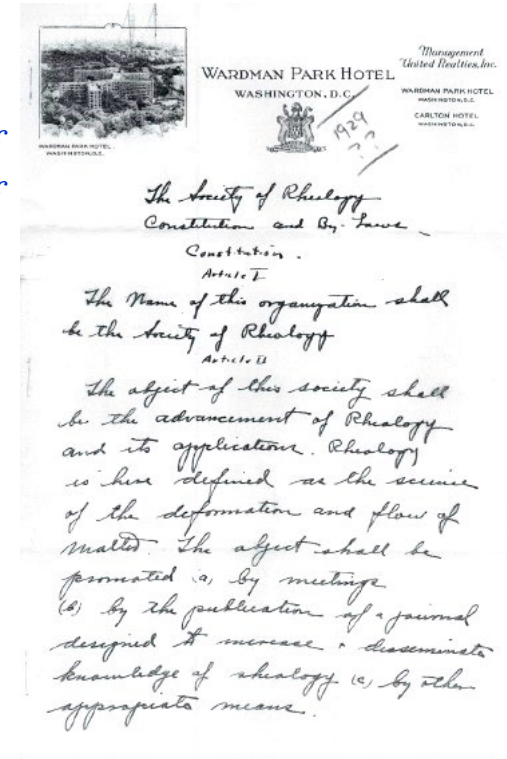
He was largely responsible for the formation of the Society of Rheology and the word of rheology in 1929.

Logo of Rheology Society



Rheology

The term deformation and flow of matter is a rather cumbersome one to cover the subjects of elasticity, viscosity and plasticity. There is no single word to cover the field, so the only recourse has been to invent one. The Greek roots to flow (ῥεω) and science (λογία), already familiar in numerous words such as rheostat and geology, made the term rheology appear to be at the same time distinctive and self explanatory.





Scientific Societies



Society of Rheology



European Society of Rheology



Austrian Group of Rheology



Belgian Group of Rheology



Canadian Society of Rheology



Chinese Society of Rheology



French Group of Rheology



German Society of Rheology



Hellenic Society of Rheology



Italian Society of Rheology



Nordic Society of Rheology



Slovenian Society of Rheology



Society of Rheology, Japan



Swiss Group of Rheology



Portuguese Society of Rheology



Southern African Society of Rheology



The British Society of Rheology



Vinogradov Society of Rheology



Iranian Society of Rheology



1st International Conference on Rheology
17-18 December 2019
Iran Polymer & Petrochemical Institute (IPPI), Tehran, Iran

ICOR 2019 Plenary & Keynote Speakers

<p>Prof. Ole Hassager Danish Polymer Center, Department of Chemical & Biochemical Engineering, Technical University of Denmark, Soltofts Plads, Bypng 229, DK2800 Kongens Lyngby, Denmark</p> <p>Polymers in fast flow: linking rheology with structure</p>	<p>Prof. James J. Feng Departments of Chemical & Biological Engineering and Mathematics, University of British Columbia, Vancouver, Canada</p> <p>Hydrodynamics and rheology of sheared two-dimensional foam</p>
<p>Prof. Peter Fischer Institute of Food, Nutrition and Health, ETH Zurich, 8092 Zurich, Switzerland</p> <p>Rheology and food digestion Controlling fat digestion with a little help from interfacial rheology</p>	<p>Prof. Bo Nyström Department of Chemistry, University of Oslo, Oslo, Norway</p> <p>Rheology and structure of associating polymers and responsive gels in aqueous media</p>
<p>Dr. Ehsan Nazockdast Department of Applied Physical Sciences, University of North Carolina, USA</p> <p>Cell nucleus as a microrheological probe to study the rheology of cytoskeleton</p>	<p>Dr. Ismaeil Moghimi IESL and Department of Material Science and Technology, FORTH and University of Crete, Heraklion, Crete 70013, Greece</p> <p>Rheology of telechelic star polymers</p>
<p>Dr. Saba Jamali Department of Mechanical and Industrial Engineering, Northeastern University, Boston, MA, USA</p> <p>Colloidal gels: rheology, thixotropy and microstructure</p>	<p>Dr. Mohammadreza Nofar Metallurgical & Materials Engineering Department, Faculty of Chemical and Metallurgical Engineering, Istanbul Technical University, Maslak, Istanbul, 34469, Turkey</p> <p>Self-reinforced PLA/PLA compounds with enhanced viscoelastic properties</p>

Dr. Mahdi Davoodi
School of Engineering, University of Liverpool, UK and Schlumberger, Cambridge Research Centre, Cambridge, CB3 0EL, UK

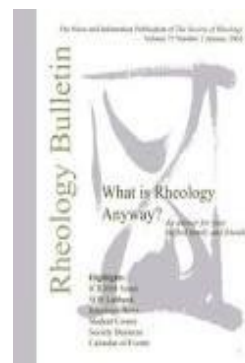
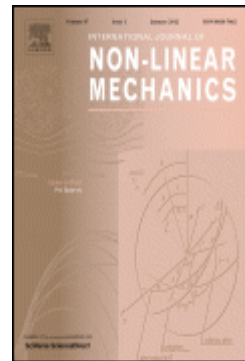
Control of a purely-elastic symmetry-breaking flow instability in cross-slot geometries

icor-isr.com





Journals





Famous Rheologists



J.C. Maxwell
(1831-1879)



L.E. Boltzmann
(1844-1906)



E.C. Bingham
(1878-1945)



K. Weissenberg
(1893-1976)



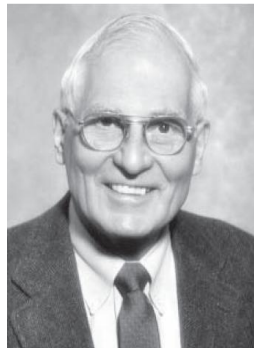
M. Reiner
(1886-1976)



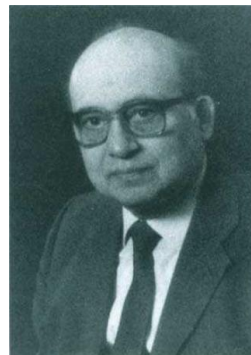
S.W. Scott Blair
(1902-1987)



J.G. Oldroyd
(1921-1982)



A. B. Metzner
(1927-2006)



H. Giesekus
(1922-1917)



R. Byron Bird



R.I. Tanner



G.H. McKinley



Newtonian Fluids



Newtonian Fluids

In Newtonian fluids, the shear stress is only depended on shear rate linearly.



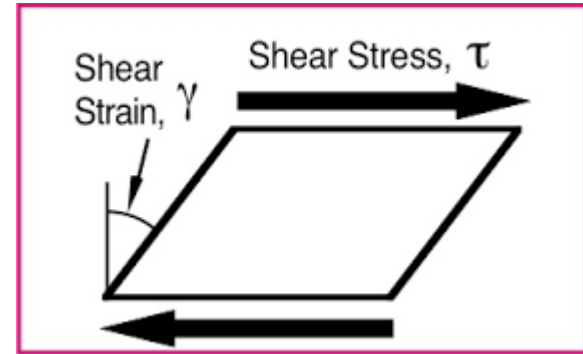


Newtonian Fluids



$$\text{---} \circ \text{---} \boxed{\eta} \text{---} \circ \text{---}$$

$$F = c\dot{x} \quad \text{or} \quad \tau = \eta \dot{\gamma}$$



Newton claimed that in shear flow of fluids: $F \propto U$
or $F = cU$

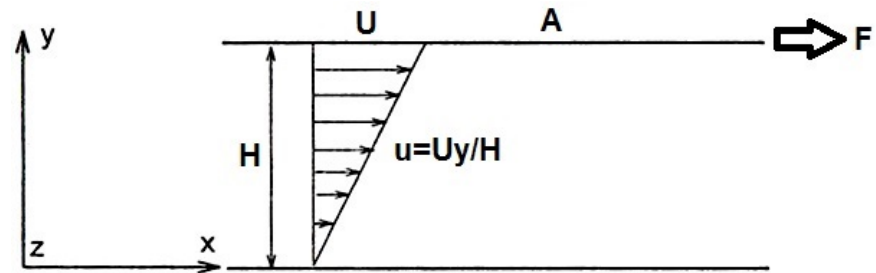
For a simple shear flow:

$$\tau_{xy} = \frac{F}{A} \quad \& \quad \dot{\gamma}_{xy} = \frac{\partial u}{\partial y} = \frac{U}{H} \Rightarrow F = \tau_{xy} A \quad \& \quad U = H \dot{\gamma}_{xy}$$

$$\text{Therefore, } \tau_{xy} = \left(\frac{cH}{A} \right) \dot{\gamma}_{xy} \Rightarrow \tau_{xy} \propto \dot{\gamma}_{xy}$$

Finally, we have: $\tau_{xy} = \eta \dot{\gamma}_{xy}$

where η is **VISCOSITY**.





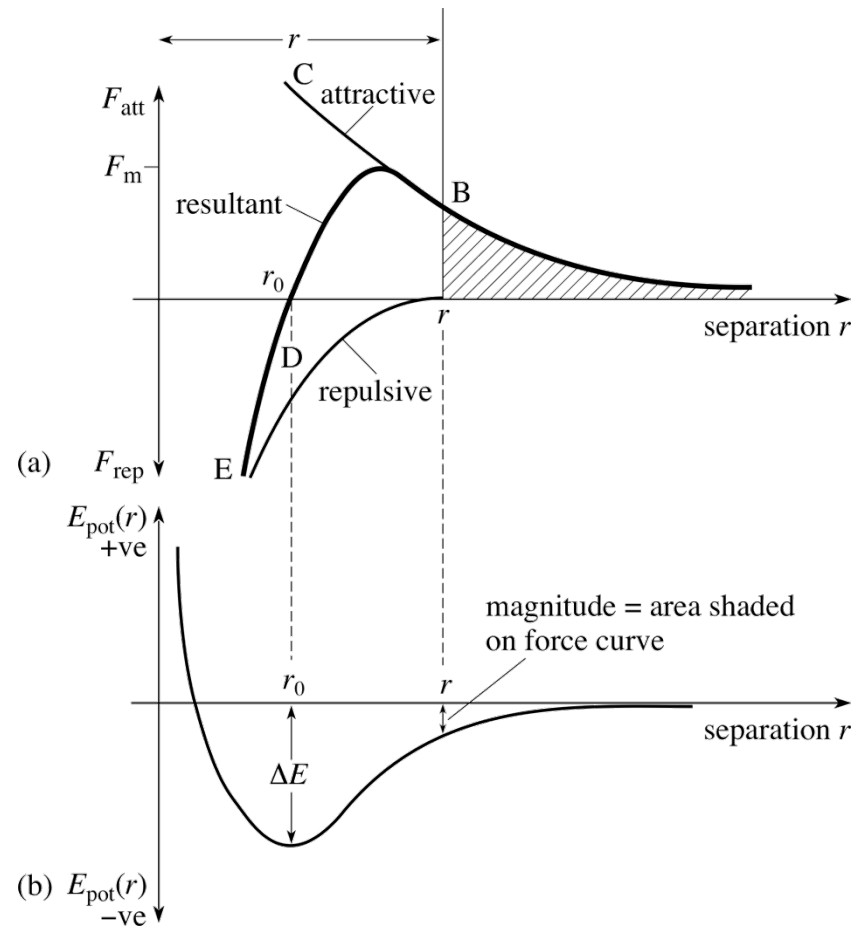
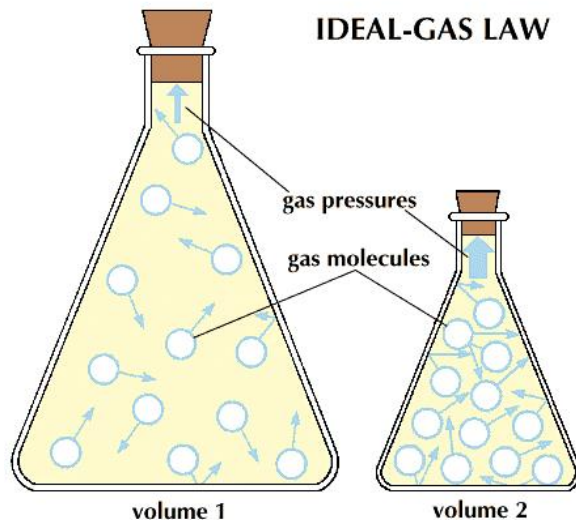
Viscosity



Molecular Theory for **Gases**:

$$\eta = \frac{5}{16} \frac{\sqrt{\pi m k T}}{\pi \sigma^2 \Omega_\mu}$$

$$\Omega_\mu = \Omega_\mu (kT / \varepsilon)$$





Viscosity



Molecular Theory for Liquids:

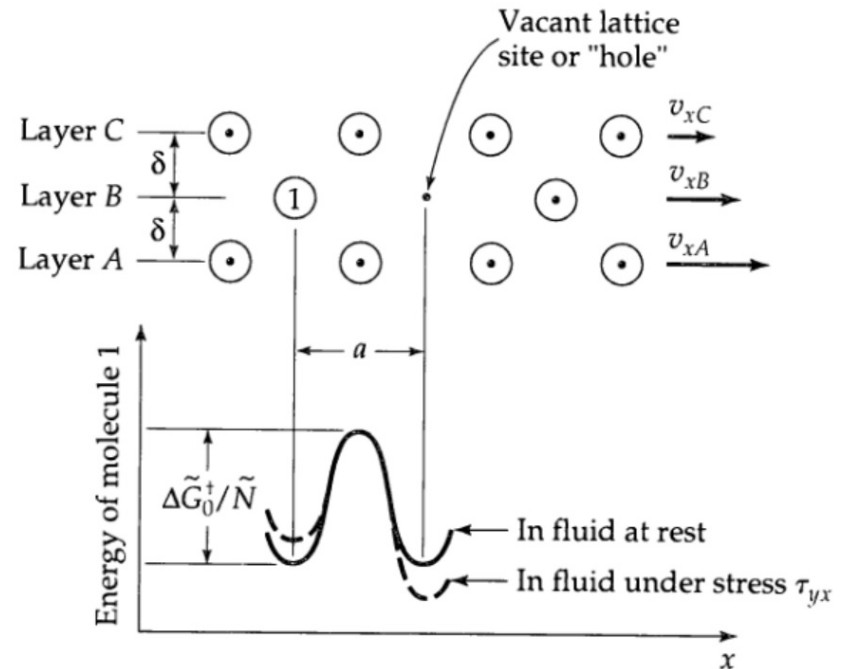
$$\eta = \frac{\tilde{N}h}{\tilde{V}} \exp\left(3.8 \frac{T_b}{T}\right)$$

The above shear-independent viscosity for a liquid is obtained subjected that:

$$\frac{\tau_{xy} \tilde{V}}{RT} \ll 1 \quad \Rightarrow \quad \tilde{V} \ll 1$$

\tilde{V} is the volume any molecule.

This indicates that the Newtonian behavior occurs in liquids with small molecular weight.





The Chemical Nature of Polymers (Macromolecules)



A **macromolecule (polymer)** is a large molecule composed of many small simple chemical unites, generally called structural units.

The polymers may be built from one structural unit (**homopolymer**) or made from two or more structural units (**copolymer**).

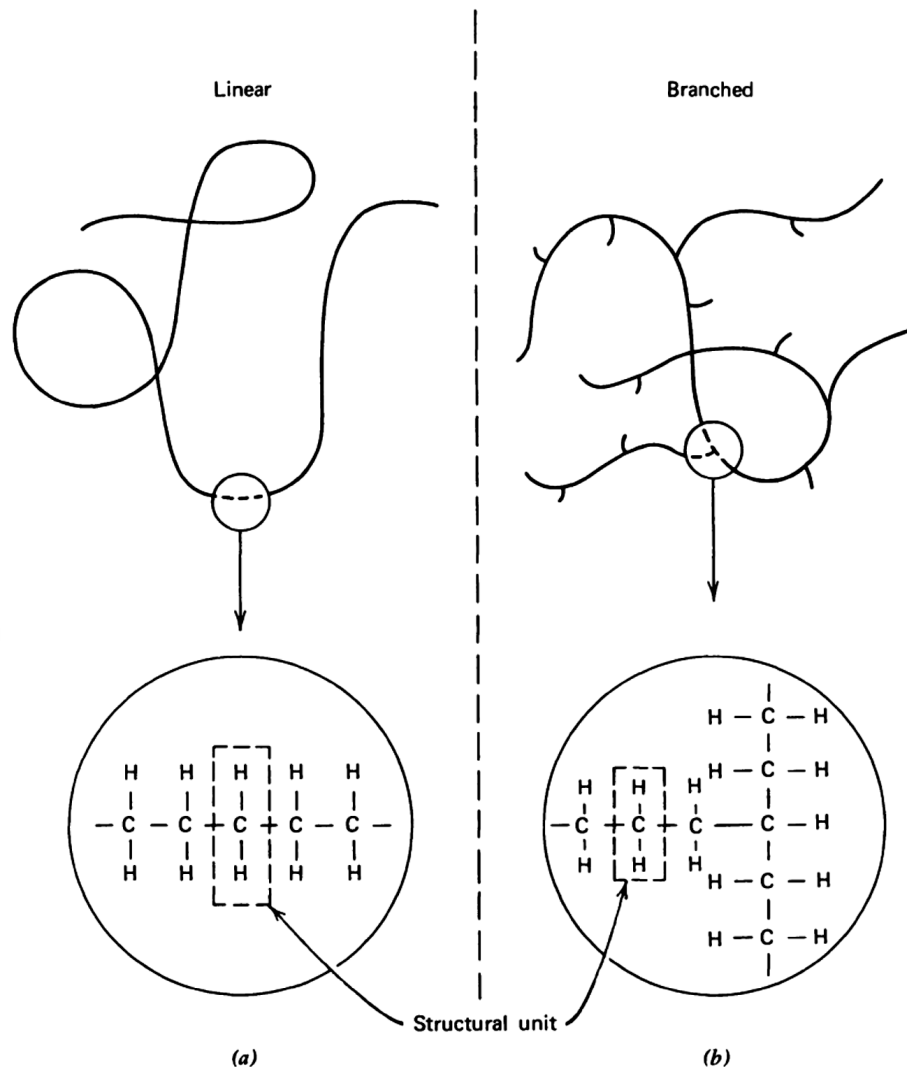
(a) ...AAABABBAABBBAB ...

(b) ...BBAAAAAAAAABBBBBBBBAA ...

(c) ...AAAAAAAAAAAAAAAAAA ...

B	B	B
B	B	B
B	B	B
⋮	⋮	⋮

Schematic representation of (a) random, (b) block and (c) graft copolymers.



(a): HDPE & (b): LDPF



Synthetic Polymers (Macromolecules)



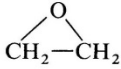


Typical synthesis polymer molecules may have molecular weights between **10,000** to **20,000,000 g/mole**.

The molecular weight of all of polymer molecules may be same in a sample. This category is called the **monodisperse**. In contrast, in **polydisperse**, the molecular weight of molecules is different.

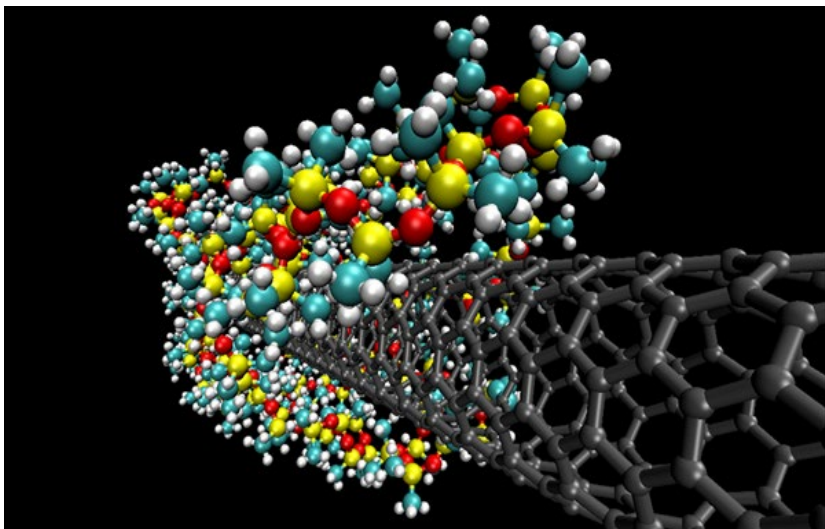
$$\bar{M}_n = \frac{\sum_i N_i M_i}{\sum_i N_i}$$

For more information refer to the section 2-1 of book of “Dynamics of Polymeric Liquids” Ed. 2 (1987).

Polymer	Monomer(s)	Structural Unit
Polyethylene	$\text{CH}_2=\text{CH}_2$	$-\text{CH}_2-$
Polyvinylchloride	$\text{CH}_2=\text{CHCl}$	$-\text{CH}_2-\text{CHCl}-$
Polystyrene	$\text{CH}_2=\text{CH}$ 	$-\text{CH}_2-\text{CH}-$ 
Polyacrylamide	$\text{CH}_2=\text{CH}$ CONH_2	$-\text{CH}_2-\text{CH}-$ CONH_2
Polyisobutylene	$\text{CH}_2=\text{C}$ CH_3	$-\text{CH}_2-\text{C}-$ CH_3
Polyisoprene (natural rubber)	$\text{CH}_2=\text{C}-\text{CH}=\text{CH}_2$ CH_3	$-\text{CH}_2-\text{C}=\text{CH}-\text{CH}_2-$ CH_3
Polydimethylsiloxane	$\text{HO}-\text{Si}-\text{OH}$ CH_3	$-\text{Si}-\text{O}-$ CH_3
Polyethyleneoxide (Polyox)		$-\text{O}-\text{CH}_2-\text{CH}_2-$
Polyhexamethylene adipamide (Nylon 66)	$\text{NH}_2-(\text{CH}_2)_6-\text{NH}_2$ and $\text{HO}-\text{C}(=\text{O})-(\text{CH}_2)_4-\text{C}(=\text{O})-\text{OH}$	$-\text{NH}-(\text{CH}_2)_6-\text{NH}-\text{C}(=\text{O})-(\text{CH}_2)_4-\text{C}(=\text{O})-$
Polyethylene terephthalate (polyester)	$\text{HO}-\text{CH}_2-\text{CH}_2-\text{OH}$ and $\text{HO}-\text{C}(=\text{O})-\text{C}_6\text{H}_4-\text{C}(=\text{O})-\text{OH}$	$-\text{O}-\text{CH}_2-\text{CH}_2-\text{O}-\text{C}(=\text{O})-\text{C}_6\text{H}_4-\text{C}(=\text{O})-$



The Chemical Nature of Macromolecules



Model for a PDMS chain adsorbed on a carbon nanotube as a new supra-molecular assembly.

A highly elastic solution (2.2% polyacrylamide in 17.8% water and 80% glycerin). The figure is captured in “The Great Australian Science Show”, Melbourne, 1993.

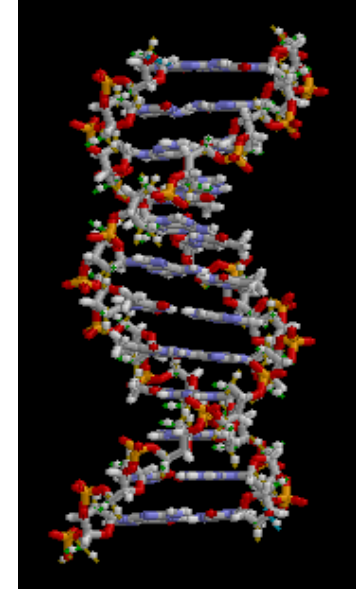
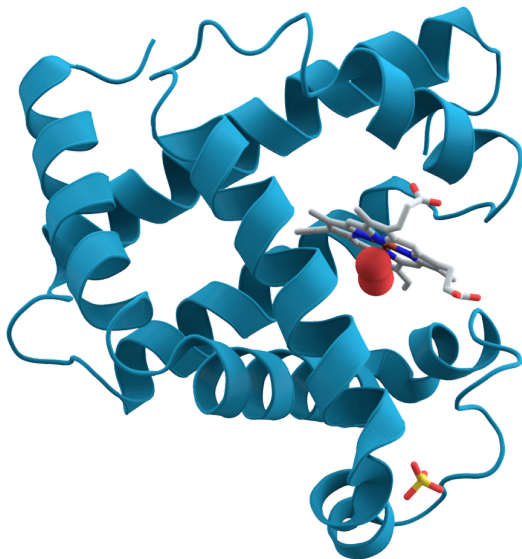




Natural Polymers



Naturally occurring polymers such as **cotton**, **starch**, and **rubber** were familiar materials for years before synthetic polymers appeared on the market. Biopolymers are polymers produced by living organisms; in other words, they are polymeric biomolecules. There are three main classes of biopolymers, classified according to the monomeric units used and the structure of the biopolymer formed: **polynucleotides** (RNA and DNA), **polypeptides**, and **polysaccharides**. There are other biopolymers such as **rubber**, **suberin**, **melanin**, and **lignin**.

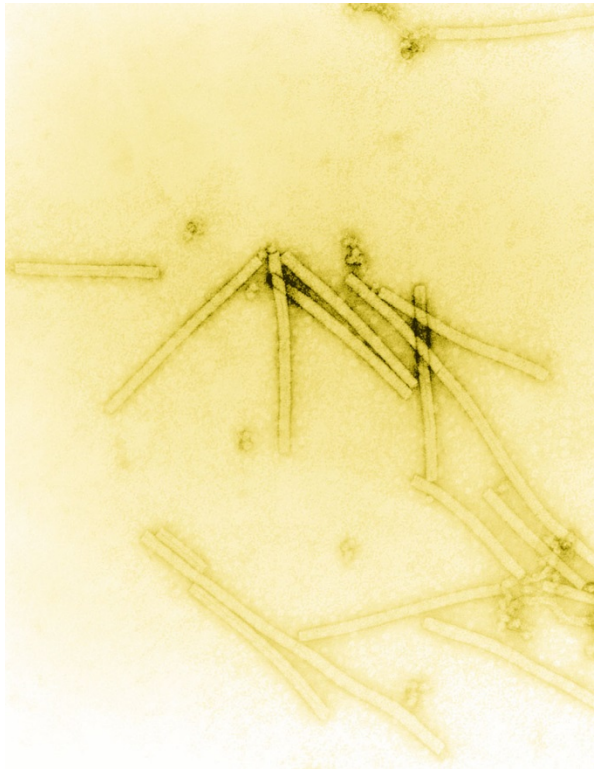




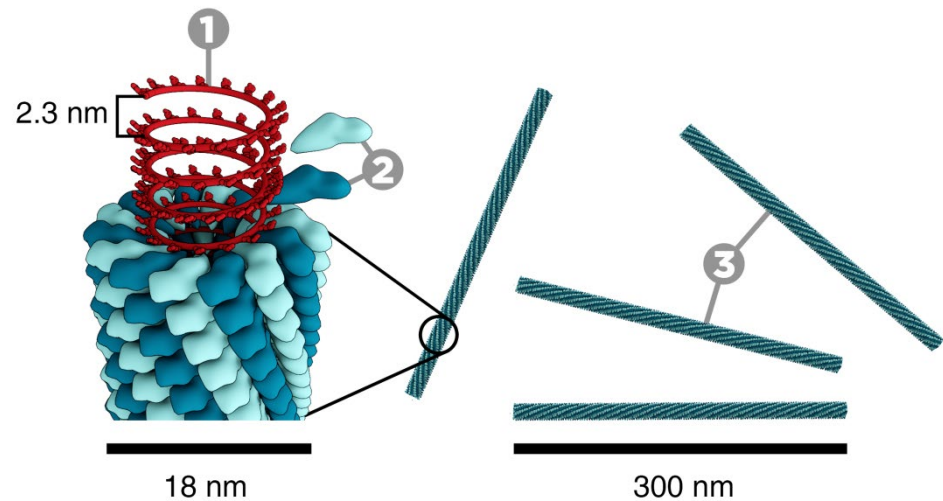
Biological Macromolecules



Biological macromolecules may have even larger molecular weight. For example, the molecular weight of tobacco mosaic virus is about **40,000,000 g/mole**.



Tobacco mosaic virus





Applications of Rheology



1. Polymer science (solutions and melts)
2. Pumping and materials transport
3. Chemical industries
4. Food production
5. Cosmetics industries
6. Bio-rheology, hemo-rheology and physiology
7. Control (smart fluids)
8. Pharmaceutical industries
9. Military industries
10. Paints and inks manufacture
11. Glass and silicates production
12. Paper production
13. Geology science and geophysics
14. Petroleum industry
15. Thickening and dewatering of mineral slurries
16. Forming materials (concrete, brick & ceramics)
17. Reactions involving mineral slurries (gold extraction)
18. Soil mechanic and chemistry
19. Separation (more viscous, less speed)



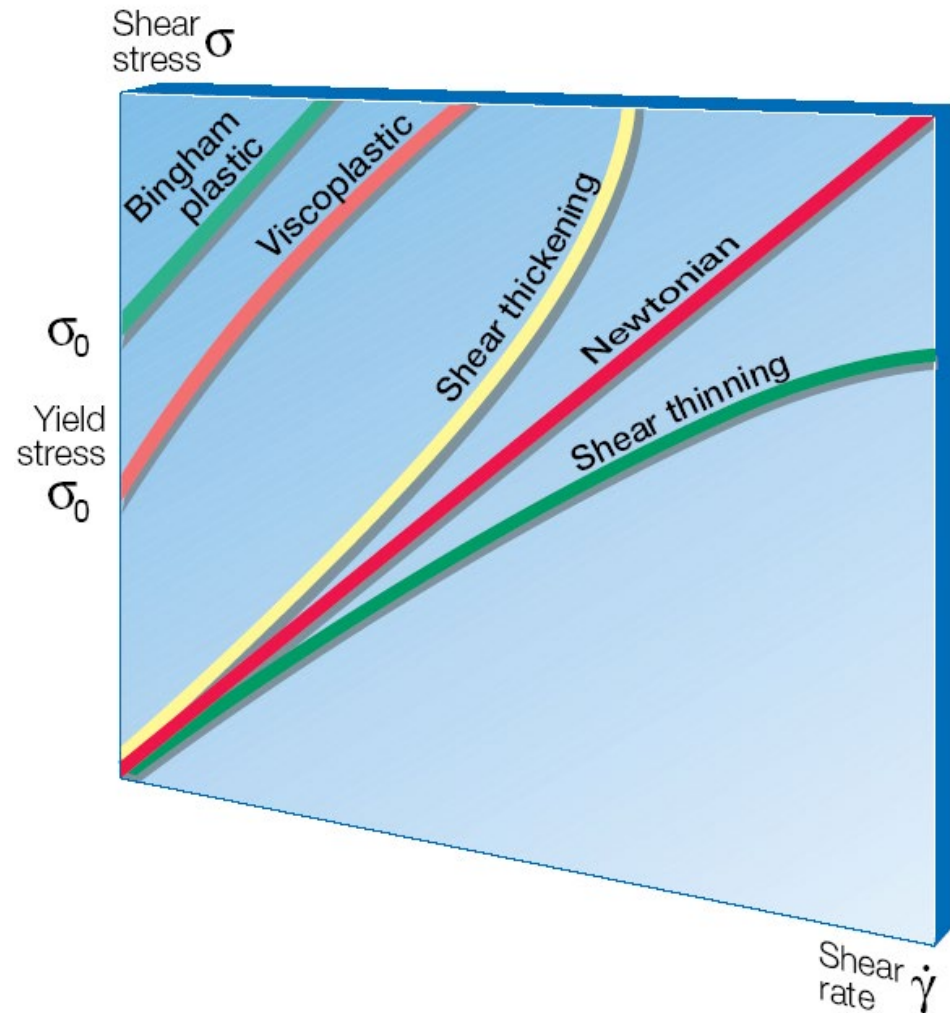


Time-Independent Fluids



Characteristics:

1. Shear dependent viscosity
2. Yield stress





Classification of Non-Newtonian Fluids



Non-Newtonian Fluids

**Time-Independent
Fluids**

**Time-Dependent
Fluids**

Viscoelastic Fluids

Smart Fluids

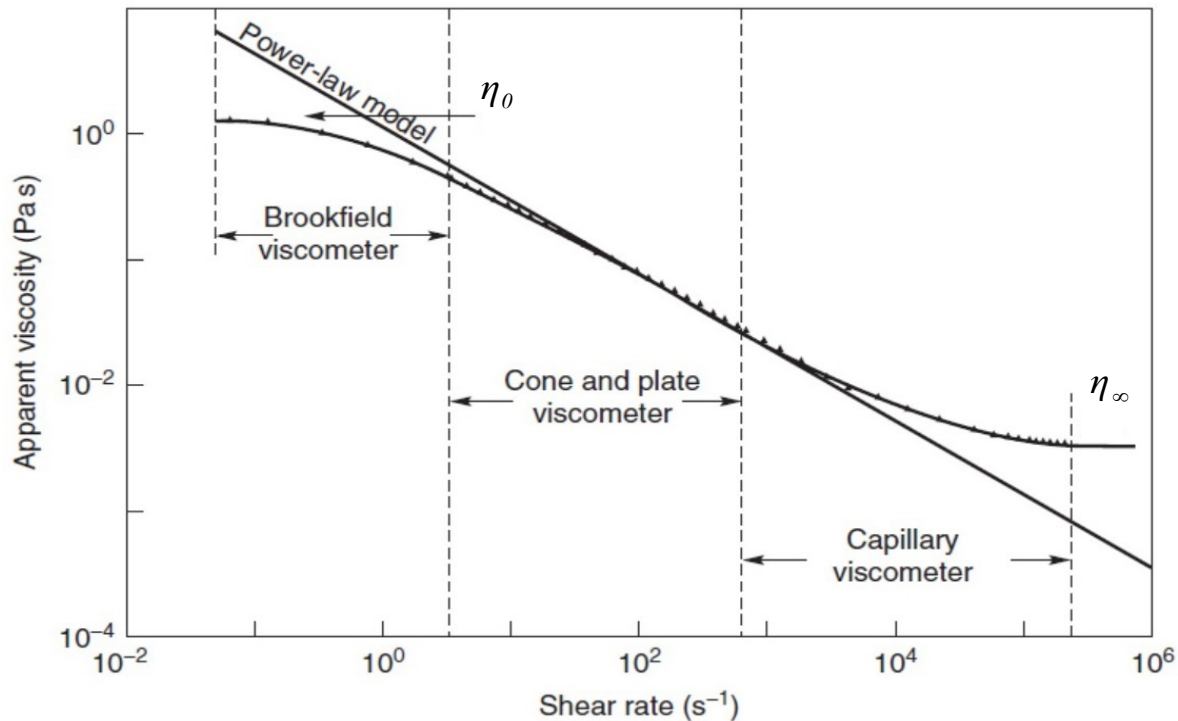




Shear-Thinning Fluids: Apparent Viscosity



Many of non-Newtonian fluids exhibit the pseudo-plastic behavior. This property is found in certain complex solutions, such as [lava](#), [ketchup](#), [whipped cream](#), [blood](#), [paints](#), [molasses](#), [syrops](#), [paper pulp in water](#), [ice-cream](#), [glue](#), some [silicon oils](#), some [silicon coatings](#), [printing inks](#), and [disperse systems](#). It is also a common property of [polymeric solutions](#) and [molten polymers](#).



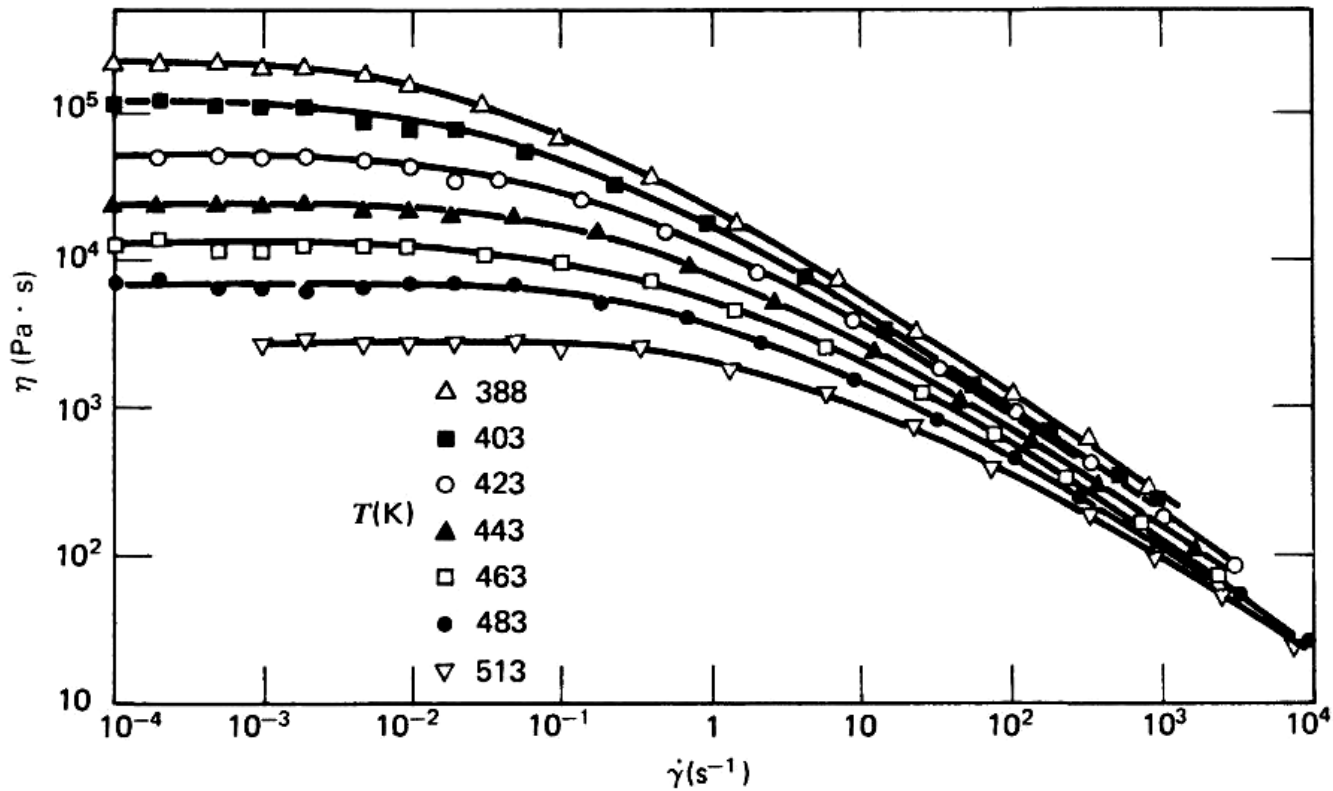
Demonstration of zero shear and infinite shear viscosities for a shear-thinning polymer solution.



Temperature Effect



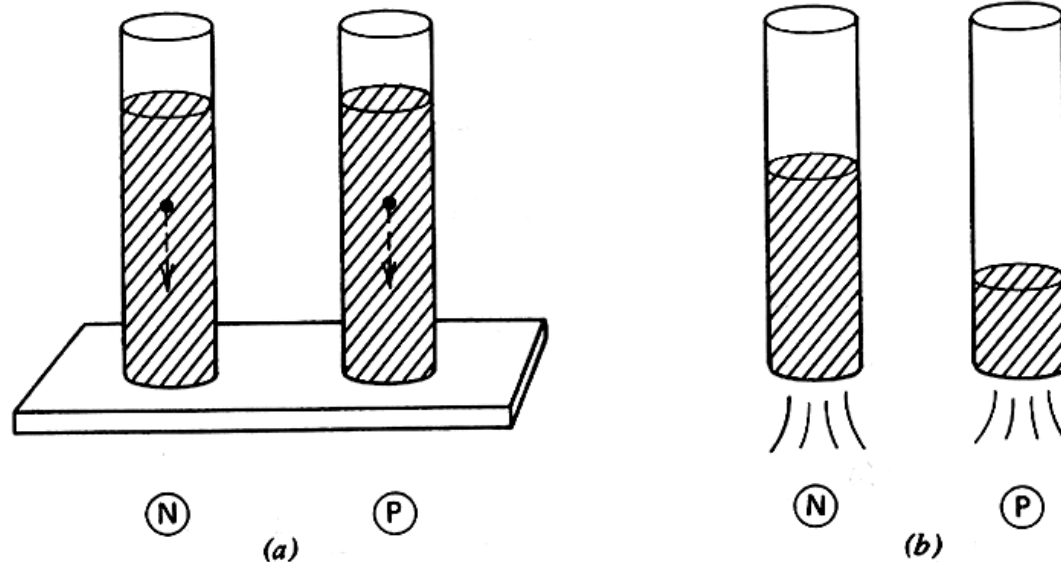
The viscosity of shear-thinning liquids has a strong dependency on **temperature**.



Viscosity of a low density polyethylene melt versus the shear rate.



Shear-Thinning Fluids – Sample of Effect



Tube flow and “shear thinning”. In each part, the Newtonian behavior is shown on the left (N); the behavior of a polymer on the right (P). (a) A tiny sphere falls at the same rate through each; (b) the polymer flows out faster than the Newtonian fluid.

[Reproduced from R. B. Bird, R. C. Armstrong and O. Hassager, *Dynamics of Polymeric Liquids. Vol I: Fluid Mechanics*, 2nd edition, Wiley-Interscience (1987), p. 61.]



Shear-Thinning Fluids: Origin



In a shear flow, the shear stress causes:

1. Alignment of polymer molecules in direction of streamlines

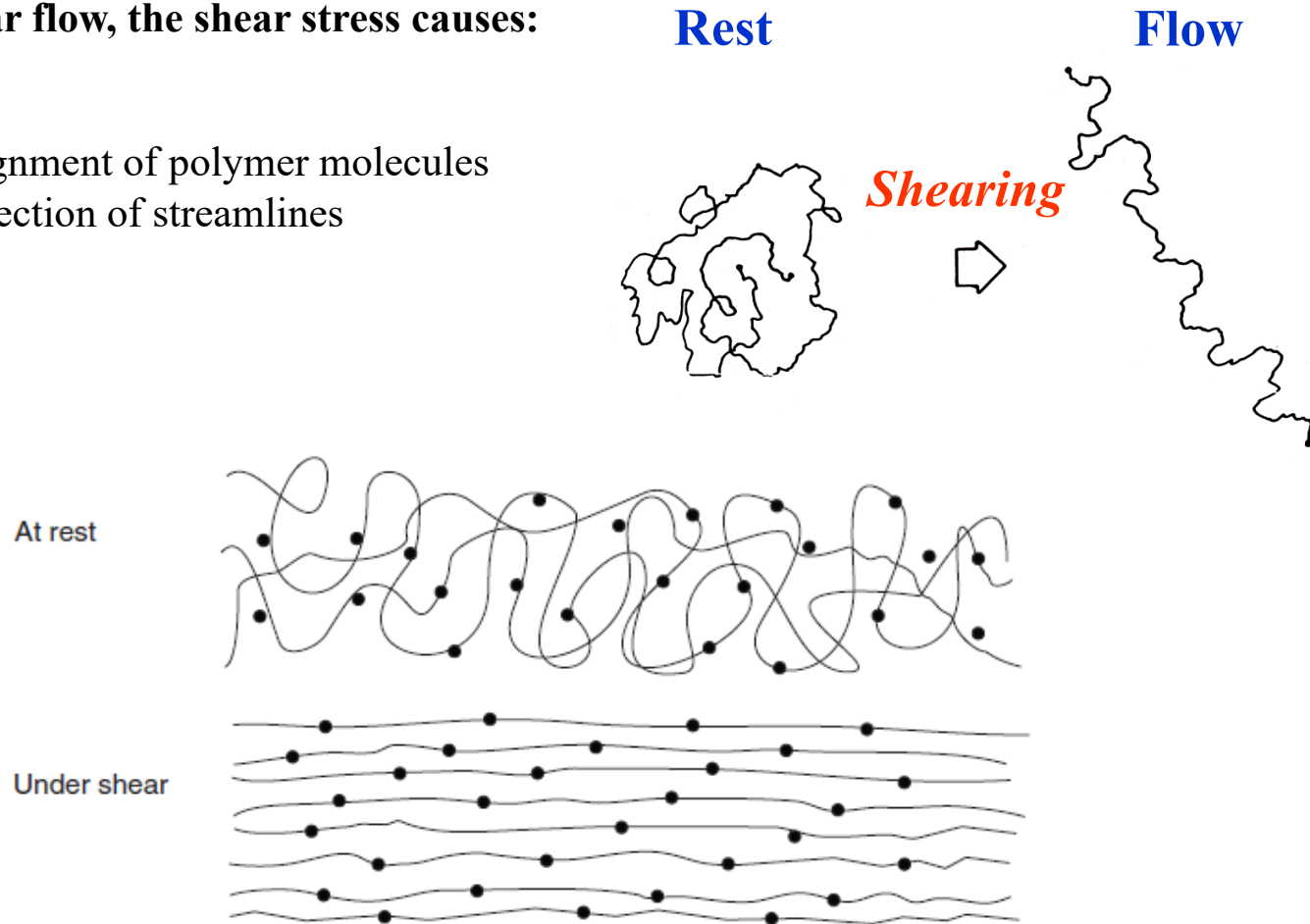


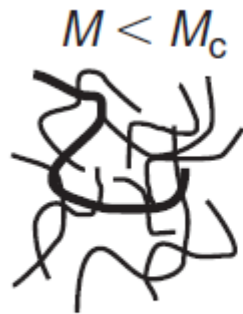
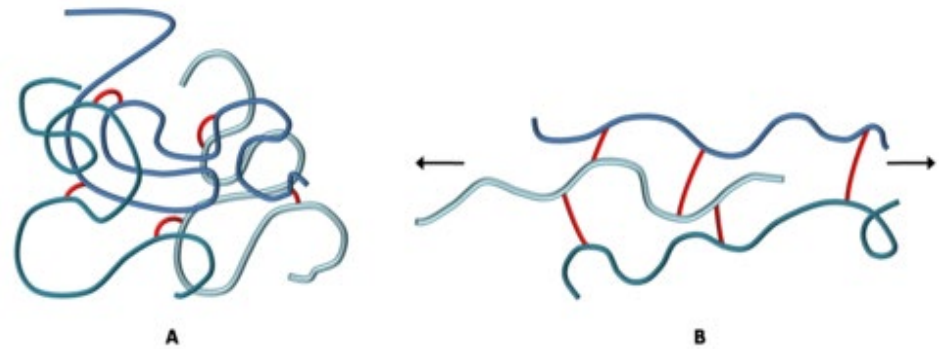
Figure 1.31 *Qualitative role of shearing in uncoiling and stretching of an entwined macromolecule*



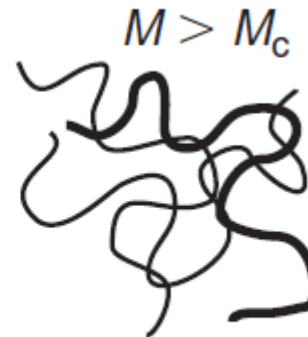
Shear-Thinning Fluids: Origin



2. Breaking the cross-links



No entanglement



Entangled system



Viscosity Modeling: Power-Law Model



Power-law equation: $\eta = K |\dot{\gamma}_{xy}|^{n-1}$

TABLE 1.6. Power-law parameters

		Range of $\dot{\gamma}(\text{s}^{-1})$	$k(\text{Pa}\cdot\text{s}^n)$	n
54.3% cement rock in water		10–200	2.51	0.153
23.3% Illinois yellow clay in water		1800–6000	5.55	0.229
Polystyrene at 422 K		0.03–3	1.6×10^5	0.4
1.5% carboxymethyl cellulose (CMC) in water		10^2 – 10^4	9.7	0.4
0.7% CMC in water		2×10^3 – 3×10^4	1.5	0.5
3% polyisobutylene in decalin		25–200	0.94	0.77
0.5% hydroxyethylcellulose in water	293 K	—	0.84	0.509
	313 K	—	0.30	0.595
	333 K	—	0.136	0.645
1% poly(ethylene-oxide) in water	293 K	—	0.994	0.532
	313 K	—	0.706	0.544
	333 K	—	0.486	0.599

All data are at room temperature (300 K) except where indicated. Data adapted from W. R. Schowalter (1978), *Mechanics of non-Newtonian fluids*, p. 139, Pergamon Press, Oxford. See also Table 1.7.



Viscosity Modeling: Cross Model



JOURNAL OF COLLOID SCIENCE **20**, 417–437 (1965)

RHEOLOGY OF NON-NEWTONIAN FLUIDS: A NEW FLOW EQUATION FOR PSEUDOPLASTIC SYSTEMS

Malcolm M. Cross

*Imperial Chemical Industries Ltd., Dyestuffs Division, Hexagon House,
Blackley, Manchester 9, England*

Received December 10, 1964

ABSTRACT

On the assumption that pseudoplastic flow is associated with the formation and rupture of structural linkages a new flow equation is derived. The equation takes the form

$$\eta = \eta_{\infty} + \left(\frac{\eta_0 - \eta_{\infty}}{1 + \alpha D^{2/3}} \right),$$

where D = rate of shear, η_0 = limiting viscosity at zero rate of shear, η_{∞} = limiting viscosity at infinite rate of shear, and α is a constant associated with the rupture of linkages.

Graphical methods for evaluating the three constants η_0 , η_{∞} , and α are presented.

Experimental data are presented on a wide range of pseudoplastic systems, ranging from suspensions to optically clear solutions, in both aqueous and nonaqueous media. In all cases the results conform to the equation with a high degree of accuracy over a wide range of shear rates.



Viscosity Modeling: Cross Model



Cross model: The cross model is based on the rupture theory of linkages due to the shear stress. Here, it is supposed that the viscosity is proportioned by the length of linkages.

Consider the equilibrium of an average chain having L linkages.

$$\frac{dL}{dt} = k_2 P - (k_0 + k_1 D^n) L. \quad [2]$$

Equilibrium is attained when $dL/dt = 0$ and $L = k_2 P / (k_0 + k_1 D^n)$. Putting $L = L_0$ when $D = 0$, we have $L_0 = k_2 P / k_0$. Hence

$$\frac{L}{L_0} = \frac{k_0}{k_0 + k_1 D^n};$$

i.e.,

$$\frac{L}{L_0} = \frac{1}{1 + \alpha D^n}, \quad [3]$$

where

$$\alpha = \frac{k_1}{k_0}. \quad [4]$$

D = applied shear rate in reciprocal seconds;

P = total number of single particles (linked or otherwise) in unit volume;

L = average number of links per chain at shear rate D ;

L_0 = average number of links per chain at zero rate of shear.



Viscosity Modeling of Shear-Thinning Fluids



Cross Equation (Modified by Barnes *et al.* (1989)):

$$\frac{\eta(\dot{\gamma}_{xy}) - \eta_{\infty}}{\eta_0 - \eta_{\infty}} = \frac{1}{1 + \lambda |\dot{\gamma}_{xy}|^n}$$

Carreau-Yasuda Equation:

$$\frac{\eta(\dot{\gamma}_{xy}) - \eta_{\infty}}{\eta_0 - \eta_{\infty}} = \frac{1}{\left(1 + (\lambda |\dot{\gamma}_{xy}|)^a\right)^{(1-n)/2}}$$

Ellis Equation:

$$\eta(\dot{\gamma}_{xy}) = \frac{\eta_0}{1 + (\tau_{xy} / \tau_{12})^{\alpha-1}}$$

- When the deviations from the power-law model are significant only at low shear rates, it is perhaps more appropriate to use the Ellis model.
- Index α is a measure of the degree of shear-thinning behavior (the greater the value of α , greater is the extent of shear-thinning ($\alpha > 1$)).
- $\tau_{1/2}$ represents the value of shear stress at which the apparent viscosity has dropped to half its zero shear value.



Viscosity Modeling



Parameters in Carreau-Yasuda Model for Some Solutions of Linear Polystyrene in 1-Chloronaphthalene^a

Properties of Solution		Parameters in Eq. 4.1-9 (η_{∞} is taken to be zero)			
\bar{M}_w (g/mol)	c (g/ml)	η_0 (Pa · s)	λ (s)	n (---)	a (---)
2×10^6	0.15	1400	1.60	0.200	1.25
2×10^6	0.088	90	3.79×10^{-1}	0.265	0.98
3.9×10^5	0.45	8080	1.109	0.304	2
3.9×10^5	0.30	135	3.61×10^{-2}	0.305	2
1.1×10^5	0.52	1180	9.24×10^{-2}	0.441	2
1.1×10^5	0.45	166	1.73×10^{-2}	0.538	2
3.7×10^4	0.62	3930	1×10^{-1}	0.217	2

^a Values of the parameters are taken from K. Yasuda, R. C. Armstrong, and R. E. Cohen, *Rheol. Acta*, **20**, 163–178 (1981).

The constant “ a ” is around 2 for many shear-thinning fluids and it describes the transition region between the zero shear rate region and power-law region.



Viscosity Modeling

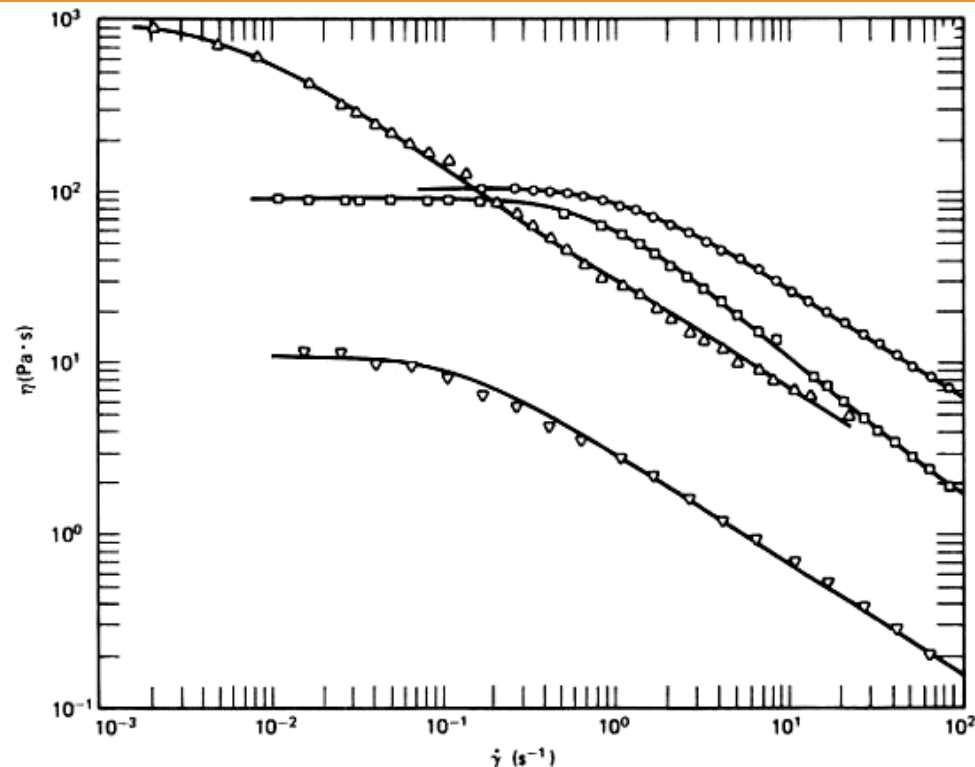


FIGURE 4.1-1. Non-Newtonian viscosity of three polymer solutions and a soap solution as fitted by the Carreau viscosity equation (Eq. 4.1-9, with $a = 2$). [R. B. Bird, O. Hassager, and S. I. Abdel-Khalik, *AIChE J.*, **20**, 1041-1066 (1974).] Δ 2% polyisobutylene in Primol 355; data of J. D. Huppler, E. Ashare, and L. A. Holmes, *Trans. Soc. Rheol.*, **11**, 159-179 (1968): $\eta_0 = 9.23 \times 10^2 \text{ Pa}\cdot\text{s}$, $\eta_\infty = 1.50 \times 10^{-1} \text{ Pa}\cdot\text{s}$, $\lambda = 191 \text{ s}$, $n = 0.358$. \circ 5% polystyrene in Aroclor 1242; data of E. Ashare, Ph.D. Thesis, University of Wisconsin, Madison (1968): $\eta_0 = 1.01 \times 10^2 \text{ Pa}\cdot\text{s}$, $\eta_\infty = 5.9 \times 10^{-2} \text{ Pa}\cdot\text{s}$, $\lambda = 0.84 \text{ s}$, $n = 0.364$. ∇ 0.75% polyacrylamide (Separan-30) in a 95/5 mixture by weight of water and glycerin; data of B. D. Marsh (1967), as cited by P. J. Carreau, I. F. Macdonald, and R. B. Bird, *Chem. Eng. Sci.*, **23**, 901-911 (1968): $\eta_0 = 10.6 \text{ Pa}\cdot\text{s}$, $\eta_\infty = 10^{-2} \text{ Pa}\cdot\text{s}$, $\lambda = 8.04 \text{ s}$, $n = 0.364$. \square 7% aluminum soap in decalin and *m*-cresol; data of J. D. Huppler, E. Ashare, and L. A. Holmes, *loc. cit.*: $\eta_0 = 89.6 \text{ Pa}\cdot\text{s}$, $\eta_\infty = 10^{-2} \text{ Pa}\cdot\text{s}$, $\lambda = 1.41 \text{ s}$, $n = 0.200$.



Modeling the Complex Flows



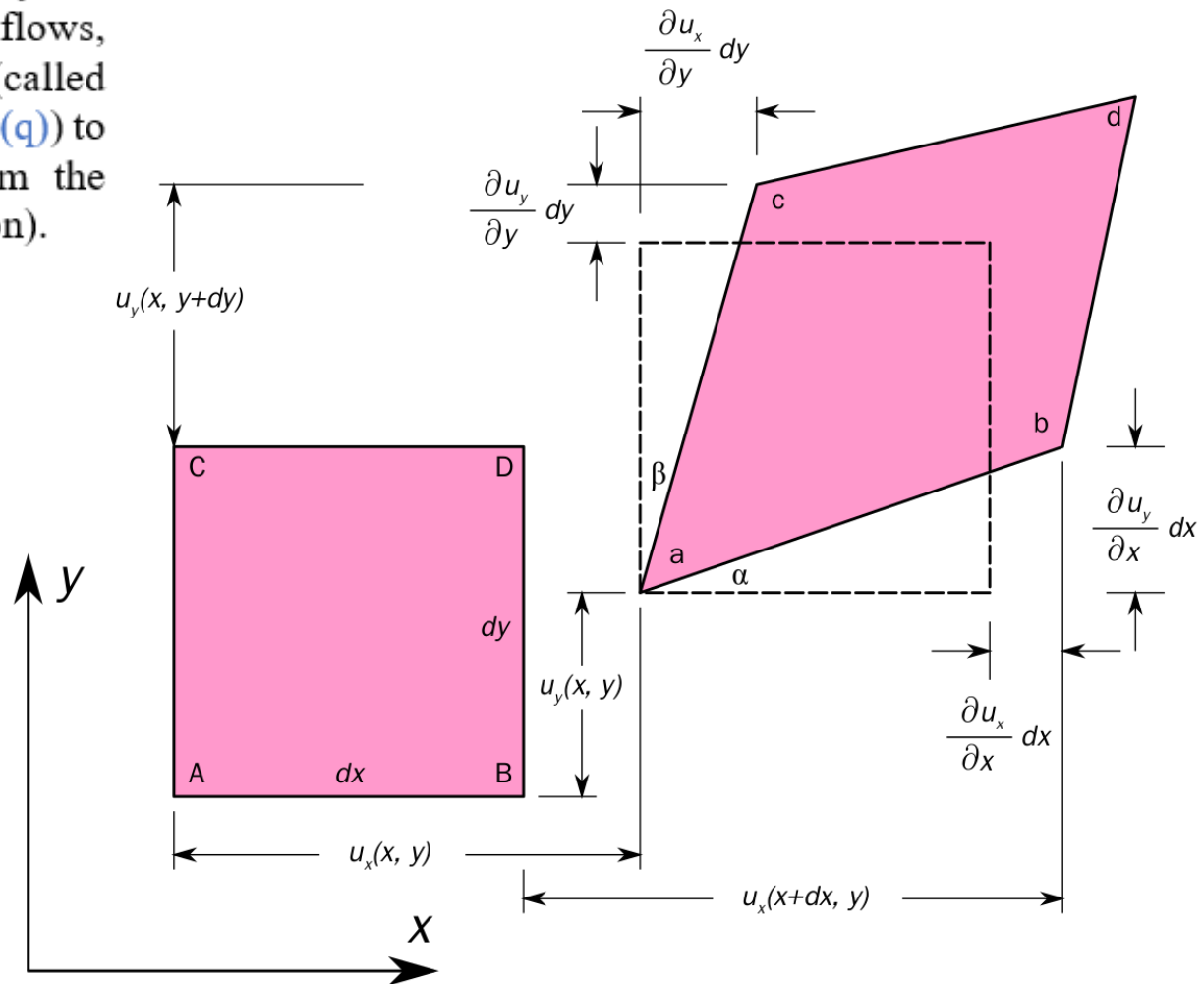
For modeling the viscosity of complex non-Newtonian flows, we need a unique shear rate (called **The Generalized Shear Rate (q)**) to calculate the viscosity from the models (constitutive equation).

For Simple Shear Flow:

$$\dot{\gamma}_{xy} = \frac{\partial u}{\partial y}$$

For 2D Flows:

$$\dot{\gamma}_{xy} = \dot{\gamma}_{yx} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$$





Modeling the Complex Flows

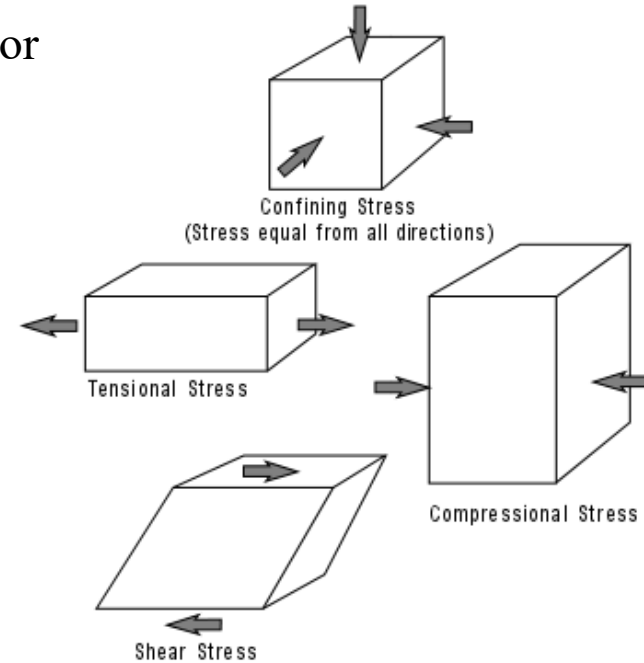


Remember that the shear rate is a cinematic tensor with nine components defined as:

$$\dot{\gamma} = \nabla \mathbf{V} + \nabla \mathbf{V}^T \quad \text{and} \quad \nabla \mathbf{V}_{ij} = \frac{\partial u_j}{\partial x_i}$$

$$\nabla \mathbf{V}_{ij} = \begin{bmatrix} \frac{\partial u}{\partial x} & \frac{\partial v}{\partial x} & \frac{\partial w}{\partial x} \\ \frac{\partial u}{\partial y} & \frac{\partial v}{\partial y} & \frac{\partial w}{\partial y} \\ \frac{\partial u}{\partial z} & \frac{\partial v}{\partial z} & \frac{\partial w}{\partial z} \end{bmatrix}$$

$$\dot{\gamma} = \begin{bmatrix} 2\frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} & \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} & 2\frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \\ \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} & \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} & 2\frac{\partial w}{\partial z} \end{bmatrix},$$



$$\begin{cases} \nabla \mathbf{V} = \mathbf{D} + \mathbf{W} \\ \mathbf{D} = \frac{1}{2}(\nabla \mathbf{V} + \nabla \mathbf{V}^T), \quad \mathbf{W} = \frac{1}{2}(\nabla \mathbf{V} - \nabla \mathbf{V}^T) \\ \mathbf{D} = \frac{1}{2}\dot{\gamma} \end{cases}$$



Modeling the Complex Flows



In order to define the **generalized shear rate**, we can use the invariants of shear rate tensor because the results must be independent from type of coordinate system and, rotation and movement of frame:

$$I = \text{tr}(\dot{\gamma}) = \sum_i \dot{\gamma}_{ii}$$

$$II = \text{tr}(\dot{\gamma}^2) = \sum_i \sum_j \dot{\gamma}_{ij} \dot{\gamma}_{ji}$$

$$III = \text{tr}(\dot{\gamma}^3) = \sum_i \sum_j \sum_k \dot{\gamma}_{ij} \dot{\gamma}_{jk} \dot{\gamma}_{ki}$$

Other combinations of these three could be played the role of invariants. For example, the following combination is usually used for this purpose:

$$I_1 = I$$

$$I_2 = \frac{1}{2}(I^2 - II)$$

$$I_3 = \frac{1}{6}(I^3 - 3I \cdot II + 2III) = |\dot{\gamma}|$$

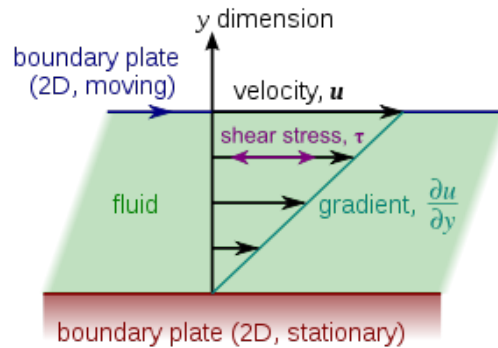
Key Question#1: Which invariants should be used as **the generalized shear rate**?



Modeling the Complex Flows



For simple shear flows, the non zero term is only: $\frac{\partial u}{\partial y}$



$$\dot{\gamma} = \begin{bmatrix} 0 & \frac{\partial u}{\partial y} & 0 \\ \frac{\partial u}{\partial y} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Therefore, $I = III = 0$ & $II = 2\dot{\gamma}_{xy}^2 \Rightarrow q = \dot{\gamma}_{xy} = \sqrt{\frac{1}{2} II}$

where q is the **generalized shear rate**.

The first invariant (I) is always equal to zero due to continuity equation. Using the above calculation, the following relation is suggested for complex flows:

$$q = \sqrt{\frac{1}{2} II} = \sqrt{\frac{1}{2} (\gamma_{xx}^2 + \gamma_{yy}^2 + \gamma_{zz}^2 + 2\gamma_{xy}^2 + 2\gamma_{yz}^2 + 2\gamma_{xz}^2)}$$

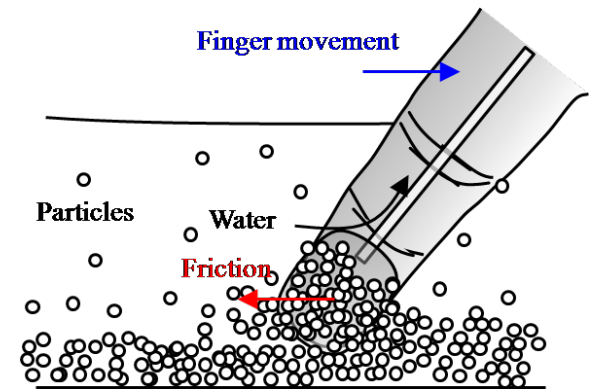
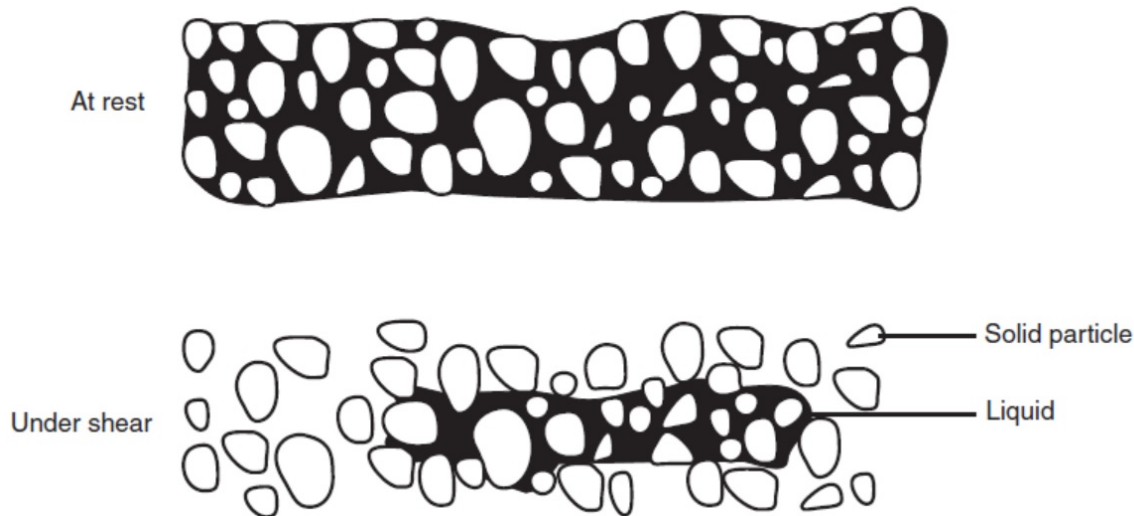
Key Question#2: What is the role of the third invariant in definition of **generalized shear rate**?



Shear-Thickening Fluids



A dilatant (also termed shear thickening) material is one in which viscosity increases with the rate of shear rate.



Schematic representation of shear-thickening behavior.

This type of fluid behavior was observed in **concentrated suspensions** and it is important in **handling and processing of systems with high solids loadings**. Typical examples of materials exhibiting dilatant behavior include concentrated suspensions of **china clay**, **titanium dioxide** and **corn flour in water**. This behavior could be modeled using power-law equation considering $n > 1$.



End of presentation

Thank You!