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FUNDAMENTALS OF

PHYSICS

Halliday & Resnick

10th edition

JEARL WALKER

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MATHEMATICAL FORMULAS*

Quadratic Formula

If $ax^2 + bx + c = 0$, then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Binomial Theorem

$$(1 + x)^n = 1 + \frac{nx}{1!} + \frac{n(n-1)x^2}{2!} + \dots \quad (x^2 < 1)$$

Products of Vectors

Let θ be the smaller of the two angles between \vec{a} and \vec{b} .

Then

$$\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a} = a_x b_x + a_y b_y + a_z b_z = ab \cos \theta$$

$$\vec{a} \times \vec{b} = -\vec{b} \times \vec{a} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

$$= \hat{i} \begin{vmatrix} a_y & a_z \\ b_y & b_z \end{vmatrix} - \hat{j} \begin{vmatrix} a_x & a_z \\ b_x & b_z \end{vmatrix} + \hat{k} \begin{vmatrix} a_x & a_y \\ b_x & b_y \end{vmatrix}$$

$$= (a_y b_z - b_y a_z) \hat{i} + (a_z b_x - b_z a_x) \hat{j} + (a_x b_y - b_x a_y) \hat{k}$$

$$|\vec{a} \times \vec{b}| = ab \sin \theta$$

Trigonometric Identities

$$\sin \alpha \pm \sin \beta = 2 \sin \frac{1}{2}(\alpha \pm \beta) \cos \frac{1}{2}(\alpha \mp \beta)$$

$$\cos \alpha + \cos \beta = 2 \cos \frac{1}{2}(\alpha + \beta) \cos \frac{1}{2}(\alpha - \beta)$$

*See Appendix E for a more complete list.

Derivatives and Integrals

$$\frac{d}{dx} \sin x = \cos x \quad \int \sin x \, dx = -\cos x$$

$$\frac{d}{dx} \cos x = -\sin x \quad \int \cos x \, dx = \sin x$$

$$\frac{d}{dx} e^x = e^x \quad \int e^x \, dx = e^x$$

$$\int \frac{dx}{\sqrt{x^2 + a^2}} = \ln(x + \sqrt{x^2 + a^2})$$

$$\int \frac{x \, dx}{(x^2 + a^2)^{3/2}} = -\frac{1}{(x^2 + a^2)^{1/2}}$$

$$\int \frac{dx}{(x^2 + a^2)^{3/2}} = \frac{x}{a^2(x^2 + a^2)^{1/2}}$$

Cramer's Rule

Two simultaneous equations in unknowns x and y ,

$$a_1 x + b_1 y = c_1 \quad \text{and} \quad a_2 x + b_2 y = c_2,$$

have the solutions

$$x = \frac{\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}} = \frac{c_1 b_2 - c_2 b_1}{a_1 b_2 - a_2 b_1}$$

and

$$y = \frac{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}} = \frac{a_1 c_2 - a_2 c_1}{a_1 b_2 - a_2 b_1}.$$

SI PREFIXES*

Factor	Prefix	Symbol	Factor	Prefix	Symbol
10^{24}	yotta	Y	10^{-1}	deci	d
10^{21}	zetta	Z	10^{-2}	centi	c
10^{18}	exa	E	10^{-3}	milli	m
10^{15}	peta	P	10^{-6}	micro	μ
10^{12}	tera	T	10^{-9}	nano	n
10^9	giga	G	10^{-12}	pico	p
10^6	mega	M	10^{-15}	femto	f
10^3	kilo	k	10^{-18}	atto	a
10^2	hecto	h	10^{-21}	zepto	z
10^1	deka	da	10^{-24}	yocto	y

*In all cases, the first syllable is accented, as in ná-no-mé-ter.

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Halliday & Resnick

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This book was set in 10/12 Times Ten by cMPreparé, CSR Francesca Monaco, and was printed and bound by Quad Graphics. The cover was printed by Quad Graphics.

This book is printed on acid free paper.

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Library of Congress Cataloging-in-Publication Data

Walker, Jearl
Fundamentals of physics / Jearl Walker, David Halliday, Robert Resnick—10th edition.
volumes cm
Includes index.
ISBN 978-1-118-23072-5 (Extended edition)
Binder-ready version ISBN 978-1-118-23061-9 (Extended edition)
1. Physics—Textbooks. I. Resnick, Robert. II. Halliday, David. III. Title.
QC21.3.H35 2014
530—dc23

2012035307

Printed in the United States of America
10 9 8 7 6 5 4 3 2 1

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WHY I WROTE THIS BOOK

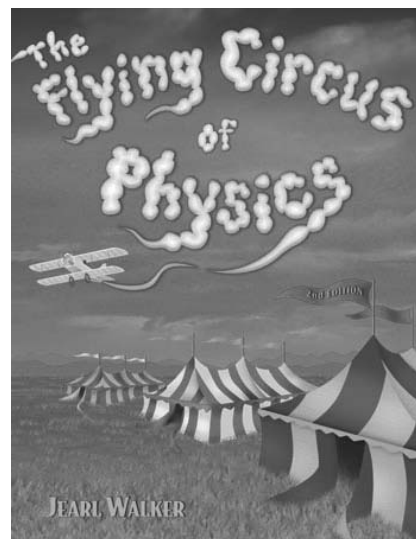
Fun with a big challenge. That is how I have regarded physics since the day when Sharon, one of the students in a class I taught as a graduate student, suddenly demanded of me, “What has any of this got to do with my life?” Of course I immediately responded, “Sharon, this has everything to do with your life—this is physics.”

She asked me for an example. I thought and thought but could not come up with a single one. That night I began writing the book *The Flying Circus of Physics* (John Wiley & Sons Inc., 1975) for Sharon but also for me because I realized her complaint was mine. I had spent six years slugging my way through many dozens of physics textbooks that were carefully written with the best of pedagogical plans, but there was something missing. Physics is the most interesting subject in the world because it is about how the world works, and yet the textbooks had been thoroughly wrung of any connection with the real world. The fun was missing.

I have packed a lot of real-world physics into *Fundamentals of Physics*, connecting it with the new edition of *The Flying Circus of Physics*. Much of the material comes from the introductory physics classes I teach, where I can judge from the faces and blunt comments what material and presentations work and what do not. The notes I make on my successes and failures there help form the basis of this book. My message here is the same as I had with every student I’ve met since Sharon so long ago: “Yes, you *can* reason from basic physics concepts all the way to valid conclusions about the real world, and that understanding of the real world is where the fun is.”

I have many goals in writing this book but the overriding one is to provide instructors with tools by which they can teach students how to effectively read scientific material, identify fundamental concepts, reason through scientific questions, and solve quantitative problems. This process is not easy for either students or instructors. Indeed, the course associated with this book may be one of the most challenging of all the courses taken by a student. However, it can also be one of the most rewarding because it reveals the world’s fundamental clockwork from which all scientific and engineering applications spring.

Many users of the ninth edition (both instructors and students) sent in comments and suggestions to improve the book. These improvements are now incorporated into the narrative and problems throughout the book. The publisher John Wiley & Sons and I regard the book as an ongoing project and encourage more input from users. You can send suggestions, corrections, and positive or negative comments to John Wiley & Sons or Jearl Walker (mail address: Physics Department, Cleveland State University, Cleveland, OH 44115 USA; or the blog site at www.flyingcircusofphysics.com). We may not be able to respond to all suggestions, but we keep and study each of them.



WHAT'S NEW?

Modules and Learning Objectives “What was I supposed to learn from this section?” Students have asked me this question for decades, from the weakest student to the strongest. The problem is that even a thoughtful student may not feel confident that the important points were captured while reading a section. I felt the same way back when I was using the first edition of Halliday and Resnick while taking first-year physics.

To ease the problem in this edition, I restructured the chapters into concept modules based on a primary theme and begin each module with a list of the module’s learning objectives. The list is an explicit statement of the skills and learning points that should be gathered in reading the module. Each list is followed by a brief summary of the key ideas that should also be gathered. For example, check out the first module in Chapter 16, where a student faces a truck load of concepts and terms. Rather than depending on the student’s ability to gather and sort those ideas, I now provide an explicit checklist that functions somewhat like the checklist a pilot works through before taxiing out to the runway for takeoff.



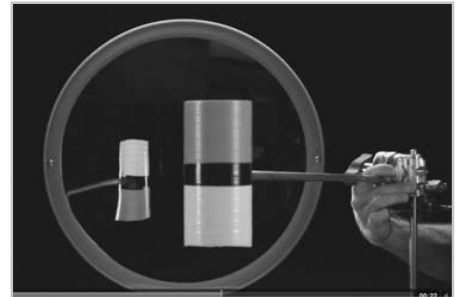
Links Between Homework Problems and Learning Objectives In *WileyPLUS*, every question and problem at the end of the chapter is linked to a learning objective, to answer the (usually unspoken) questions, “Why am I working this problem? What am I supposed to learn from it?” By being explicit about a problem’s purpose, I believe that a student might better transfer the learning objective to other problems with a different wording but the same key idea. Such transference would help defeat the common trouble that a student learns to work a particular problem but cannot then apply its key idea to a problem in a different setting.

Rewritten Chapters My students have continued to be challenged by several key chapters and by spots in several other chapters and so, in this edition, I rewrote a lot of the material. For example, I redesigned the chapters on Gauss’ law and electric potential, which have proved to be tough-going for my students. The presentations are now smoother and more direct to the key points. In the quantum chapters, I expanded the coverage of the Schrödinger equation, including reflection of matter waves from a step potential. At the request of several instructors, I decoupled the discussion of the Bohr atom from the Schrödinger solution for the hydrogen atom so that the historical account of Bohr’s work can be bypassed. Also, there is now a module on Planck’s blackbody radiation.

New Sample Problems and Homework Questions and Problems Sixteen new sample problems have been added to the chapters, written so as to spotlight some of the difficult areas for my students. Also, about 250 problems and 50 questions have been added to the homework sections of the chapters. Some of these problems come from earlier editions of the book, as requested by several instructors.



Video Illustrations In the eVersion of the text available in *WileyPLUS*, David Maiullo of Rutgers University has created video versions of approximately 30 of the photographs and figures from the text. Much of physics is the study of things that move and video can often provide a better representation than a static photo or figure.



Online Aid *WileyPLUS* is not just an online grading program. Rather, it is a dynamic learning center stocked with many different learning aids, including just-in-time problem-solving tutorials, embedded reading quizzes to encourage reading, animated figures, hundreds of sample problems, loads of simulations and demonstrations, and over 1500 videos ranging from math reviews to mini-lectures to examples. More of these learning aids are added every semester. For this 10th edition of HRW, some of the photos involving motion have been converted into videos so that the motion can be slowed and analyzed.

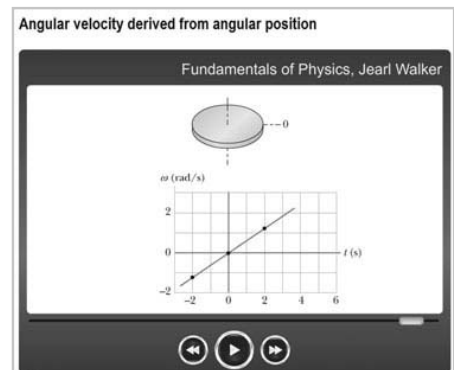
These thousands of learning aids are available 24/7 and can be repeated as many times as desired. Thus, if a student gets stuck on a homework problem at, say, 2:00 AM (which appears to be a popular time for doing physics homework), friendly and helpful resources are available at the click of a mouse.

LEARNINGS TOOLS

When I learned first-year physics in the first edition of Halliday and Resnick, I caught on by repeatedly rereading a chapter. These days we better understand that students have a wide range of learning styles. So, I have produced a wide range of learning tools, both in this new edition and online in *WileyPLUS*:



Animations of one of the key figures in each chapter. Here in the book, those figures are flagged with the swirling icon. In the online chapter in *WileyPLUS*, a mouse click begins the animation. I have chosen the figures that are rich in information so that a student can see the physics in action and played out over a minute or two



instead of just being flat on a printed page. Not only does this give life to the physics, but the animation can be repeated as many times as a student wants.



Videos I have made well over 1500 instructional videos, with more coming each semester. Students can watch me draw or type on the screen as they hear me talk about a solution, tutorial, sample problem, or review, very much as they would experience were they sitting next to me in my office while I worked out something on a notepad. An instructor's lectures and tutoring will always be the most valuable learning tools, but my videos are available 24 hours a day, 7 days a week, and can be repeated indefinitely.

- **Video tutorials on subjects in the chapters.** I chose the subjects that challenge the students the most, the ones that my students scratch their heads about.

- **Video reviews of high school math**, such as basic algebraic manipulations, trig functions, and simultaneous equations.

- **Video introductions to math**, such as vector multiplication, that will be new to the students.

- **Video presentations of every Sample Problem** in the textbook chapters. My intent is to work out the physics, starting with the Key Ideas instead of just grabbing a formula. However, I also want to demonstrate how to read a sample problem, that is, how to read technical material to learn problem-solving procedures that can be transferred to other types of problems.

- **Video solutions to 20% of the end-of chapter problems.** The availability and timing of these solutions are controlled by the instructor. For example, they might be available after a homework deadline or a quiz. Each solution is not simply a plug-and-chug recipe. Rather I build a solution from the Key Ideas to the first step of reasoning and to a final solution. The student learns not just how to solve a particular problem but how to tackle any problem, even those that require *physics courage*.

- **Video examples of how to read data from graphs** (more than simply reading off a number with no comprehension of the physics).



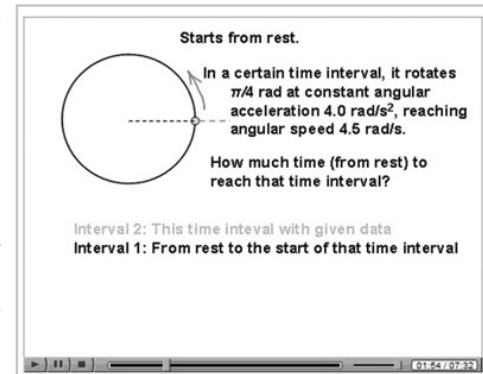
Problem-Solving Help I have written a large number of resources for WileyPLUS designed to help build the students' problem-solving skills.

- **Every sample problem in the textbook** is available online in both reading and video formats.

- **Hundreds of additional sample problems.** These are available as stand-alone resources but (at the discretion of the instructor) they are also linked out of the homework problems. So, if a homework problem deals with, say, forces on a block on a ramp, a link to a related sample problem is provided. However, the sample problem is not just a replica of the homework problem and thus does not provide a solution that can be merely duplicated without comprehension.

- **GO Tutorials** for 15% of the end-of-chapter homework problems. In multiple steps, I lead a student through a homework problem, starting with the Key Ideas and giving hints when wrong answers are submitted. However, I purposely leave the last step (for the final answer) to the student so that they are responsible at the end. Some online tutorial systems trap a student when wrong answers are given, which can generate a lot of frustration. My GO Tutorials are not traps, because at any step along the way, a student can return to the main problem.

- **Hints on every end-of-chapter homework problem** are available (at the discretion of the instructor). I wrote these as true hints about the main ideas and the general procedure for a solution, not as recipes that provide an answer without any comprehension.



GO Tutorial [Close]

This GO Tutorial will provide you with a step-by-step guide on how to approach this problem. When you are finished, go back and try the problem again on your own. To view the original question while you work, you can just drag this screen to the side. (This GO Tutorial consists of 4 steps).

Step 1: Solution Step 1 of GO Tutorial 10-30

KEY IDEAS:

(1) When an object rotates at constant angular acceleration, we can use the constant-acceleration equations of Table 10-1, modified for angular motion:

(1) $\omega = \omega_0 + \alpha t$

(2) $\theta - \theta_0 = \omega_0 t + \frac{1}{2} \alpha t^2$

(3) $\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$

(4) $\theta - \theta_0 = \frac{1}{2}(\omega_0 + \omega)t$

(5) $\theta - \theta_0 = \omega t - \frac{1}{2} \alpha t^2$

Counterclockwise is the positive direction of rotation, and clockwise is the negative direction.

(2) If a particle moves around a rotation axis at radius r , the magnitude of its radial (centripetal) acceleration a_r at any moment is related to its tangential speed v (the speed along the circular path) and its angular speed at that moment by

$a_r = \frac{v^2}{r} = \omega^2 r$

(3) If a particle moves around a rotation axis at radius r , the magnitude of its tangential acceleration a_t (the acceleration along the circular path) at any moment is related to the angular acceleration α at that moment by

$a_t = r\alpha$

(4) If a particle moves around a rotation axis at radius r , the angular displacement through which it rotates is related to the distance s it moves along its circular path by

$s = r\Delta\theta$

GETTING STARTED: What is the radius of rotation (in meters) of a point on the rim of the flywheel?

Number: Unit:

exact number, no tolerance

[Check Your Input]

Step 2: Solution Step 2 of GO Tutorial 10-30

What is the final angular speed in radians per second?

Number: Unit:

the tolerance is +/-2%

[Check Your Input]

Step 3: Solution Step 3 of GO Tutorial 10-30

What was the initial angular speed?

Number: Unit:

exact number, no tolerance

[Check Your Input]

Step 4: Solution Step 4 of GO Tutorial 10-30

Through what angular distance does the flywheel rotate to reach the final angular speed?

Number: Unit:

the tolerance is +/-2%

[Check Your Input]

Now that you know how to solve the problem, go back and try again on your own. [Close]



Evaluation Materials

- **Reading questions are available within each online section.** I wrote these so that they do not require analysis or any deep understanding; rather they simply test whether a student has read the section. When a student opens up a section, a randomly chosen reading question (from a bank of questions) appears at the end. The instructor can decide whether the question is part of the grading for that section or whether it is just for the benefit of the student.
- **Checkpoints are available within most sections.** I wrote these so that they require analysis and decisions about the physics in the section. *Answers to all checkpoints are in the back of the book.*



Checkpoint 1

Here are three pairs of initial and final positions, respectively, along an x axis. Which pairs give a negative displacement: (a) -3 m, $+5$ m; (b) -3 m, -7 m; (c) 7 m, -3 m?

- **All end-of-chapter homework Problems** in the book (and many more problems) are available in *WileyPLUS*. The instructor can construct a homework assignment and control how it is graded when the answers are submitted online. For example, the instructor controls the deadline for submission and how many attempts a student is allowed on an answer. The instructor also controls which, if any, learning aids are available with each homework problem. Such links can include hints, sample problems, in-chapter reading materials, video tutorials, video math reviews, and even video solutions (which can be made available to the students after, say, a homework deadline).
- **Symbolic notation problems** that require algebraic answers are available in every chapter.
- **All end-of-chapter homework Questions** in the book are available for assignment in *WileyPLUS*. These Questions (in a multiple choice format) are designed to evaluate the students' conceptual understanding.

Icons for Additional Help When worked-out solutions are provided either in print or electronically for certain of the odd-numbered problems, the statements for those problems include an icon to alert both student and instructor as to where the solutions are located. There are also icons indicating which problems have GO Tutorial, an Interactive LearningWare, or a link to the *The Flying Circus of Physics*. An icon guide is provided here and at the beginning of each set of problems.



Tutoring problem available (at instructor's discretion) in *WileyPLUS* and WebAssign



Worked-out solution available in Student Solutions Manual



Worked-out solution is at



Number of dots indicates level of problem difficulty



Interactive solution is at



Additional information available in *The Flying Circus of Physics* and at flyingcircusofphysics.com

<http://www.wiley.com/college/halliday>

VERSIONS OF THE TEXT

To accommodate the individual needs of instructors and students, the ninth edition of *Fundamentals of Physics* is available in a number of different versions.

The **Regular Edition** consists of Chapters 1 through 37 (ISBN 9781118230718).

The **Extended Edition** contains seven additional chapters on quantum physics and cosmology, Chapters 1–44 (ISBN 9781118230725).

Volume 1 — Chapters 1–20 (Mechanics and Thermodynamics), hardcover, ISBN 9781118233764

Volume 2 — Chapters 21–44 (E&M, Optics, and Quantum Physics), hardcover, ISBN 9781118230732

INSTRUCTOR SUPPLEMENTS

Instructor's Solutions Manual by Sen-Ben Liao, Lawrence Livermore National Laboratory. This manual provides worked-out solutions for all problems found at the end of each chapter. It is available in both MSWord and PDF.

Instructor Companion Site <http://www.wiley.com/college/halliday>

- **Instructor's Manual** This resource contains lecture notes outlining the most important topics of each chapter; demonstration experiments; laboratory and computer projects; film and video sources; answers to all Questions, Exercises, Problems, and Checkpoints; and a correlation guide to the Questions, Exercises, and Problems in the previous edition. It also contains a complete list of all problems for which solutions are available to students (SSM, WWW, and ILW).
- **Lecture PowerPoint Slides** These PowerPoint slides serve as a helpful starter pack for instructors, outlining key concepts and incorporating figures and equations from the text.
- **Classroom Response Systems ("Clicker") Questions** by David Marx, Illinois State University. There are two sets of questions available: Reading Quiz questions and Interactive Lecture questions. The Reading Quiz questions are intended to be relatively straightforward for any student who reads the assigned material. The Interactive Lecture questions are intended for use in an interactive lecture setting.
- **Wiley Physics Simulations** by Andrew Duffy, Boston University and John Gastineau, Vernier Software. This is a collection of 50 interactive simulations (Java applets) that can be used for classroom demonstrations.
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- **Test Bank** For the 10th edition, the Test Bank has been completely over-hauled by Suzanne Willis, Northern Illinois University. The Test Bank includes more than 2200 multiple-choice questions. These items are also available in the Computerized Test Bank which provides full editing features to help you customize tests (available in both IBM and Macintosh versions).
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Student Study Guide (ISBN 9781118230787) by Thomas Barrett of Ohio State University. The Student Study Guide consists of an overview of the chapter's important concepts, problem solving techniques and detailed examples.

Student Solutions Manual (ISBN 9781118230664) by Sen-Ben Liao, Lawrence Livermore National Laboratory. This manual provides students with complete worked-out solutions to 15 percent of the problems found at the end of each chapter within the text. The Student Solutions Manual for the 10th edition is written using an innovative approach called TEAL which stands for Think, Express, Analyze, and Learn. This learning strategy was originally developed at the Massachusetts Institute of Technology and has proven to be an effective learning tool for students. These problems with TEAL solutions are indicated with an SSM icon in the text.

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ACKNOWLEDGMENTS

A great many people have contributed to this book. Sen-Ben Liao of Lawrence Livermore National Laboratory, James Whitenton of Southern Polytechnic State University, and Jerry Shi, of Pasadena City College, performed the Herculean task of working out solutions for every one of the homework problems in the book. At John Wiley publishers, the book received support from Stuart Johnson, Geraldine Osnato and Aly Rentrop, the editors who oversaw the entire project from start to finish. We thank Elizabeth Swain, the production editor, for pulling all the pieces together during the complex production process. We also thank Maddy Lesure for her design of the text and the cover; Lee Goldstein for her page make-up; Helen Walden for her copyediting; and Lilian Brady for her proofreading. Jennifer Atkins was inspired in the search for unusual and interesting photographs. Both the publisher John Wiley & Sons, Inc. and Jearl Walker would like to thank the following for comments and ideas about the recent editions:

Jonathan Abramson, *Portland State University*; Omar Adawi, *Parkland College*; Edward Adelson, *The Ohio State University*; Steven R. Baker, *Naval Postgraduate School*; George Caplan, *Wellesley College*; Richard Kass, *The Ohio State University*; M. R. Khoshbin-e-Khoshnazar, *Research Institution for Curriculum Development & Educational Innovations (Tehran)*; Craig Kletzing, *University of Iowa*; Stuart Loucks, *American River College*; Laurence Lurio, *Northern Illinois University*; Ponn Maheswaranathan, *Winthrop University*; Joe McCullough, *Cabrillo College*; Carl E. Mungan, *U. S. Naval Academy*; Don N. Page, *University of Alberta*; Elie Riachi, *Fort Scott Community College*; Andrew G. Rinzler, *University of Florida*; Dubravka Rupnik, *Louisiana State University*; Robert Schabinger, *Rutgers University*; Ruth Schwartz, *Milwaukee School of Engineering*; Carol Strong, *University of Alabama at Huntsville*; Nora Thornber, *Raritan Valley Community College*; Frank Wang, *LaGuardia Community College*; Graham W. Wilson, *University of Kansas*; Roland Winkler, *Northern Illinois University*; William Zacharias, *Cleveland State University*; Ulrich Zurcher, *Cleveland State University*.

Finally, our external reviewers have been outstanding and we acknowledge here our debt to each member of that team.

Maris A. Abolins, *Michigan State University*
Edward Adelson, *Ohio State University*
Nural Akchurin, *Texas Tech*
Yildirim Aktas, *University of North Carolina-Charlotte*
Barbara Andereck, *Ohio Wesleyan University*
Tetyana Antimirova, *Ryerson University*
Mark Arnett, *Kirkwood Community College*
Arun Bansil, *Northeastern University*
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Dan Styer, *Oberlin College*
Frank Wang, *LaGuardia Community College*
Robert Webb, *Texas A&M University*
Suzanne Willis, *Northern Illinois University*
Shannon Willoughby, *Montana State University*

C H A P T E R 1

Measurement

1-1 MEASURING THINGS, INCLUDING LENGTHS

Learning Objectives

After reading this module, you should be able to . . .

- 1.01 Identify the base quantities in the SI system.
- 1.02 Name the most frequently used prefixes for SI units.

- 1.03 Change units (here for length, area, and volume) by using chain-link conversions.

- 1.04 Explain that the meter is defined in terms of the speed of light in vacuum.

Key Ideas

- Physics is based on measurement of physical quantities. Certain physical quantities have been chosen as base quantities (such as length, time, and mass); each has been defined in terms of a standard and given a unit of measure (such as meter, second, and kilogram). Other physical quantities are defined in terms of the base quantities and their standards and units.
- The unit system emphasized in this book is the International System of Units (SI). The three physical quantities displayed in Table 1-1 are used in the early chapters. Standards, which must be both accessible and invariable, have been established for these base quantities by international agreement.

These standards are used in all physical measurement, for both the base quantities and the quantities derived from them. Scientific notation and the prefixes of Table 1-2 are used to simplify measurement notation.

- Conversion of units may be performed by using chain-link conversions in which the original data are multiplied successively by conversion factors written as unity and the units are manipulated like algebraic quantities until only the desired units remain.
- The meter is defined as the distance traveled by light during a precisely specified time interval.

What Is Physics?

Science and engineering are based on measurements and comparisons. Thus, we need rules about how things are measured and compared, and we need experiments to establish the units for those measurements and comparisons. One purpose of physics (and engineering) is to design and conduct those experiments.

For example, physicists strive to develop clocks of extreme accuracy so that any time or time interval can be precisely determined and compared. You may wonder whether such accuracy is actually needed or worth the effort. Here is one example of the worth: Without clocks of extreme accuracy, the Global Positioning System (GPS) that is now vital to worldwide navigation would be useless.

Measuring Things

We discover physics by learning how to measure the quantities involved in physics. Among these quantities are length, time, mass, temperature, pressure, and electric current.

We measure each physical quantity in its own units, by comparison with a **standard**. The **unit** is a unique name we assign to measures of that quantity—for example, meter (m) for the quantity length. The standard corresponds to exactly 1.0 unit of the quantity. As you will see, the standard for length, which corresponds

to exactly 1.0 m, is the distance traveled by light in a vacuum during a certain fraction of a second. We can define a unit and its standard in any way we care to. However, the important thing is to do so in such a way that scientists around the world will agree that our definitions are both sensible and practical.

Once we have set up a standard—say, for length—we must work out procedures by which any length whatever, be it the radius of a hydrogen atom, the wheelbase of a skateboard, or the distance to a star, can be expressed in terms of the standard. Rulers, which approximate our length standard, give us one such procedure for measuring length. However, many of our comparisons must be indirect. You cannot use a ruler, for example, to measure the radius of an atom or the distance to a star.

Base Quantities. There are so many physical quantities that it is a problem to organize them. Fortunately, they are not all independent; for example, speed is the ratio of a length to a time. Thus, what we do is pick out—by international agreement—a small number of physical quantities, such as length and time, and assign standards to them alone. We then define all other physical quantities in terms of these *base quantities* and their standards (called *base standards*). Speed, for example, is defined in terms of the base quantities length and time and their base standards.

Base standards must be both accessible and invariable. If we define the length standard as the distance between one's nose and the index finger on an outstretched arm, we certainly have an accessible standard—but it will, of course, vary from person to person. The demand for precision in science and engineering pushes us to aim first for invariability. We then exert great effort to make duplicates of the base standards that are accessible to those who need them.

Table 1-1 Units for Three SI Base Quantities

Quantity	Unit Name	Unit Symbol
Length	meter	m
Time	second	s
Mass	kilogram	kg

Table 1-2 Prefixes for SI Units

Factor	Prefix ^a	Symbol
10 ²⁴	yotta-	Y
10 ²¹	zetta-	Z
10 ¹⁸	exa-	E
10 ¹⁵	peta-	P
10 ¹²	tera-	T
10⁹	giga-	G
10⁶	mega-	M
10³	kilo-	k
10 ²	hecto-	h
10 ¹	deka-	da
10 ⁻¹	deci-	d
10⁻²	centi-	c
10⁻³	milli-	m
10⁻⁶	micro-	μ
10⁻⁹	nano-	n
10⁻¹²	pico-	p
10 ⁻¹⁵	femto-	f
10 ⁻¹⁸	atto-	a
10 ⁻²¹	zepto-	z
10 ⁻²⁴	yocto-	y

^aThe most frequently used prefixes are shown in bold type.

The International System of Units

In 1971, the 14th General Conference on Weights and Measures picked seven quantities as base quantities, thereby forming the basis of the International System of Units, abbreviated SI from its French name and popularly known as the *metric system*. Table 1-1 shows the units for the three base quantities—length, mass, and time—that we use in the early chapters of this book. These units were defined to be on a “human scale.”

Many SI *derived units* are defined in terms of these base units. For example, the SI unit for power, called the **watt** (W), is defined in terms of the base units for mass, length, and time. Thus, as you will see in Chapter 7,

$$1 \text{ watt} = 1 \text{ W} = 1 \text{ kg} \cdot \text{m}^2/\text{s}^3, \quad (1-1)$$

where the last collection of unit symbols is read as kilogram-meter squared per second cubed.

To express the very large and very small quantities we often run into in physics, we use *scientific notation*, which employs powers of 10. In this notation,

$$3\,560\,000\,000 \text{ m} = 3.56 \times 10^9 \text{ m} \quad (1-2)$$

$$\text{and} \quad 0.000\,000\,492 \text{ s} = 4.92 \times 10^{-7} \text{ s}. \quad (1-3)$$

Scientific notation on computers sometimes takes on an even briefer look, as in 3.56 E9 and 4.92 E-7, where E stands for “exponent of ten.” It is briefer still on some calculators, where E is replaced with an empty space.

As a further convenience when dealing with very large or very small measurements, we use the prefixes listed in Table 1-2. As you can see, each prefix represents a certain power of 10, to be used as a multiplication factor. Attaching a prefix to an SI unit has the effect of multiplying by the associated factor. Thus, we can express a particular electric power as

$$1.27 \times 10^9 \text{ watts} = 1.27 \text{ gigawatts} = 1.27 \text{ GW} \quad (1-4)$$

or a particular time interval as

$$2.35 \times 10^{-9} \text{ s} = 2.35 \text{ nanoseconds} = 2.35 \text{ ns.} \quad (1-5)$$

Some prefixes, as used in milliliter, centimeter, kilogram, and megabyte, are probably familiar to you.

Changing Units

We often need to change the units in which a physical quantity is expressed. We do so by a method called *chain-link conversion*. In this method, we multiply the original measurement by a **conversion factor** (a ratio of units that is equal to unity). For example, because 1 min and 60 s are identical time intervals, we have

$$\frac{1 \text{ min}}{60 \text{ s}} = 1 \quad \text{and} \quad \frac{60 \text{ s}}{1 \text{ min}} = 1.$$

Thus, the ratios (1 min)/(60 s) and (60 s)/(1 min) can be used as conversion factors. This is *not* the same as writing $\frac{1}{60} = 1$ or $60 = 1$; each *number* and its *unit* must be treated together.

Because multiplying any quantity by unity leaves the quantity unchanged, we can introduce conversion factors wherever we find them useful. In chain-link conversion, we use the factors to cancel unwanted units. For example, to convert 2 min to seconds, we have

$$2 \text{ min} = (2 \text{ min})(1) = (2 \text{ min})\left(\frac{60 \text{ s}}{1 \text{ min}}\right) = 120 \text{ s.} \quad (1-6)$$

If you introduce a conversion factor in such a way that unwanted units do *not* cancel, invert the factor and try again. In conversions, the units obey the same algebraic rules as variables and numbers.

Appendix D gives conversion factors between SI and other systems of units, including non-SI units still used in the United States. However, the conversion factors are written in the style of “1 min = 60 s” rather than as a ratio. So, you need to decide on the numerator and denominator in any needed ratio.

Length

In 1792, the newborn Republic of France established a new system of weights and measures. Its cornerstone was the meter, defined to be one ten-millionth of the distance from the north pole to the equator. Later, for practical reasons, this Earth standard was abandoned and the meter came to be defined as the distance between two fine lines engraved near the ends of a platinum–iridium bar, the **standard meter bar**, which was kept at the International Bureau of Weights and Measures near Paris. Accurate copies of the bar were sent to standardizing laboratories throughout the world. These **secondary standards** were used to produce other, still more accessible standards, so that ultimately every measuring device derived its authority from the standard meter bar through a complicated chain of comparisons.

Eventually, a standard more precise than the distance between two fine scratches on a metal bar was required. In 1960, a new standard for the meter, based on the wavelength of light, was adopted. Specifically, the standard for the meter was redefined to be 1 650 763.73 wavelengths of a particular orange-red light emitted by atoms of krypton-86 (a particular isotope, or type, of krypton) in a gas discharge tube that can be set up anywhere in the world. This awkward number of wavelengths was chosen so that the new standard would be close to the old meter-bar standard.

By 1983, however, the demand for higher precision had reached such a point that even the krypton-86 standard could not meet it, and in that year a bold step was taken. The meter was redefined as the distance traveled by light in a specified time interval. In the words of the 17th General Conference on Weights and Measures:



The meter is the length of the path traveled by light in a vacuum during a time interval of $1/299\,792\,458$ of a second.

This time interval was chosen so that the speed of light c is exactly

$$c = 299\,792\,458 \text{ m/s.}$$

Measurements of the speed of light had become extremely precise, so it made sense to adopt the speed of light as a defined quantity and to use it to redefine the meter.

Table 1-3 shows a wide range of lengths, from that of the universe (top line) to those of some very small objects.

Table 1-3 Some Approximate Lengths

Measurement	Length in Meters
Distance to the first galaxies formed	2×10^{26}
Distance to the Andromeda galaxy	2×10^{22}
Distance to the nearby star Proxima Centauri	4×10^{16}
Distance to Pluto	6×10^{12}
Radius of Earth	6×10^6
Height of Mt. Everest	9×10^3
Thickness of this page	1×10^{-4}
Length of a typical virus	1×10^{-8}
Radius of a hydrogen atom	5×10^{-11}
Radius of a proton	1×10^{-15}

Significant Figures and Decimal Places

Suppose that you work out a problem in which each value consists of two digits. Those digits are called **significant figures** and they set the number of digits that you can use in reporting your final answer. With data given in two significant figures, your final answer should have only two significant figures. However, depending on the mode setting of your calculator, many more digits might be displayed. Those extra digits are meaningless.

In this book, final results of calculations are often rounded to match the least number of significant figures in the given data. (However, sometimes an extra significant figure is kept.) When the leftmost of the digits to be discarded is 5 or more, the last remaining digit is rounded up; otherwise it is retained as is. For example, 11.3516 is rounded to three significant figures as 11.4 and 11.3279 is rounded to three significant figures as 11.3. (The answers to sample problems in this book are usually presented with the symbol $=$ instead of \approx even if rounding is involved.)

When a number such as 3.15 or 3.15×10^3 is provided in a problem, the number of significant figures is apparent, but how about the number 3000? Is it known to only one significant figure (3×10^3)? Or is it known to as many as four significant figures (3.000×10^3)? In this book, we assume that all the zeros in such given numbers as 3000 are significant, but you had better not make that assumption elsewhere.

Don't confuse *significant figures* with *decimal places*. Consider the lengths 35.6 mm, 3.56 m, and 0.00356 m. They all have three significant figures but they have one, two, and five decimal places, respectively.

Sample Problem 1.01 Estimating order of magnitude, ball of string

The world's largest ball of string is about 2 m in radius. To the nearest order of magnitude, what is the total length L of the string in the ball?

KEY IDEA

We could, of course, take the ball apart and measure the total length L , but that would take great effort and make the

ball's builder most unhappy. Instead, because we want only the nearest order of magnitude, we can estimate any quantities required in the calculation.

Calculations: Let us assume the ball is spherical with radius $R = 2$ m. The string in the ball is not closely packed (there are uncountable gaps between adjacent sections of string). To allow for these gaps, let us somewhat overestimate

the cross-sectional area of the string by assuming the cross section is square, with an edge length $d = 4$ mm. Then, with a cross-sectional area of d^2 and a length L , the string occupies a total volume of

$$V = (\text{cross-sectional area})(\text{length}) = d^2 L.$$

This is approximately equal to the volume of the ball, given by $\frac{4}{3}\pi R^3$, which is about $4R^3$ because π is about 3. Thus, we have the following

$$\begin{aligned} d^2 L &= 4R^3, \\ \text{or } L &= \frac{4R^3}{d^2} = \frac{4(2 \text{ m})^3}{(4 \times 10^{-3} \text{ m})^2} \\ &= 2 \times 10^6 \text{ m} \approx 10^6 \text{ m} = 10^3 \text{ km.} \end{aligned} \quad (\text{Answer})$$

(Note that you do not need a calculator for such a simplified calculation.) To the nearest order of magnitude, the ball contains about 1000 km of string!



Additional examples, video, and practice available at WileyPLUS



1-2 TIME

Learning Objectives

After reading this module, you should be able to . . .

1.05 Change units for time by using chain-link conversions.

1.06 Use various measures of time, such as for motion or as determined on different clocks.

Key Idea

● The second is defined in terms of the oscillations of light emitted by an atomic (cesium-133) source. Accurate time

signals are sent worldwide by radio signals keyed to atomic clocks in standardizing laboratories.

Time

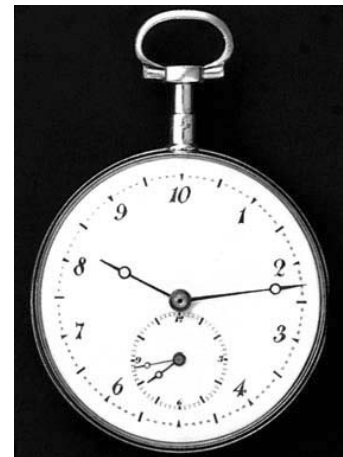
Time has two aspects. For civil and some scientific purposes, we want to know the time of day so that we can order events in sequence. In much scientific work, we want to know how long an event lasts. Thus, any time standard must be able to answer two questions: “When did it happen?” and “What is its *duration*?” Table 1-4 shows some time intervals.

Any phenomenon that repeats itself is a possible time standard. Earth’s rotation, which determines the length of the day, has been used in this way for centuries; Fig. 1-1 shows one novel example of a watch based on that rotation. A quartz clock, in which a quartz ring is made to vibrate continuously, can be calibrated against Earth’s rotation via astronomical observations and used to measure time intervals in the laboratory. However, the calibration cannot be carried out with the accuracy called for by modern scientific and engineering technology.

Table 1-4 Some Approximate Time Intervals

Measurement	Time Interval in Seconds	Measurement	Time Interval in Seconds
Lifetime of the proton (predicted)	3×10^{40}	Time between human heartbeats	8×10^{-1}
Age of the universe	5×10^{17}	Lifetime of the muon	2×10^{-6}
Age of the pyramid of Cheops	1×10^{11}	Shortest lab light pulse	1×10^{-16}
Human life expectancy	2×10^9	Lifetime of the most unstable particle	1×10^{-23}
Length of a day	9×10^4	The Planck time ^a	1×10^{-43}

^aThis is the earliest time after the big bang at which the laws of physics as we know them can be applied.



Steven Pitkin

Figure 1-1 When the metric system was proposed in 1792, the hour was redefined to provide a 10-hour day. The idea did not catch on. The maker of this 10-hour watch wisely provided a small dial that kept conventional 12-hour time. Do the two dials indicate the same time?

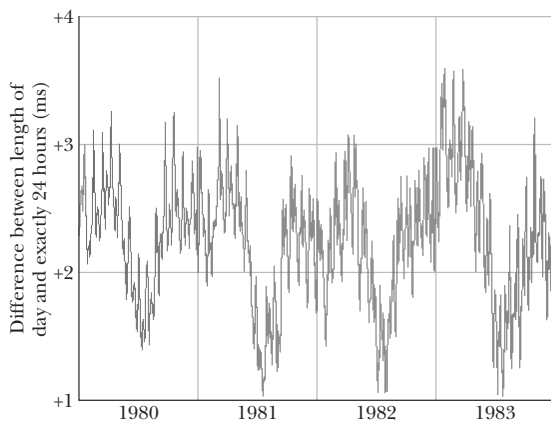


Figure 1-2 Variations in the length of the day over a 4-year period. Note that the entire vertical scale amounts to only 3 ms (= 0.003 s).

To meet the need for a better time standard, atomic clocks have been developed. An atomic clock at the National Institute of Standards and Technology (NIST) in Boulder, Colorado, is the standard for Coordinated Universal Time (UTC) in the United States. Its time signals are available by shortwave radio (stations WWV and WWVH) and by telephone (303-499-7111). Time signals (and related information) are also available from the United States Naval Observatory at website <http://tycho.usno.navy.mil/time.html>. (To set a clock extremely accurately at your particular location, you would have to account for the travel time required for these signals to reach you.)

Figure 1-2 shows variations in the length of one day on Earth over a 4-year period, as determined by comparison with a cesium (atomic) clock. Because the variation displayed by Fig. 1-2 is seasonal and repetitious, we suspect the rotating Earth when there is a difference between Earth and atom as timekeepers. The variation is due to tidal effects caused by the Moon and to large-scale winds.

The 13th General Conference on Weights and Measures in 1967 adopted a standard second based on the cesium clock:



One second is the time taken by 9 192 631 770 oscillations of the light (of a specified wavelength) emitted by a cesium-133 atom.

Atomic clocks are so consistent that, in principle, two cesium clocks would have to run for 6000 years before their readings would differ by more than 1 s. Even such accuracy pales in comparison with that of clocks currently being developed; their precision may be 1 part in 10^{18} —that is, 1 s in 1×10^{18} s (which is about 3×10^{10} y).

1-3 MASS

Learning Objectives

After reading this module, you should be able to . . .

1.07 Change units for mass by using chain-link conversions.

1.08 Relate density to mass and volume when the mass is uniformly distributed.

Key Ideas

● The kilogram is defined in terms of a platinum–iridium standard mass kept near Paris. For measurements on an atomic scale, the atomic mass unit, defined in terms of the atom carbon-12, is usually used.

● The density ρ of a material is the mass per unit volume:

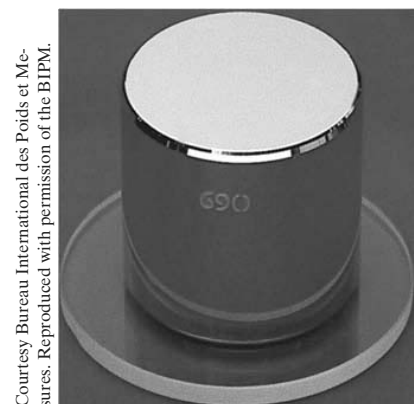
$$\rho = \frac{m}{V}.$$

Mass

The Standard Kilogram

The SI standard of mass is a cylinder of platinum and iridium (Fig. 1-3) that is kept at the International Bureau of Weights and Measures near Paris and assigned, by

Figure 1-3 The international 1 kg standard of mass, a platinum–iridium cylinder 3.9 cm in height and in diameter.



international agreement, a mass of 1 kilogram. Accurate copies have been sent to standardizing laboratories in other countries, and the masses of other bodies can be determined by balancing them against a copy. Table 1-5 shows some masses expressed in kilograms, ranging over about 83 orders of magnitude.

The U.S. copy of the standard kilogram is housed in a vault at NIST. It is removed, no more than once a year, for the purpose of checking duplicate copies that are used elsewhere. Since 1889, it has been taken to France twice for recomparison with the primary standard.

A Second Mass Standard

The masses of atoms can be compared with one another more precisely than they can be compared with the standard kilogram. For this reason, we have a second mass standard. It is the carbon-12 atom, which, by international agreement, has been assigned a mass of 12 **atomic mass units** (u). The relation between the two units is

$$1 \text{ u} = 1.660\,538\,86 \times 10^{-27} \text{ kg}, \quad (1-7)$$

with an uncertainty of ± 10 in the last two decimal places. Scientists can, with reasonable precision, experimentally determine the masses of other atoms relative to the mass of carbon-12. What we presently lack is a reliable means of extending that precision to more common units of mass, such as a kilogram.

Density

As we shall discuss further in Chapter 14, **density** ρ (lowercase Greek letter rho) is the mass per unit volume:

$$\rho = \frac{m}{V}. \quad (1-8)$$

Densities are typically listed in kilograms per cubic meter or grams per cubic centimeter. The density of water (1.00 gram per cubic centimeter) is often used as a comparison. Fresh snow has about 10% of that density; platinum has a density that is about 21 times that of water.


Table 1-5 Some Approximate Masses

Object	Mass in Kilograms
Known universe	1×10^{53}
Our galaxy	2×10^{41}
Sun	2×10^{30}
Moon	7×10^{22}
Asteroid Eros	5×10^{15}
Small mountain	1×10^{12}
Ocean liner	7×10^7
Elephant	5×10^3
Grape	3×10^{-3}
Speck of dust	7×10^{-10}
Penicillin molecule	5×10^{-17}
Uranium atom	4×10^{-25}
Proton	2×10^{-27}
Electron	9×10^{-31}

Sample Problem 1.02 Density and liquefaction

A heavy object can sink into the ground during an earthquake if the shaking causes the ground to undergo *liquefaction*, in which the soil grains experience little friction as they slide over one another. The ground is then effectively quicksand. The possibility of liquefaction in sandy ground can be predicted in terms of the *void ratio* e for a sample of the ground:

$$e = \frac{V_{\text{voids}}}{V_{\text{grains}}}. \quad (1-9)$$

Here, V_{grains} is the total volume of the sand grains in the sample and V_{voids} is the total volume between the grains (in the *voids*). If e exceeds a critical value of 0.80, liquefaction can occur during an earthquake. What is the corresponding sand density ρ_{sand} ? Solid silicon dioxide (the primary component of sand) has a density of $\rho_{\text{SiO}_2} = 2.600 \times 10^3 \text{ kg/m}^3$. 

KEY IDEA

The density of the sand ρ_{sand} in a sample is the mass per unit volume—that is, the ratio of the total mass m_{sand} of the sand grains to the total volume V_{total} of the sample:

$$\rho_{\text{sand}} = \frac{m_{\text{sand}}}{V_{\text{total}}}. \quad (1-10)$$

Calculations: The total volume V_{total} of a sample is

$$V_{\text{total}} = V_{\text{grains}} + V_{\text{voids}}.$$

Substituting for V_{voids} from Eq. 1-9 and solving for V_{grains} lead to

$$V_{\text{grains}} = \frac{V_{\text{total}}}{1 + e}. \quad (1-11)$$



From Eq. 1-8, the total mass m_{sand} of the sand grains is the product of the density of silicon dioxide and the total volume of the sand grains:

$$m_{\text{sand}} = \rho_{\text{SiO}_2} V_{\text{grains}} \quad (1-12)$$

Substituting this expression into Eq. 1-10 and then substituting for V_{grains} from Eq. 1-11 lead to

$$\rho_{\text{sand}} = \frac{\rho_{\text{SiO}_2}}{V_{\text{total}}} \frac{V_{\text{total}}}{1+e} = \frac{\rho_{\text{SiO}_2}}{1+e}. \quad (1-13)$$

Substituting $\rho_{\text{SiO}_2} = 2.600 \times 10^3 \text{ kg/m}^3$ and the critical value of $e = 0.80$, we find that liquefaction occurs when the sand density is less than

$$\rho_{\text{sand}} = \frac{2.600 \times 10^3 \text{ kg/m}^3}{1.80} = 1.4 \times 10^3 \text{ kg/m}^3. \quad (\text{Answer})$$

A building can sink several meters in such liquefaction.



Additional examples, video, and practice available at WileyPLUS

Review & Summary

Measurement in Physics Physics is based on measurement of physical quantities. Certain physical quantities have been chosen as **base quantities** (such as length, time, and mass); each has been defined in terms of a **standard** and given a **unit** of measure (such as meter, second, and kilogram). Other physical quantities are defined in terms of the base quantities and their standards and units.

SI Units The unit system emphasized in this book is the International System of Units (SI). The three physical quantities displayed in Table 1-1 are used in the early chapters. Standards, which must be both accessible and invariable, have been established for these base quantities by international agreement. These standards are used in all physical measurement, for both the base quantities and the quantities derived from them. Scientific notation and the prefixes of Table 1-2 are used to simplify measurement notation.

Changing Units Conversion of units may be performed by using *chain-link conversions* in which the original data are multiplied

successively by conversion factors written as unity and the units are manipulated like algebraic quantities until only the desired units remain.

Length The meter is defined as the distance traveled by light during a precisely specified time interval.

Time The second is defined in terms of the oscillations of light emitted by an atomic (cesium-133) source. Accurate time signals are sent worldwide by radio signals keyed to atomic clocks in standardizing laboratories.

Mass The kilogram is defined in terms of a platinum-iridium standard mass kept near Paris. For measurements on an atomic scale, the atomic mass unit, defined in terms of the atom carbon-12, is usually used.

Density The density ρ of a material is the mass per unit volume:

$$\rho = \frac{m}{V}. \quad (1-8)$$

Problems



Tutoring problem available (at instructor's discretion) in WileyPLUS and WebAssign



Worked-out solution available in Student Solutions Manual

WWW Worked-out solution is at



Number of dots indicates level of problem difficulty

ILW Interactive solution is at



Additional information available in *The Flying Circus of Physics* and at flyingcircusofphysics.com

<http://www.wiley.com/college/halliday>

Module 1-1 Measuring Things, Including Lengths

•1 **SSM** Earth is approximately a sphere of radius $6.37 \times 10^6 \text{ m}$. What are (a) its circumference in kilometers, (b) its surface area in square kilometers, and (c) its volume in cubic kilometers?

•2 A *gry* is an old English measure for length, defined as 1/10 of a line, where *line* is another old English measure for length, defined as 1/12 inch. A common measure for length in the publishing business is a *point*, defined as 1/72 inch. What is an area of 0.50 gry² in points squared (points²)?

•3 The micrometer (1 μm) is often called the *micron*. (a) How

many microns make up 1.0 km? (b) What fraction of a centimeter equals 1.0 μm ? (c) How many microns are in 1.0 yd?

•4 Spacing in this book was generally done in units of points and picas: 12 points = 1 pica, and 6 picas = 1 inch. If a figure was misplaced in the page proofs by 0.80 cm, what was the misplacement in (a) picas and (b) points?

•5 **SSM** **WWW** Horses are to race over a certain English meadow for a distance of 4.0 furlongs. What is the race distance in (a) rods and (b) chains? (1 furlong = 201.168 m, 1 rod = 5.0292 m, and 1 chain = 20.117 m.)

••6 You can easily convert common units and measures electronically, but you still should be able to use a conversion table, such as those in Appendix D. Table 1-6 is part of a conversion table for a system of volume measures once common in Spain; a volume of 1 fanega is equivalent to 55.501 dm^3 (cubic decimeters). To complete the table, what numbers (to three significant figures) should be entered in (a) the cahiz column, (b) the fanega column, (c) the cuartilla column, and (d) the almude column, starting with the top blank? Express 7.00 almudes in (e) medios, (f) cahizes, and (g) cubic centimeters (cm^3).

Table 1-6 Problem 6

	cahiz	fanega	cuartilla	almude	medio
1 cahiz =	1	12	48	144	288
1 fanega =		1	4	12	24
1 cuartilla =			1	3	6
1 almude =				1	2
1 medio =					1

••7 **ILW** Hydraulic engineers in the United States often use, as a unit of volume of water, the *acre-foot*, defined as the volume of water that will cover 1 acre of land to a depth of 1 ft. A severe thunderstorm dumped 2.0 in. of rain in 30 min on a town of area 26 km^2 . What volume of water, in acre-feet, fell on the town?

••8 **GO** Harvard Bridge, which connects MIT with its fraternities across the Charles River, has a length of 364.4 Smoots plus one ear. The unit of one Smoot is based on the length of Oliver Reed Smoot, Jr., class of 1962, who was carried or dragged length by length across the bridge so that other pledge members of the Lambda Chi Alpha fraternity could mark off (with paint) 1-Smoot lengths along the bridge. The marks have been repainted biannually by fraternity pledges since the initial measurement, usually during times of traffic congestion so that the police cannot easily interfere. (Presumably, the police were originally upset because the Smoot is not an SI base unit, but these days they seem to have accepted the unit.) Figure 1-4 shows three parallel paths, measured in Smoots (S), Willies (W), and Zeldas (Z). What is the length of 50.0 Smoots in (a) Willies and (b) Zeldas?



Figure 1-4 Problem 8.

••9 Antarctica is roughly semicircular, with a radius of 2000 km (Fig. 1-5). The average thickness of its ice cover is 3000 m. How many cubic centimeters of ice does Antarctica contain? (Ignore the curvature of Earth.)

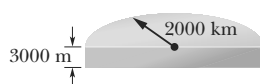


Figure 1-5 Problem 9.

Module 1-2 Time

•10 Until 1883, every city and town in the United States kept its own local time. Today, travelers reset their watches only when the time change equals 1.0 h. How far, on the average, must you travel in degrees of longitude between the time-zone boundaries at which your watch must be reset by 1.0 h? (*Hint*: Earth rotates 360° in about 24 h.)

•11 For about 10 years after the French Revolution, the French government attempted to base measures of time on multiples of ten: One week consisted of 10 days, one day consisted of 10 hours, one hour consisted of 100 minutes, and one minute consisted of 100 seconds. What are the ratios of (a) the French decimal week to the standard week and (b) the French decimal second to the standard second?

•12 The fastest growing plant on record is a *Hesperoyucca whipplei* that grew 3.7 m in 14 days. What was its growth rate in micrometers per second?

•13 **GO** Three digital clocks A, B, and C run at different rates and do not have simultaneous readings of zero. Figure 1-6 shows simultaneous readings on pairs of the clocks for four occasions. (At the earliest occasion, for example, B reads 25.0 s and C reads 92.0 s.) If two events are 600 s apart on clock A, how far apart are they on (a) clock B and (b) clock C? (c) When clock A reads 400 s, what does clock B read? (d) When clock C reads 15.0 s, what does clock B read? (Assume negative readings for prezero times.)

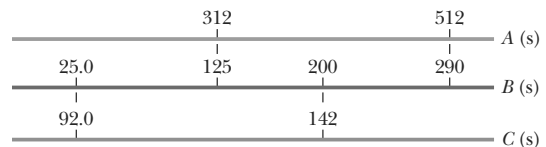


Figure 1-6 Problem 13.

•14 A lecture period (50 min) is close to 1 microcentury. (a) How long is a microcentury in minutes? (b) Using

$$\text{percentage difference} = \left(\frac{\text{actual} - \text{approximation}}{\text{actual}} \right) 100,$$

find the percentage difference from the approximation.

•15 A fortnight is a charming English measure of time equal to 2.0 weeks (the word is a contraction of “fourteen nights”). That is a nice amount of time in pleasant company but perhaps a painful string of microseconds in unpleasant company. How many microseconds are in a fortnight?

•16 Time standards are now based on atomic clocks. A promising second standard is based on *pulsars*, which are rotating neutron stars (highly compact stars consisting only of neutrons). Some rotate at a rate that is highly stable, sending out a radio beacon that sweeps briefly across Earth once with each rotation, like a lighthouse beacon. Pulsar PSR 1937 + 21 is an example; it rotates once every $1.557\,806\,448\,872\,75 \pm 3 \text{ ms}$, where the trailing ± 3 indicates the uncertainty in the last decimal place (it does *not* mean $\pm 3 \text{ ms}$). (a) How many rotations does PSR 1937 + 21 make in 7.00 days? (b) How much time does the pulsar take to rotate exactly one million times and (c) what is the associated uncertainty?

•17 **SSM** Five clocks are being tested in a laboratory. Exactly at noon, as determined by the WWV time signal, on successive days of a week the clocks read as in the following table. Rank the five clocks according to their relative value as good timekeepers, best to worst. Justify your choice.

Clock	Sun.	Mon.	Tues.	Wed.	Thurs.	Fri.	Sat.
A	12:36:40	12:36:56	12:37:12	12:37:27	12:37:44	12:37:59	12:38:14
B	11:59:59	12:00:02	11:59:57	12:00:07	12:00:02	11:59:56	12:00:03
C	15:50:45	15:51:43	15:52:41	15:53:39	15:54:37	15:55:35	15:56:33
D	12:03:59	12:02:52	12:01:45	12:00:38	11:59:31	11:58:24	11:57:17
E	12:03:59	12:02:49	12:01:54	12:01:52	12:01:32	12:01:22	12:01:12

•18 Because Earth's rotation is gradually slowing, the length of each day increases: The day at the end of 1.0 century is 1.0 ms longer than the day at the start of the century. In 20 centuries, what is the total of the daily increases in time?

•19 Suppose that, while lying on a beach near the equator watching the Sun set over a calm ocean, you start a stopwatch just as the top of the Sun disappears. You then stand, elevating your eyes by a height $H = 1.70$ m, and stop the watch when the top of the Sun again disappears. If the elapsed time is $t = 11.1$ s, what is the radius r of Earth?

Module 1-3 Mass

•20 **GO** The record for the largest glass bottle was set in 1992 by a team in Millville, New Jersey—they blew a bottle with a volume of 193 U.S. fluid gallons. (a) How much short of 1.0 million cubic centimeters is that? (b) If the bottle were filled with water at the leisurely rate of 1.8 g/min, how long would the filling take? Water has a density of 1000 kg/m^3 .

•21 Earth has a mass of $5.98 \times 10^{24} \text{ kg}$. The average mass of the atoms that make up Earth is 40 u. How many atoms are there in Earth?

•22 Gold, which has a density of 19.32 g/cm^3 , is the most ductile metal and can be pressed into a thin leaf or drawn out into a long fiber. (a) If a sample of gold, with a mass of 27.63 g, is pressed into a leaf of $1.000 \mu\text{m}$ thickness, what is the area of the leaf? (b) If, instead, the gold is drawn out into a cylindrical fiber of radius $2.500 \mu\text{m}$, what is the length of the fiber?

•23 **SSM** (a) Assuming that water has a density of exactly 1 g/cm^3 , find the mass of one cubic meter of water in kilograms. (b) Suppose that it takes 10.0 h to drain a container of 5700 m^3 of water. What is the “mass flow rate,” in kilograms per second, of water from the container?

•24 **GO** Grains of fine California beach sand are approximately spheres with an average radius of $50 \mu\text{m}$ and are made of silicon dioxide, which has a density of 2600 kg/m^3 . What mass of sand grains would have a total surface area (the total area of all the individual spheres) equal to the surface area of a cube 1.00 m on an edge?

•25 **GO** During heavy rain, a section of a mountainside measuring 2.5 km horizontally, 0.80 km up along the slope, and 2.0 m deep slips into a valley in a mud slide. Assume that the mud ends up uniformly distributed over a surface area of the valley measuring $0.40 \text{ km} \times 0.40 \text{ km}$ and that mud has a density of 1900 kg/m^3 . What is the mass of the mud sitting above a 4.0 m^2 area of the valley floor?

•26 One cubic centimeter of a typical cumulus cloud contains 50 to 500 water drops, which have a typical radius of $10 \mu\text{m}$. For

that range, give the lower value and the higher value, respectively, for the following. (a) How many cubic meters of water are in a cylindrical cumulus cloud of height 3.0 km and radius 1.0 km? (b) How many 1-liter pop bottles would that water fill? (c) Water has a density of 1000 kg/m^3 . How much mass does the water in the cloud have?

•27 Iron has a density of 7.87 g/cm^3 , and the mass of an iron atom is $9.27 \times 10^{-26} \text{ kg}$. If the atoms are spherical and tightly packed, (a) what is the volume of an iron atom and (b) what is the distance between the centers of adjacent atoms?

•28 A mole of atoms is 6.02×10^{23} atoms. To the nearest order of magnitude, how many moles of atoms are in a large domestic cat? The masses of a hydrogen atom, an oxygen atom, and a carbon atom are 1.0 u, 16 u, and 12 u, respectively. (Hint: Cats are sometimes known to kill a mole.)

•29 On a spending spree in Malaysia, you buy an ox with a weight of 28.9 piculs in the local unit of weights: 1 picul = 100 gins, 1 gin = 16 tahils, 1 tahlil = 10 chees, and 1 chee = 10 hoons. The weight of 1 hoon corresponds to a mass of 0.3779 g. When you arrange to ship the ox home to your astonished family, how much mass in kilograms must you declare on the shipping manifest? (Hint: Set up multiple chain-link conversions.)

•30 **GO** Water is poured into a container that has a small leak. The mass m of the water is given as a function of time t by $m = 5.00t^{0.8} - 3.00t + 20.00$, with $t \geq 0$, m in grams, and t in seconds. (a) At what time is the water mass greatest, and (b) what is that greatest mass? In kilograms per minute, what is the rate of mass change at (c) $t = 2.00$ s and (d) $t = 5.00$ s?

•31 A vertical container with base area measuring 14.0 cm by 17.0 cm is being filled with identical pieces of candy, each with a volume of 50.0 mm^3 and a mass of 0.0200 g . Assume that the volume of the empty spaces between the candies is negligible. If the height of the candies in the container increases at the rate of 0.250 cm/s , at what rate (kilograms per minute) does the mass of the candies in the container increase?

Additional Problems

32 In the United States, a doll house has the scale of 1:12 of a real house (that is, each length of the doll house is $\frac{1}{12}$ that of the real house) and a miniature house (a doll house to fit within a doll house) has the scale of 1:144 of a real house. Suppose a real house (Fig. 1-7) has a front length of 20 m, a depth of 12 m, a height of 6.0 m, and a standard sloped roof (vertical triangular faces on the ends) of height 3.0 m. In cubic meters, what are the volumes of the corresponding (a) doll house and (b) miniature house?

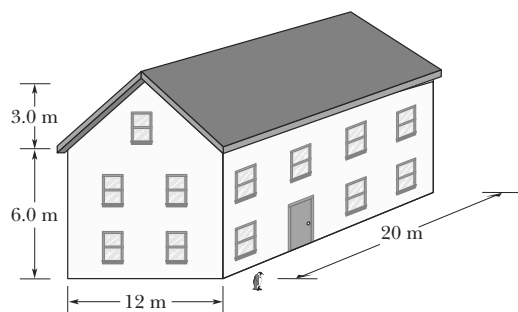


Figure 1-7 Problem 32.

33 SSM A ton is a measure of volume frequently used in shipping, but that use requires some care because there are at least three types of tons: A *displacement ton* is equal to 7 barrels bulk, a *freight ton* is equal to 8 barrels bulk, and a *register ton* is equal to 20 barrels bulk. A *barrel bulk* is another measure of volume: 1 barrel bulk = 0.1415 m^3 . Suppose you spot a shipping order for “73 tons” of M&M candies, and you are certain that the client who sent the order intended “ton” to refer to volume (instead of weight or mass, as discussed in Chapter 5). If the client actually meant displacement tons, how many extra U.S. bushels of the candies will you erroneously ship if you interpret the order as (a) 73 freight tons and (b) 73 register tons? ($1 \text{ m}^3 = 28.378 \text{ U.S. bushels}$.)

34 Two types of *barrel* units were in use in the 1920s in the United States. The apple barrel had a legally set volume of 7056 cubic inches; the cranberry barrel, 5826 cubic inches. If a merchant sells 20 cranberry barrels of goods to a customer who thinks he is receiving apple barrels, what is the discrepancy in the shipment volume in liters?

35 An old English children’s rhyme states, “Little Miss Muffet sat on a tuffet, eating her curds and whey, when along came a spider who sat down beside her. . . .” The spider sat down not because of the curds and whey but because Miss Muffet had a stash of 11 tuffets of dried flies. The volume measure of a tuffet is given by $1 \text{ tuffet} = 2 \text{ pecks} = 0.50 \text{ Imperial bushel}$, where $1 \text{ Imperial bushel} = 36.3687 \text{ liters (L)}$. What was Miss Muffet’s stash in (a) pecks, (b) Imperial bushels, and (c) liters?

36 Table 1-7 shows some old measures of liquid volume. To complete the table, what numbers (to three significant figures) should be entered in (a) the wey column, (b) the chaldron column, (c) the bag column, (d) the pottle column, and (e) the gill column, starting from the top down? (f) The volume of 1 bag is equal to 0.1091 m^3 . If an old story has a witch cooking up some vile liquid in a cauldron of volume 1.5 chaldrons, what is the volume in cubic meters?

Table 1-7 Problem 36

	wey	chaldron	bag	pottle	gill
1 wey =	1	10/9	40/3	640	120 240
1 chaldron =					
1 bag =					
1 pottle =					
1 gill =					

37 A typical sugar cube has an edge length of 1 cm. If you had a cubical box that contained a mole of sugar cubes, what would its edge length be? (One mole = 6.02×10^{23} units.)

38 An old manuscript reveals that a landowner in the time of King Arthur held 3.00 acres of plowed land plus a livestock area of 25.0 perches by 4.00 perches. What was the total area in (a) the old unit of roods and (b) the more modern unit of square meters? Here, 1 acre is an area of 40 perches by 4 perches, 1 rood is an area of 40 perches by 1 perch, and 1 perch is the length 16.5 ft.

39 SSM A tourist purchases a car in England and ships it home to the United States. The car sticker advertised that the car’s fuel consumption was at the rate of 40 miles per gallon on the open road.

The tourist does not realize that the U.K. gallon differs from the U.S. gallon:

$$\begin{aligned} 1 \text{ U.K. gallon} &= 4.546\,090\,0 \text{ liters} \\ 1 \text{ U.S. gallon} &= 3.785\,411\,8 \text{ liters} \end{aligned}$$

For a trip of 750 miles (in the United States), how many gallons of fuel does (a) the mistaken tourist believe she needs and (b) the car actually require?

40 Using conversions and data in the chapter, determine the number of hydrogen atoms required to obtain 1.0 kg of hydrogen. A hydrogen atom has a mass of 1.0 u.

41 SSM A *cord* is a volume of cut wood equal to a stack 8 ft long, 4 ft wide, and 4 ft high. How many cords are in 1.0 m^3 ?

42 One molecule of water (H_2O) contains two atoms of hydrogen and one atom of oxygen. A hydrogen atom has a mass of 1.0 u and an atom of oxygen has a mass of 16 u, approximately. (a) What is the mass in kilograms of one molecule of water? (b) How many molecules of water are in the world’s oceans, which have an estimated total mass of $1.4 \times 10^{21} \text{ kg}$?

43 A person on a diet might lose 2.3 kg per week. Express the mass loss rate in milligrams per second, as if the dieter could sense the second-by-second loss.

44 What mass of water fell on the town in Problem 7? Water has a density of $1.0 \times 10^3 \text{ kg/m}^3$.

45 (a) A unit of time sometimes used in microscopic physics is the *shake*. One shake equals 10^{-8} s . Are there more shakes in a second than there are seconds in a year? (b) Humans have existed for about 10^6 years, whereas the universe is about 10^{10} years old. If the age of the universe is defined as 1 “universe day,” where a universe day consists of “universe seconds” as a normal day consists of normal seconds, how many universe seconds have humans existed?

46 A unit of area often used in measuring land areas is the *hectare*, defined as 10^4 m^2 . An open-pit coal mine consumes 75 hectares of land, down to a depth of 26 m, each year. What volume of earth, in cubic kilometers, is removed in this time?

47 SSM An astronomical unit (AU) is the average distance between Earth and the Sun, approximately $1.50 \times 10^8 \text{ km}$. The speed of light is about $3.0 \times 10^8 \text{ m/s}$. Express the speed of light in astronomical units per minute.

48 The common Eastern mole, a mammal, typically has a mass of 75 g, which corresponds to about 7.5 moles of atoms. (A mole of atoms is 6.02×10^{23} atoms.) In atomic mass units (u), what is the average mass of the atoms in the common Eastern mole?

49 A traditional unit of length in Japan is the ken (1 ken = 1.97 m). What are the ratios of (a) square kens to square meters and (b) cubic kens to cubic meters? What is the volume of a cylindrical water tank of height 5.50 kens and radius 3.00 kens in (c) cubic kens and (d) cubic meters?

50 You receive orders to sail due east for 24.5 mi to put your salvage ship directly over a sunken pirate ship. However, when your divers probe the ocean floor at that location and find no evidence of a ship, you radio back to your source of information, only to discover that the sailing distance was supposed to be 24.5 *nautical miles*, not regular miles. Use the Length table in Appendix D to calculate how far horizontally you are from the pirate ship in kilometers.

51 The cubit is an ancient unit of length based on the distance between the elbow and the tip of the middle finger of the measurer. Assume that the distance ranged from 43 to 53 cm, and suppose that ancient drawings indicate that a cylindrical pillar was to have a length of 9 cubits and a diameter of 2 cubits. For the stated range, what are the lower value and the upper value, respectively, for (a) the cylinder's length in meters, (b) the cylinder's length in millimeters, and (c) the cylinder's volume in cubic meters?

52 As a contrast between the old and the modern and between the large and the small, consider the following: In old rural England 1 hide (between 100 and 120 acres) was the area of land needed to sustain one family with a single plough for one year. (An area of 1 acre is equal to 4047 m^2 .) Also, 1 wapentake was the area of land needed by 100 such families. In quantum physics, the cross-sectional area of a nucleus (defined in terms of the chance of a particle hitting and being absorbed by it) is measured in units of barns, where 1 barn is $1 \times 10^{-28} \text{ m}^2$. (In nuclear physics jargon, if a nucleus is "large," then shooting a particle at it is like shooting a bullet at a barn door, which can hardly be missed.) What is the ratio of 25 wapentakes to 11 barns?

53 SSM An *astronomical unit* (AU) is equal to the average distance from Earth to the Sun, about $92.9 \times 10^6 \text{ mi}$. A *parsec* (pc) is the distance at which a length of 1 AU would subtend an angle of exactly 1 second of arc (Fig. 1-8). A *light-year* (ly) is the distance that light, traveling through a vacuum with a speed of $186\,000 \text{ mi/s}$, would cover in 1.0 year. Express the Earth–Sun distance in (a) parsecs and (b) light-years.

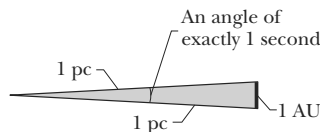


Figure 1-8 Problem 53.

54 The description for a certain brand of house paint claims a coverage of $460 \text{ ft}^2/\text{gal}$. (a) Express this quantity in square meters per liter. (b) Express this quantity in an SI unit (see Appendices A and D). (c) What is the inverse of the original quantity, and (d) what is its physical significance?

55 Strangely, the wine for a large wedding reception is to be served in a stunning cut-glass receptacle with the interior dimensions of $40 \text{ cm} \times 40 \text{ cm} \times 30 \text{ cm}$ (height). The receptacle is to be initially filled to the top. The wine can be purchased in bottles of the sizes given in the following table. Purchasing a larger bottle instead of multiple smaller bottles decreases the overall cost of the wine. To minimize the cost, (a) which bottle sizes should be purchased and how many of each should be purchased and, once the receptacle is filled, how much wine is left over in terms of (b) standard bottles and (c) liters?

1 standard bottle

1 magnum = 2 standard bottles

1 jeroboam = 4 standard bottles

1 rehoboam = 6 standard bottles

1 methuselah = 8 standard bottles

1 salmanazar = 12 standard bottles

1 balthazar = 16 standard bottles = 11.356 L

1 nebuchadnezzar = 20 standard bottles

56 The *corn–hog ratio* is a financial term used in the pig market and presumably is related to the cost of feeding a pig until it is large enough for market. It is defined as the ratio of the market price of a pig with a mass of 3.108 slugs to the market price of a U.S. bushel of corn. (The word “slug” is derived from an old German word that means “to hit”; we have the same meaning for “slug” as a verb in modern English.) A U.S. bushel is equal to 35.238 L . If the corn–hog ratio is listed as 5.7 on the market exchange, what is it in the metric units of

$$\frac{\text{price of 1 kilogram of pig}}{\text{price of 1 liter of corn}} ?$$

(Hint: See the Mass table in Appendix D.)

57 You are to fix dinners for 400 people at a convention of Mexican food fans. Your recipe calls for 2 jalapeño peppers per serving (one serving per person). However, you have only habanero peppers on hand. The spiciness of peppers is measured in terms of the *scoville heat unit* (SHU). On average, one jalapeño pepper has a spiciness of 4000 SHU and one habanero pepper has a spiciness of 300 000 SHU. To get the desired spiciness, how many habanero peppers should you substitute for the jalapeño peppers in the recipe for the 400 dinners?

58 A standard interior staircase has steps each with a rise (height) of 19 cm and a run (horizontal depth) of 23 cm. Research suggests that the stairs would be safer for descent if the run were, instead, 28 cm. For a particular staircase of total height 4.57 m, how much farther into the room would the staircase extend if this change in run were made?

59 In purchasing food for a political rally, you erroneously order shucked medium-size Pacific oysters (which come 8 to 12 per U.S. pint) instead of shucked medium-size Atlantic oysters (which come 26 to 38 per U.S. pint). The filled oyster container shipped to you has the interior measure of $1.0 \text{ m} \times 12 \text{ cm} \times 20 \text{ cm}$, and a U.S. pint is equivalent to 0.4732 liter. By how many oysters is the order short of your anticipated count?

60 An old English cookbook carries this recipe for cream of nettle soup: “Boil stock of the following amount: 1 breakfastcup plus 1 teacup plus 6 tablespoons plus 1 dessertspoon. Using gloves, separate nettle tops until you have 0.5 quart; add the tops to the boiling stock. Add 1 tablespoon of cooked rice and 1 saltspoon of salt. Simmer for 15 min.” The following table gives some of the conversions among old (premetric) British measures and among common (still premetric) U.S. measures. (These measures just scream for metrication.) For liquid measures, 1 British teaspoon = 1 U.S. teaspoon. For dry measures, 1 British teaspoon = 2 U.S. teaspoons and 1 British quart = 1 U.S. quart. In U.S. measures, how much (a) stock, (b) nettle tops, (c) rice, and (d) salt are required in the recipe?

Old British Measures	U.S. Measures
teaspoon = 2 saltspoons	tablespoon = 3 teaspoons
dessertspoon = 2 teaspoons	half cup = 8 tablespoons
tablespoon = 2 dessertspoons	cup = 2 half cups
teacup = 8 tablespoons	
breakfastcup = 2 teacups	

CHAPTER 2

Motion Along a Straight Line

2-1 POSITION, DISPLACEMENT, AND AVERAGE VELOCITY

Learning Objectives

After reading this module, you should be able to ...

- 2.01** Identify that if all parts of an object move in the same direction and at the same rate, we can treat the object as if it were a (point-like) particle. (This chapter is about the motion of such objects.)
- 2.02** Identify that the position of a particle is its location as read on a scaled axis, such as an x axis.
- 2.03** Apply the relationship between a particle's displacement and its initial and final positions.
- 2.04** Apply the relationship between a particle's average velocity, its displacement, and the time interval for that displacement.
- 2.05** Apply the relationship between a particle's average speed, the total distance it moves, and the time interval for the motion.
- 2.06** Given a graph of a particle's position versus time, determine the average velocity between any two particular times.

Key Ideas

- The position x of a particle on an x axis locates the particle with respect to the origin, or zero point, of the axis.
- The position is either positive or negative, according to which side of the origin the particle is on, or zero if the particle is at the origin. The positive direction on an axis is the direction of increasing positive numbers; the opposite direction is the negative direction on the axis.
- The displacement Δx of a particle is the change in its position:

$$\Delta x = x_2 - x_1.$$

- Displacement is a vector quantity. It is positive if the particle has moved in the positive direction of the x axis and negative if the particle has moved in the negative direction.

- When a particle has moved from position x_1 to position x_2 during a time interval $\Delta t = t_2 - t_1$, its average velocity during that interval is

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}.$$

- The algebraic sign of v_{avg} indicates the direction of motion (v_{avg} is a vector quantity). Average velocity does not depend on the actual distance a particle moves, but instead depends on its original and final positions.
- On a graph of x versus t , the average velocity for a time interval Δt is the slope of the straight line connecting the points on the curve that represent the two ends of the interval.
- The average speed s_{avg} of a particle during a time interval Δt depends on the total distance the particle moves in that time interval:

$$s_{\text{avg}} = \frac{\text{total distance}}{\Delta t}.$$

What Is Physics?

One purpose of physics is to study the motion of objects—how fast they move, for example, and how far they move in a given amount of time. NASCAR engineers are fanatical about this aspect of physics as they determine the performance of their cars before and during a race. Geologists use this physics to measure tectonic-plate motion as they attempt to predict earthquakes. Medical researchers need this physics to map the blood flow through a patient when diagnosing a partially closed artery, and motorists use it to determine how they might slow sufficiently when their radar detector sounds a warning. There are countless other examples. In this chapter, we study the basic physics of motion where the object (race car, tectonic plate, blood cell, or any other object) moves along a single axis. Such motion is called *one-dimensional motion*.

Motion

The world, and everything in it, moves. Even seemingly stationary things, such as a roadway, move with Earth's rotation, Earth's orbit around the Sun, the Sun's orbit around the center of the Milky Way galaxy, and that galaxy's migration relative to other galaxies. The classification and comparison of motions (called **kinematics**) is often challenging. What exactly do you measure, and how do you compare?

Before we attempt an answer, we shall examine some general properties of motion that is restricted in three ways.

1. The motion is along a straight line only. The line may be vertical, horizontal, or slanted, but it must be straight.
2. Forces (pushes and pulls) cause motion but will not be discussed until Chapter 5. In this chapter we discuss only the motion itself and changes in the motion. Does the moving object speed up, slow down, stop, or reverse direction? If the motion does change, how is time involved in the change?
3. The moving object is either a **particle** (by which we mean a point-like object such as an electron) or an object that moves like a particle (such that every portion moves in the same direction and at the same rate). A stiff pig slipping down a straight playground slide might be considered to be moving like a particle; however, a tumbling tumbleweed would not.

Position and Displacement

To locate an object means to find its position relative to some reference point, often the **origin** (or zero point) of an axis such as the x axis in Fig. 2-1. The **positive direction** of the axis is in the direction of increasing numbers (coordinates), which is to the right in Fig. 2-1. The opposite is the **negative direction**.

For example, a particle might be located at $x = 5$ m, which means it is 5 m in the positive direction from the origin. If it were at $x = -5$ m, it would be just as far from the origin but in the opposite direction. On the axis, a coordinate of -5 m is less than a coordinate of -1 m, and both coordinates are less than a coordinate of $+5$ m. A plus sign for a coordinate need not be shown, but a minus sign must always be shown.

A change from position x_1 to position x_2 is called a **displacement** Δx , where

$$\Delta x = x_2 - x_1. \quad (2-1)$$

(The symbol Δ , the Greek uppercase delta, represents a change in a quantity, and it means the final value of that quantity minus the initial value.) When numbers are inserted for the position values x_1 and x_2 in Eq. 2-1, a displacement in the positive direction (to the right in Fig. 2-1) always comes out positive, and a displacement in the opposite direction (left in the figure) always comes out negative. For example, if the particle moves from $x_1 = 5$ m to $x_2 = 12$ m, then the displacement is $\Delta x = (12 \text{ m}) - (5 \text{ m}) = +7 \text{ m}$. The positive result indicates that the motion is in the positive direction. If, instead, the particle moves from $x_1 = 5$ m to $x_2 = 1$ m, then $\Delta x = (1 \text{ m}) - (5 \text{ m}) = -4 \text{ m}$. The negative result indicates that the motion is in the negative direction.

The actual number of meters covered for a trip is irrelevant; displacement involves only the original and final positions. For example, if the particle moves from $x = 5$ m out to $x = 200$ m and then back to $x = 5$ m, the displacement from start to finish is $\Delta x = (5 \text{ m}) - (5 \text{ m}) = 0$.

Signs. A plus sign for a displacement need not be shown, but a minus sign must always be shown. If we ignore the sign (and thus the direction) of a displacement, we are left with the **magnitude** (or absolute value) of the displacement. For example, a displacement of $\Delta x = -4$ m has a magnitude of 4 m.

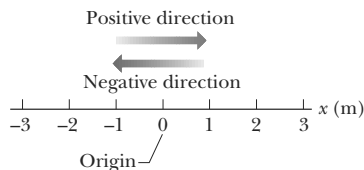
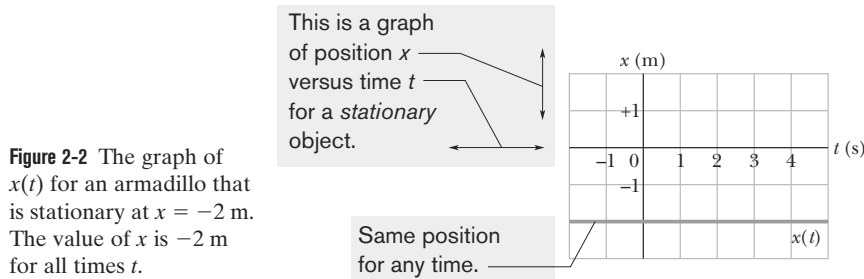


Figure 2-1 Position is determined on an axis that is marked in units of length (here meters) and that extends indefinitely in opposite directions. The axis name, here x , is always on the positive side of the origin.



Displacement is an example of a **vector quantity**, which is a quantity that has both a direction and a magnitude. We explore vectors more fully in Chapter 3, but here all we need is the idea that displacement has two features: (1) Its *magnitude* is the distance (such as the number of meters) between the original and final positions. (2) Its *direction*, from an original position to a final position, can be represented by a plus sign or a minus sign if the motion is along a single axis.

Here is the first of many checkpoints where you can check your understanding with a bit of reasoning. The answers are in the back of the book.



Checkpoint 1

Here are three pairs of initial and final positions, respectively, along an x axis. Which pairs give a negative displacement: (a) -3 m, $+5$ m; (b) -3 m, -7 m; (c) 7 m, -3 m?

Average Velocity and Average Speed

A compact way to describe position is with a graph of position x plotted as a function of time t —a graph of $x(t)$. (The notation $x(t)$ represents a function x of t , not the product x times t .) As a simple example, Fig. 2-2 shows the position function $x(t)$ for a stationary armadillo (which we treat as a particle) over a 7 s time interval. The animal's position stays at $x = -2$ m.

Figure 2-3 is more interesting, because it involves motion. The armadillo is apparently first noticed at $t = 0$ when it is at the position $x = -5$ m. It moves

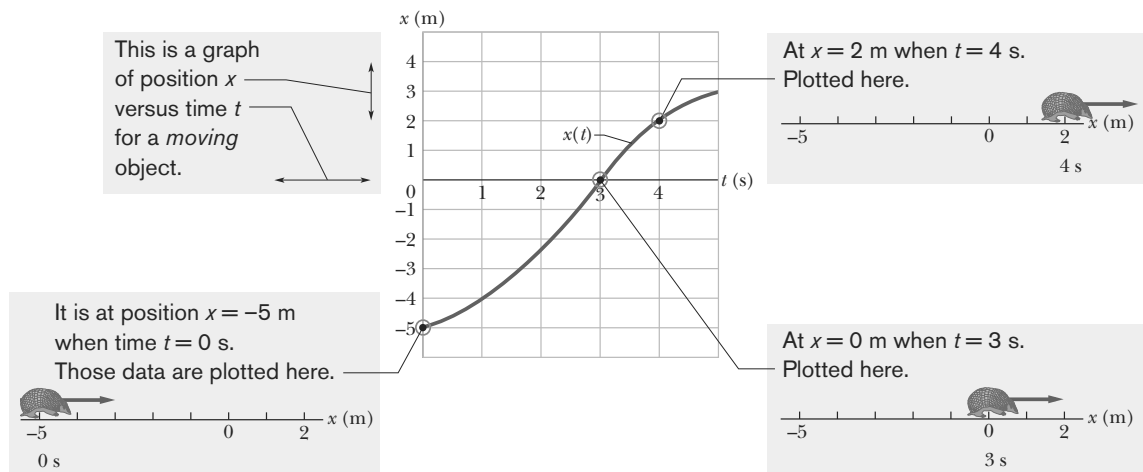


Figure 2-3 The graph of $x(t)$ for a moving armadillo. The path associated with the graph is also shown, at three times.

toward $x = 0$, passes through that point at $t = 3$ s, and then moves on to increasingly larger positive values of x . Figure 2-3 also depicts the straight-line motion of the armadillo (at three times) and is something like what you would see. The graph in Fig. 2-3 is more abstract, but it reveals how fast the armadillo moves.

Actually, several quantities are associated with the phrase “how fast.” One of them is the **average velocity** v_{avg} , which is the ratio of the displacement Δx that occurs during a particular time interval Δt to that interval:

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}. \quad (2-2)$$

The notation means that the position is x_1 at time t_1 and then x_2 at time t_2 . A common unit for v_{avg} is the meter per second (m/s). You may see other units in the problems, but they are always in the form of length/time.

Graphs. On a graph of x versus t , v_{avg} is the **slope** of the straight line that connects two particular points on the $x(t)$ curve: one is the point that corresponds to x_2 and t_2 , and the other is the point that corresponds to x_1 and t_1 . Like displacement, v_{avg} has both magnitude and direction (it is another vector quantity). Its magnitude is the magnitude of the line’s slope. A positive v_{avg} (and slope) tells us that the line slants upward to the right; a negative v_{avg} (and slope) tells us that the line slants downward to the right. The average velocity v_{avg} always has the same sign as the displacement Δx because Δt in Eq. 2-2 is always positive.

Figure 2-4 shows how to find v_{avg} in Fig. 2-3 for the time interval $t = 1$ s to $t = 4$ s. We draw the straight line that connects the point on the position curve at the beginning of the interval and the point on the curve at the end of the interval. Then we find the slope $\Delta x/\Delta t$ of the straight line. For the given time interval, the average velocity is

$$v_{\text{avg}} = \frac{6 \text{ m}}{3 \text{ s}} = 2 \text{ m/s}.$$

Average speed s_{avg} is a different way of describing “how fast” a particle moves. Whereas the average velocity involves the particle’s displacement Δx , the average speed involves the total distance covered (for example, the number of meters moved), independent of direction; that is,

$$s_{\text{avg}} = \frac{\text{total distance}}{\Delta t}. \quad (2-3)$$

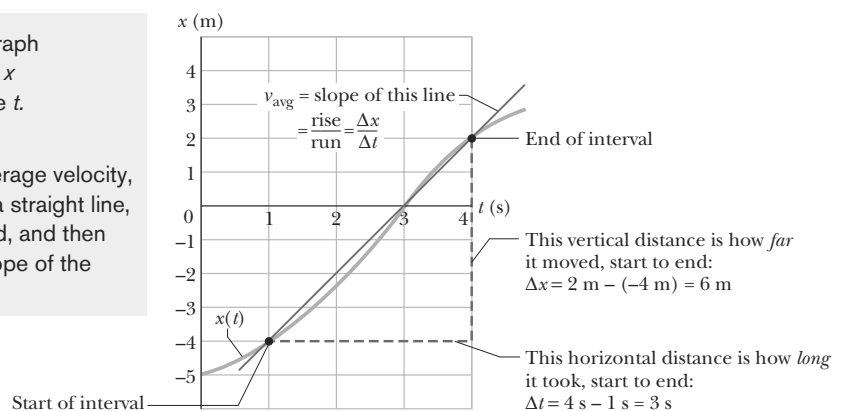
Because average speed does *not* include direction, it lacks any algebraic sign. Sometimes s_{avg} is the same (except for the absence of a sign) as v_{avg} . However, the two can be quite different.



Figure 2-4 Calculation of the average velocity between $t = 1$ s and $t = 4$ s as the slope of the line that connects the points on the $x(t)$ curve representing those times. The swirling icon indicates that a figure is available in *WileyPLUS* as an animation with voiceover.

This is a graph of position x versus time t .

To find average velocity, first draw a straight line, start to end, and then find the slope of the line.





Sample Problem 2.01 Average velocity, beat-up pickup truck

You drive a beat-up pickup truck along a straight road for 8.4 km at 70 km/h, at which point the truck runs out of gasoline and stops. Over the next 30 min, you walk another 2.0 km farther along the road to a gasoline station.

(a) What is your overall displacement from the beginning of your drive to your arrival at the station?

KEY IDEA

Assume, for convenience, that you move in the positive direction of an x axis, from a first position of $x_1 = 0$ to a second position of x_2 at the station. That second position must be at $x_2 = 8.4 \text{ km} + 2.0 \text{ km} = 10.4 \text{ km}$. Then your displacement Δx along the x axis is the second position minus the first position.

Calculation: From Eq. 2-1, we have

$$\Delta x = x_2 - x_1 = 10.4 \text{ km} - 0 = 10.4 \text{ km}. \quad (\text{Answer})$$

Thus, your overall displacement is 10.4 km in the positive direction of the x axis.

(b) What is the time interval Δt from the beginning of your drive to your arrival at the station?

KEY IDEA

We already know the walking time interval $\Delta t_{\text{wlk}} (= 0.50 \text{ h})$, but we lack the driving time interval Δt_{dr} . However, we know that for the drive the displacement Δx_{dr} is 8.4 km and the average velocity $v_{\text{avg,dr}}$ is 70 km/h. Thus, this average velocity is the ratio of the displacement for the drive to the time interval for the drive.

Calculations: We first write

$$v_{\text{avg,dr}} = \frac{\Delta x_{\text{dr}}}{\Delta t_{\text{dr}}}.$$

Rearranging and substituting data then give us

$$\Delta t_{\text{dr}} = \frac{\Delta x_{\text{dr}}}{v_{\text{avg,dr}}} = \frac{8.4 \text{ km}}{70 \text{ km/h}} = 0.12 \text{ h}.$$

$$\begin{aligned} \text{So, } \Delta t &= \Delta t_{\text{dr}} + \Delta t_{\text{wlk}} \\ &= 0.12 \text{ h} + 0.50 \text{ h} = 0.62 \text{ h}. \quad (\text{Answer}) \end{aligned}$$

(c) What is your average velocity v_{avg} from the beginning of your drive to your arrival at the station? Find it both numerically and graphically.

KEY IDEA

From Eq. 2-2 we know that v_{avg} for the entire trip is the ratio of the displacement of 10.4 km for the entire trip to the time interval of 0.62 h for the entire trip.

Calculation: Here we find

$$\begin{aligned} v_{\text{avg}} &= \frac{\Delta x}{\Delta t} = \frac{10.4 \text{ km}}{0.62 \text{ h}} \\ &= 16.8 \text{ km/h} \approx 17 \text{ km/h}. \quad (\text{Answer}) \end{aligned}$$

To find v_{avg} graphically, first we graph the function $x(t)$ as shown in Fig. 2-5, where the beginning and arrival points on the graph are the origin and the point labeled as “Station.” Your average velocity is the slope of the straight line connecting those points; that is, v_{avg} is the ratio of the *rise* ($\Delta x = 10.4 \text{ km}$) to the *run* ($\Delta t = 0.62 \text{ h}$), which gives us $v_{\text{avg}} = 16.8 \text{ km/h}$.

(d) Suppose that to pump the gasoline, pay for it, and walk back to the truck takes you another 45 min. What is your average speed from the beginning of your drive to your return to the truck with the gasoline?

KEY IDEA

Your average speed is the ratio of the total distance you move to the total time interval you take to make that move.

Calculation: The total distance is $8.4 \text{ km} + 2.0 \text{ km} + 2.0 \text{ km} = 12.4 \text{ km}$. The total time interval is $0.12 \text{ h} + 0.50 \text{ h} + 0.75 \text{ h} = 1.37 \text{ h}$. Thus, Eq. 2-3 gives us

$$s_{\text{avg}} = \frac{12.4 \text{ km}}{1.37 \text{ h}} = 9.1 \text{ km/h}. \quad (\text{Answer})$$

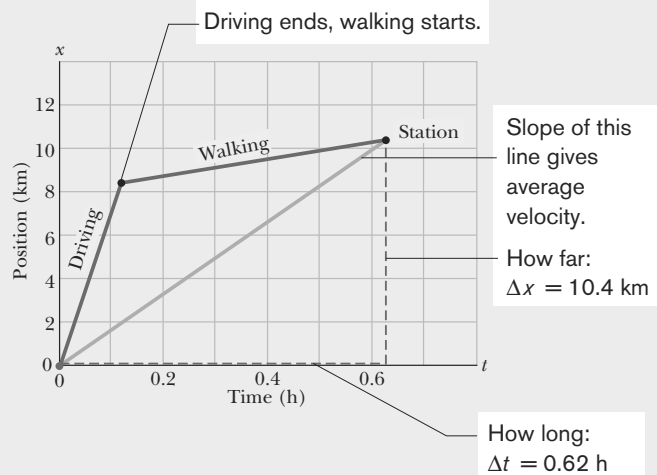


Figure 2-5 The lines marked “Driving” and “Walking” are the position–time plots for the driving and walking stages. (The plot for the walking stage assumes a constant rate of walking.) The slope of the straight line joining the origin and the point labeled “Station” is the average velocity for the trip, from the beginning to the station.



Additional examples, video, and practice available at WileyPLUS



2-2 INSTANTANEOUS VELOCITY AND SPEED

Learning Objectives

After reading this module, you should be able to . . .

2.07 Given a particle's position as a function of time, calculate the instantaneous velocity for any particular time.

2.08 Given a graph of a particle's position versus time, determine the instantaneous velocity for any particular time.

2.09 Identify speed as the magnitude of the instantaneous velocity.

Key Ideas

● The instantaneous velocity (or simply velocity) v of a moving particle is

$$v = \lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt},$$

where $\Delta x = x_2 - x_1$ and $\Delta t = t_2 - t_1$.

● The instantaneous velocity (at a particular time) may be found as the slope (at that particular time) of the graph of x versus t .

● Speed is the magnitude of instantaneous velocity.

Instantaneous Velocity and Speed

You have now seen two ways to describe how fast something moves: average velocity and average speed, both of which are measured over a time interval Δt . However, the phrase “how fast” more commonly refers to how fast a particle is moving at a given instant—its **instantaneous velocity** (or simply **velocity**) v .

The velocity at any instant is obtained from the average velocity by shrinking the time interval Δt closer and closer to 0. As Δt dwindles, the average velocity approaches a limiting value, which is the velocity at that instant:

$$v = \lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}. \quad (2-4)$$

Note that v is the rate at which position x is changing with time at a given instant; that is, v is the derivative of x with respect to t . Also note that v at any instant is the slope of the position–time curve at the point representing that instant. Velocity is another vector quantity and thus has an associated direction.

Speed is the magnitude of velocity; that is, speed is velocity that has been stripped of any indication of direction, either in words or via an algebraic sign. (*Caution:* Speed and average speed can be quite different.) A velocity of +5 m/s and one of –5 m/s both have an associated speed of 5 m/s. The speedometer in a car measures speed, not velocity (it cannot determine the direction).



Checkpoint 2

The following equations give the position $x(t)$ of a particle in four situations (in each equation, x is in meters, t is in seconds, and $t > 0$): (1) $x = 3t - 2$; (2) $x = -4t^2 - 2$; (3) $x = 2/t^2$; and (4) $x = -2$. (a) In which situation is the velocity v of the particle constant? (b) In which is v in the negative x direction?



Sample Problem 2.02 Velocity and slope of x versus t , elevator cab

Figure 2-6a is an $x(t)$ plot for an elevator cab that is initially stationary, then moves upward (which we take to be the positive direction of x), and then stops. Plot $v(t)$.

KEY IDEA

We can find the velocity at any time from the slope of the $x(t)$ curve at that time.

Calculations: The slope of $x(t)$, and so also the velocity, is zero in the intervals from 0 to 1 s and from 9 s on, so then the cab is stationary. During the interval bc , the slope is constant and nonzero, so then the cab moves with constant velocity. We calculate the slope of $x(t)$ then as

$$\frac{\Delta x}{\Delta t} = v = \frac{24 \text{ m} - 4.0 \text{ m}}{8.0 \text{ s} - 3.0 \text{ s}} = +4.0 \text{ m/s}. \quad (2-5)$$

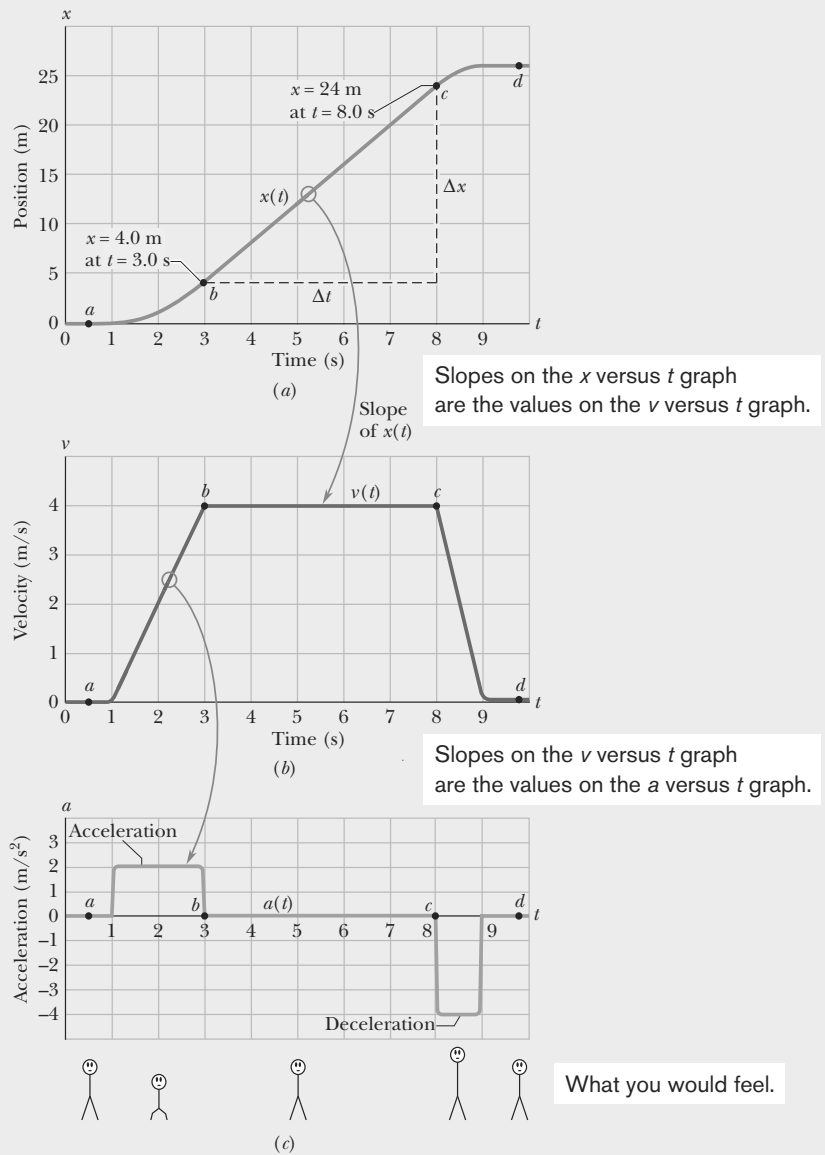


Figure 2-6 (a) The $x(t)$ curve for an elevator cab that moves upward along an x axis. (b) The $v(t)$ curve for the cab. Note that it is the derivative of the $x(t)$ curve ($v = dx/dt$). (c) The $a(t)$ curve for the cab. It is the derivative of the $v(t)$ curve ($a = dv/dt$). The stick figures along the bottom suggest how a passenger's body might feel during the accelerations.

The plus sign indicates that the cab is moving in the positive x direction. These intervals (where $v = 0$ and $v = 4$ m/s) are plotted in Fig. 2-6b. In addition, as the cab initially begins to move and then later slows to a stop, v varies as indicated in the intervals 1 s to 3 s and 8 s to 9 s. Thus, Fig. 2-6b is the required plot. (Figure 2-6c is considered in Module 2-3.)

Given a $v(t)$ graph such as Fig. 2-6b, we could “work backward” to produce the shape of the associated $x(t)$ graph (Fig. 2-6a). However, we would not know the actual values for x at various times, because the $v(t)$ graph indicates only *changes* in x . To find such a change in x during any in-

terval, we must, in the language of calculus, calculate the area “under the curve” on the $v(t)$ graph for that interval. For example, during the interval 3 s to 8 s in which the cab has a velocity of 4.0 m/s, the change in x is

$$\Delta x = (4.0 \text{ m/s})(8.0 \text{ s} - 3.0 \text{ s}) = +20 \text{ m.} \quad (2-6)$$

(This area is positive because the $v(t)$ curve is above the t axis.) Figure 2-6a shows that x does indeed increase by 20 m in that interval. However, Fig. 2-6b does not tell us the *values* of x at the beginning and end of the interval. For that, we need additional information, such as the value of x at some instant.



2-3 ACCELERATION

Learning Objectives

After reading this module, you should be able to . . .

- 2.10** Apply the relationship between a particle's average acceleration, its change in velocity, and the time interval for that change.
- 2.11** Given a particle's velocity as a function of time, calculate the instantaneous acceleration for any particular time.

- 2.12** Given a graph of a particle's velocity versus time, determine the instantaneous acceleration for any particular time and the average acceleration between any two particular times.

Key Ideas

- Average acceleration is the ratio of a change in velocity Δv to the time interval Δt in which the change occurs:

$$a_{\text{avg}} = \frac{\Delta v}{\Delta t}.$$

The algebraic sign indicates the direction of a_{avg} .

- Instantaneous acceleration (or simply acceleration) a is the first time derivative of velocity $v(t)$ and the second time derivative of position $x(t)$:

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2}.$$

- On a graph of v versus t , the acceleration a at any time t is the slope of the curve at the point that represents t .

Acceleration

When a particle's velocity changes, the particle is said to undergo **acceleration** (or to accelerate). For motion along an axis, the **average acceleration** a_{avg} over a time interval Δt is

$$a_{\text{avg}} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}, \quad (2-7)$$

where the particle has velocity v_1 at time t_1 and then velocity v_2 at time t_2 . The **instantaneous acceleration** (or simply **acceleration**) is

$$a = \frac{dv}{dt}. \quad (2-8)$$

In words, the acceleration of a particle at any instant is the rate at which its velocity is changing at that instant. Graphically, the acceleration at any point is the slope of the curve of $v(t)$ at that point. We can combine Eq. 2-8 with Eq. 2-4 to write

$$a = \frac{dv}{dt} = \frac{d}{dt} \left(\frac{dx}{dt} \right) = \frac{d^2x}{dt^2}. \quad (2-9)$$

In words, the acceleration of a particle at any instant is the second derivative of its position $x(t)$ with respect to time.

A common unit of acceleration is the meter per second per second: $\text{m}/(\text{s} \cdot \text{s})$ or m/s^2 . Other units are in the form of $\text{length}/(\text{time} \cdot \text{time})$ or $\text{length}/\text{time}^2$. Acceleration has both magnitude and direction (it is yet another vector quantity). Its algebraic sign represents its direction on an axis just as for displacement and velocity; that is, acceleration with a positive value is in the positive direction of an axis, and acceleration with a negative value is in the negative direction.

Figure 2-6 gives plots of the position, velocity, and acceleration of an elevator moving up a shaft. Compare the $a(t)$ curve with the $v(t)$ curve—each point on the $a(t)$ curve shows the derivative (slope) of the $v(t)$ curve at the corresponding time. When v is constant (at either 0 or 4 m/s), the derivative is zero and so also is the acceleration. When the cab first begins to move, the $v(t)$

curve has a positive derivative (the slope is positive), which means that $a(t)$ is positive. When the cab slows to a stop, the derivative and slope of the $v(t)$ curve are negative; that is, $a(t)$ is negative.

Next compare the slopes of the $v(t)$ curve during the two acceleration periods. The slope associated with the cab's slowing down (commonly called "deceleration") is steeper because the cab stops in half the time it took to get up to speed. The steeper slope means that the magnitude of the deceleration is larger than that of the acceleration, as indicated in Fig. 2-6c.

Sensations. The sensations you would feel while riding in the cab of Fig. 2-6 are indicated by the sketched figures at the bottom. When the cab first accelerates, you feel as though you are pressed downward; when later the cab is braked to a stop, you seem to be stretched upward. In between, you feel nothing special. In other words, your body reacts to accelerations (it is an accelerometer) but not to velocities (it is not a speedometer). When you are in a car traveling at 90 km/h or an airplane traveling at 900 km/h, you have no bodily awareness of the motion. However, if the car or plane quickly changes velocity, you may become keenly aware of the change, perhaps even frightened by it. Part of the thrill of an amusement park ride is due to the quick changes of velocity that you undergo (you pay for the accelerations, not for the speed). A more extreme example is shown in the photographs of Fig. 2-7, which were taken while a rocket sled was rapidly accelerated along a track and then rapidly braked to a stop.

g Units. Large accelerations are sometimes expressed in terms of g units, with

$$1g = 9.8 \text{ m/s}^2 \quad (g \text{ unit}). \quad (2-10)$$

(As we shall discuss in Module 2-5, g is the magnitude of the acceleration of a falling object near Earth's surface.) On a roller coaster, you may experience brief accelerations up to $3g$, which is $(3)(9.8 \text{ m/s}^2)$, or about 29 m/s^2 , more than enough to justify the cost of the ride.

Signs. In common language, the sign of an acceleration has a nonscientific meaning: positive acceleration means that the speed of an object is increasing, and negative acceleration means that the speed is decreasing (the object is decelerating). In this book, however, the sign of an acceleration indicates a direction, not

Figure 2-7

Colonel J. P. Stapp in a rocket sled as it is brought up to high speed (acceleration out of the page) and then very rapidly braked (acceleration into the page).



Courtesy U.S. Air Force

whether an object's speed is increasing or decreasing. For example, if a car with an initial velocity $v = -25 \text{ m/s}$ is braked to a stop in 5.0 s, then $a_{\text{avg}} = +5.0 \text{ m/s}^2$. The acceleration is *positive*, but the car's speed has decreased. The reason is the difference in signs: the direction of the acceleration is opposite that of the velocity.

Here then is the proper way to interpret the signs:



If the signs of the velocity and acceleration of a particle are the same, the speed of the particle increases. If the signs are opposite, the speed decreases.



Checkpoint 3

A wombat moves along an x axis. What is the sign of its acceleration if it is moving (a) in the positive direction with increasing speed, (b) in the positive direction with decreasing speed, (c) in the negative direction with increasing speed, and (d) in the negative direction with decreasing speed?

Sample Problem 2.03 Acceleration and dv/dt

A particle's position on the x axis of Fig. 2-1 is given by

$$x = 4 - 27t + t^3,$$

with x in meters and t in seconds.

(a) Because position x depends on time t , the particle must be moving. Find the particle's velocity function $v(t)$ and acceleration function $a(t)$.

KEY IDEAS

(1) To get the velocity function $v(t)$, we differentiate the position function $x(t)$ with respect to time. (2) To get the acceleration function $a(t)$, we differentiate the velocity function $v(t)$ with respect to time.

Calculations: Differentiating the position function, we find

$$v = -27 + 3t^2, \quad (\text{Answer})$$

with v in meters per second. Differentiating the velocity function then gives us

$$a = +6t, \quad (\text{Answer})$$

with a in meters per second squared.

(b) Is there ever a time when $v = 0$?

Calculation: Setting $v(t) = 0$ yields

$$0 = -27 + 3t^2,$$

which has the solution

$$t = \pm 3 \text{ s}. \quad (\text{Answer})$$

Thus, the velocity is zero both 3 s before and 3 s after the clock reads 0.

(c) Describe the particle's motion for $t \geq 0$.

Reasoning: We need to examine the expressions for $x(t)$, $v(t)$, and $a(t)$.

At $t = 0$, the particle is at $x(0) = +4 \text{ m}$ and is moving with a velocity of $v(0) = -27 \text{ m/s}$ —that is, in the negative direction of the x axis. Its acceleration is $a(0) = 0$ because just then the particle's velocity is not changing (Fig. 2-8a).

For $0 < t < 3 \text{ s}$, the particle still has a negative velocity, so it continues to move in the negative direction. However, its acceleration is no longer 0 but is increasing and positive. Because the signs of the velocity and the acceleration are opposite, the particle must be slowing (Fig. 2-8b).

Indeed, we already know that it stops momentarily at $t = 3 \text{ s}$. Just then the particle is as far to the left of the origin in Fig. 2-1 as it will ever get. Substituting $t = 3 \text{ s}$ into the expression for $x(t)$, we find that the particle's position just then is $x = -50 \text{ m}$ (Fig. 2-8c). Its acceleration is still positive.

For $t > 3 \text{ s}$, the particle moves to the right on the axis. Its acceleration remains positive and grows progressively larger in magnitude. The velocity is now positive, and it too grows progressively larger in magnitude (Fig. 2-8d).

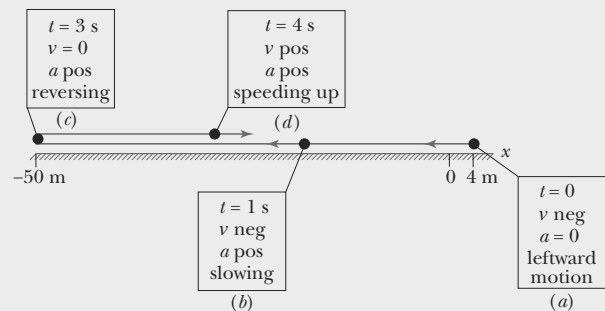


Figure 2-8 Four stages of the particle's motion.

2-4 CONSTANT ACCELERATION

Learning Objectives

After reading this module, you should be able to . . .

2.13 For constant acceleration, apply the relationships between position, displacement, velocity, acceleration, and elapsed time (Table 2-1).

2.14 Calculate a particle's change in velocity by integrating its acceleration function with respect to time.

2.15 Calculate a particle's change in position by integrating its velocity function with respect to time.

Key Ideas

● The following five equations describe the motion of a particle with constant acceleration:

$$\begin{aligned} v &= v_0 + at, & x - x_0 &= v_0 t + \frac{1}{2}at^2, \\ v^2 &= v_0^2 + 2a(x - x_0), & x - x_0 &= \frac{1}{2}(v_0 + v)t, & x - x_0 &= vt - \frac{1}{2}at^2. \end{aligned}$$

These are *not* valid when the acceleration is not constant.

Constant Acceleration: A Special Case

In many types of motion, the acceleration is either constant or approximately so. For example, you might accelerate a car at an approximately constant rate when a traffic light turns from red to green. Then graphs of your position, velocity, and acceleration would resemble those in Fig. 2-9. (Note that $a(t)$ in Fig. 2-9c is constant, which requires that $v(t)$ in Fig. 2-9b have a constant slope.) Later when you brake the car to a stop, the acceleration (or deceleration in common language) might also be approximately constant.

Such cases are so common that a special set of equations has been derived for dealing with them. One approach to the derivation of these equations is given in this section. A second approach is given in the next section. Throughout both sections and later when you work on the homework problems, keep in mind that *these equations are valid only for constant acceleration (or situations in which you can approximate the acceleration as being constant)*.

First Basic Equation. When the acceleration is constant, the average acceleration and instantaneous acceleration are equal and we can write Eq. 2-7, with some changes in notation, as

$$a = a_{\text{avg}} = \frac{v - v_0}{t - 0}.$$

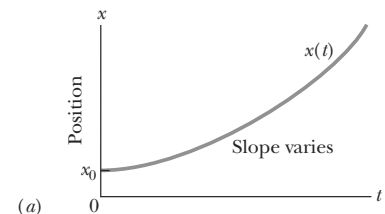
Here v_0 is the velocity at time $t = 0$ and v is the velocity at any later time t . We can recast this equation as

$$v = v_0 + at. \quad (2-11)$$

As a check, note that this equation reduces to $v = v_0$ for $t = 0$, as it must. As a further check, take the derivative of Eq. 2-11. Doing so yields $dv/dt = a$, which is the definition of a . Figure 2-9b shows a plot of Eq. 2-11, the $v(t)$ function; the function is linear and thus the plot is a straight line.

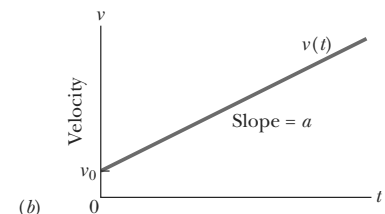
Second Basic Equation. In a similar manner, we can rewrite Eq. 2-2 (with a few changes in notation) as

$$v_{\text{avg}} = \frac{x - x_0}{t - 0}$$



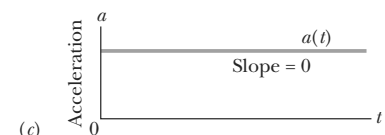
(a)

Slopes of the position graph are plotted on the velocity graph.



(b)

Slope of the velocity graph is plotted on the acceleration graph.



(c)

Figure 2-9 (a) The position $x(t)$ of a particle moving with constant acceleration. (b) Its velocity $v(t)$, given at each point by the slope of the curve of $x(t)$. (c) Its (constant) acceleration, equal to the (constant) slope of the curve of $v(t)$.

and then as

$$x = x_0 + v_{\text{avg}}t, \quad (2-12)$$

in which x_0 is the position of the particle at $t = 0$ and v_{avg} is the average velocity between $t = 0$ and a later time t .

For the linear velocity function in Eq. 2-11, the *average* velocity over any time interval (say, from $t = 0$ to a later time t) is the average of the velocity at the beginning of the interval ($= v_0$) and the velocity at the end of the interval ($= v$). For the interval from $t = 0$ to the later time t then, the average velocity is

$$v_{\text{avg}} = \frac{1}{2}(v_0 + v). \quad (2-13)$$

Substituting the right side of Eq. 2-11 for v yields, after a little rearrangement,

$$v_{\text{avg}} = v_0 + \frac{1}{2}at. \quad (2-14)$$

Finally, substituting Eq. 2-14 into Eq. 2-12 yields

$$x - x_0 = v_0t + \frac{1}{2}at^2. \quad (2-15)$$

As a check, note that putting $t = 0$ yields $x = x_0$, as it must. As a further check, taking the derivative of Eq. 2-15 yields Eq. 2-11, again as it must. Figure 2-9a shows a plot of Eq. 2-15; the function is quadratic and thus the plot is curved.

Three Other Equations. Equations 2-11 and 2-15 are the *basic equations for constant acceleration*; they can be used to solve any constant acceleration problem in this book. However, we can derive other equations that might prove useful in certain specific situations. First, note that as many as five quantities can possibly be involved in any problem about constant acceleration—namely, $x - x_0$, v , t , a , and v_0 . Usually, one of these quantities is *not* involved in the problem, *either as a given or as an unknown*. We are then presented with three of the remaining quantities and asked to find the fourth.

Equations 2-11 and 2-15 each contain four of these quantities, but not the same four. In Eq. 2-11, the “missing ingredient” is the displacement $x - x_0$. In Eq. 2-15, it is the velocity v . These two equations can also be combined in three ways to yield three additional equations, each of which involves a different “missing variable.” First, we can eliminate t to obtain

$$v^2 = v_0^2 + 2a(x - x_0). \quad (2-16)$$

This equation is useful if we do not know t and are not required to find it. Second, we can eliminate the acceleration a between Eqs. 2-11 and 2-15 to produce an equation in which a does not appear:

$$x - x_0 = \frac{1}{2}(v_0 + v)t. \quad (2-17)$$

Finally, we can eliminate v_0 , obtaining

$$x - x_0 = vt - \frac{1}{2}at^2. \quad (2-18)$$

Note the subtle difference between this equation and Eq. 2-15. One involves the initial velocity v_0 ; the other involves the velocity v at time t .

Table 2-1 lists the basic constant acceleration equations (Eqs. 2-11 and 2-15) as well as the specialized equations that we have derived. To solve a simple constant acceleration problem, you can usually use an equation from this list (*if* you have the list with you). Choose an equation for which the only unknown variable is the variable requested in the problem. A simpler plan is to remember only Eqs. 2-11 and 2-15, and then solve them as simultaneous equations whenever needed.

Table 2-1 Equations for Motion with Constant Acceleration^a

Equation Number	Equation	Missing Quantity
2-11	$v = v_0 + at$	$x - x_0$
2-15	$x - x_0 = v_0t + \frac{1}{2}at^2$	v
2-16	$v^2 = v_0^2 + 2a(x - x_0)$	t
2-17	$x - x_0 = \frac{1}{2}(v_0 + v)t$	a
2-18	$x - x_0 = vt - \frac{1}{2}at^2$	v_0

^aMake sure that the acceleration is indeed constant before using the equations in this table.

**Checkpoint 4**

The following equations give the position $x(t)$ of a particle in four situations: (1) $x = 3t - 4$; (2) $x = -5t^3 + 4t^2 + 6$; (3) $x = 2/t^2 - 4/t$; (4) $x = 5t^2 - 3$. To which of these situations do the equations of Table 2-1 apply?

Sample Problem 2.04 Drag race of car and motorcycle

A popular web video shows a jet airplane, a car, and a motorcycle racing from rest along a runway (Fig. 2-10). Initially the motorcycle takes the lead, but then the jet takes the lead, and finally the car blows past the motorcycle. Here let's focus on the car and motorcycle and assign some reasonable values to the motion. The motorcycle first takes the lead because its (constant) acceleration $a_m = 8.40 \text{ m/s}^2$ is greater than the car's (constant) acceleration $a_c = 5.60 \text{ m/s}^2$, but it soon loses to the car because it reaches its greatest speed $v_m = 58.8 \text{ m/s}$ before the car reaches its greatest speed $v_c = 106 \text{ m/s}$. How long does the car take to reach the motorcycle?

KEY IDEAS

We can apply the equations of constant acceleration to both vehicles, but for the motorcycle we must consider the motion in two stages: (1) First it travels through distance x_{m1} with zero initial velocity and acceleration $a_m = 8.40 \text{ m/s}^2$, reaching speed $v_m = 58.8 \text{ m/s}$. (2) Then it travels through distance x_{m2} with constant velocity $v_m = 58.8 \text{ m/s}$ and zero acceleration (that, too, is a constant acceleration). (Note that we symbolized the distances even though we do not know their values. Symbolizing unknown quantities is often helpful in solving physics problems, but introducing such unknowns sometimes takes *physics courage*.)

Calculations: So that we can draw figures and do calculations, let's assume that the vehicles race along the positive direction of an x axis, starting from $x = 0$ at time $t = 0$. (We can

choose any initial numbers because we are looking for the elapsed time, not a particular time in, say, the afternoon, but let's stick with these easy numbers.) We want the car to pass the motorcycle, but what does that mean mathematically?

It means that at some time t , the side-by-side vehicles are at the same coordinate: x_c for the car and the sum $x_{m1} + x_{m2}$ for the motorcycle. We can write this statement mathematically as

$$x_c = x_{m1} + x_{m2}. \quad (2-19)$$

(Writing this first step is the hardest part of the problem. That is true of most physics problems. How do you go from the problem statement (in words) to a mathematical expression? One purpose of this book is for you to build up that ability of writing the first step — it takes lots of practice just as in learning, say, tae-kwon-do.)

Now let's fill out both sides of Eq. 2-19, left side first. To reach the passing point at x_c , the car accelerates from rest. From Eq. 2-15 ($x - x_0 = v_0 t + \frac{1}{2}at^2$), with x_0 and $v_0 = 0$, we have

$$x_c = \frac{1}{2}a_c t^2. \quad (2-20)$$

To write an expression for x_{m1} for the motorcycle, we first find the time t_m it takes to reach its maximum speed v_m , using Eq. 2-11 ($v = v_0 + at$). Substituting $v_0 = 0$, $v = v_m = 58.8 \text{ m/s}$, and $a = a_m = 8.40 \text{ m/s}^2$, that time is

$$\begin{aligned} t_m &= \frac{v_m}{a_m} \\ &= \frac{58.8 \text{ m/s}}{8.40 \text{ m/s}^2} = 7.00 \text{ s}. \end{aligned} \quad (2-21)$$

To get the distance x_{m1} traveled by the motorcycle during the first stage, we again use Eq. 2-15 with $x_0 = 0$ and $v_0 = 0$, but we also substitute from Eq. 2-21 for the time. We find

$$x_{m1} = \frac{1}{2}a_m t_m^2 = \frac{1}{2}a_m \left(\frac{v_m}{a_m} \right)^2 = \frac{1}{2} \frac{v_m^2}{a_m}. \quad (2-22)$$

For the remaining time of $t - t_m$, the motorcycle travels at its maximum speed with zero acceleration. To get the distance, we use Eq. 2-15 for this second stage of the motion, but now the initial velocity is $v_0 = v_m$ (the speed at the end of the first stage) and the acceleration is $a = 0$. So, the distance traveled during the second stage is

$$x_{m2} = v_m(t - t_m) = v_m(t - 7.00 \text{ s}). \quad (2-23)$$

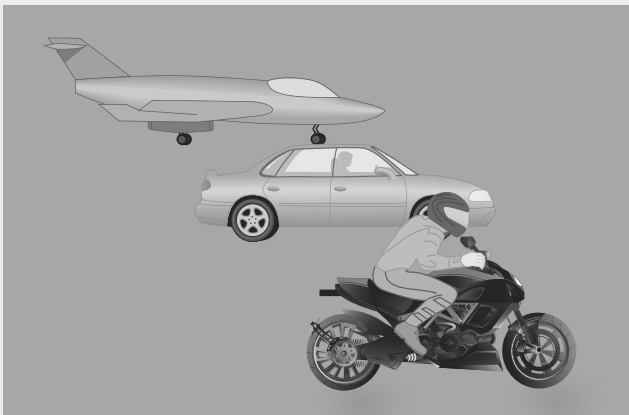


Figure 2-10 A jet airplane, a car, and a motorcycle just after accelerating from rest.

To finish the calculation, we substitute Eqs. 2-20, 2-22, and 2-23 into Eq. 2-19, obtaining

$$\frac{1}{2}a_c t^2 = \frac{1}{2} \frac{v_m^2}{a_m} + v_m(t - 7.00 \text{ s}). \quad (2-24)$$

This is a quadratic equation. Substituting in the given data, we solve the equation (by using the usual quadratic-equation formula or a polynomial solver on a calculator), finding $t = 4.44 \text{ s}$ and $t = 16.6 \text{ s}$.

But what do we do with two answers? Does the car pass the motorcycle twice? No, of course not, as we can see in the video. So, one of the answers is mathematically correct but not physically meaningful. Because we know that the car passes the motorcycle *after* the motorcycle reaches its maximum speed at $t = 7.00 \text{ s}$, we discard the solution with $t < 7.00 \text{ s}$ as being the unphysical answer and conclude that the passing occurs at

$$t = 16.6 \text{ s.} \quad (\text{Answer})$$

Figure 2-11 is a graph of the position versus time for the two vehicles, with the passing point marked. Notice

that at $t = 7.00 \text{ s}$ the plot for the motorcycle switches from being curved (because the speed had been increasing) to being straight (because the speed is thereafter constant).

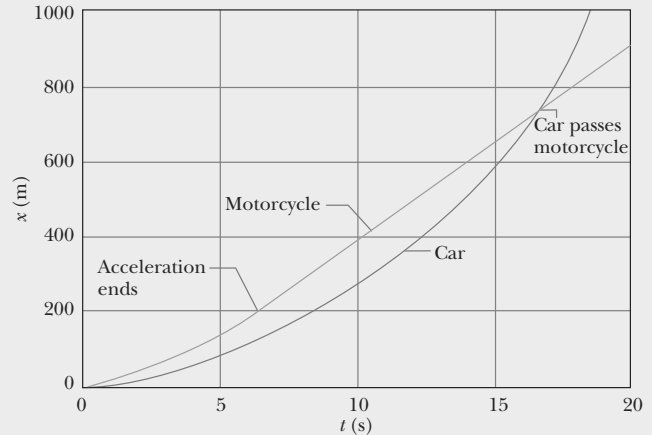


Figure 2-11 Graph of position versus time for car and motorcycle.



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Another Look at Constant Acceleration*

The first two equations in Table 2-1 are the basic equations from which the others are derived. Those two can be obtained by integration of the acceleration with the condition that a is constant. To find Eq. 2-11, we rewrite the definition of acceleration (Eq. 2-8) as

$$dv = a dt.$$

We next write the *indefinite integral* (or *antiderivative*) of both sides:

$$\int dv = \int a dt.$$

Since acceleration a is a constant, it can be taken outside the integration. We obtain

$$\int dv = a \int dt$$

or

$$v = at + C. \quad (2-25)$$

To evaluate the constant of integration C , we let $t = 0$, at which time $v = v_0$. Substituting these values into Eq. 2-25 (which must hold for all values of t , including $t = 0$) yields

$$v_0 = (a)(0) + C = C.$$

Substituting this into Eq. 2-25 gives us Eq. 2-11.

To derive Eq. 2-15, we rewrite the definition of velocity (Eq. 2-4) as

$$dx = v dt$$

and then take the indefinite integral of both sides to obtain

$$\int dx = \int v dt.$$

*This section is intended for students who have had integral calculus.

Next, we substitute for v with Eq. 2-11:

$$\int dx = \int (v_0 + at) dt.$$

Since v_0 is a constant, as is the acceleration a , this can be rewritten as

$$\int dx = v_0 \int dt + a \int t dt.$$

Integration now yields

$$x = v_0 t + \frac{1}{2} at^2 + C', \quad (2-26)$$

where C' is another constant of integration. At time $t = 0$, we have $x = x_0$. Substituting these values in Eq. 2-26 yields $x_0 = C'$. Replacing C' with x_0 in Eq. 2-26 gives us Eq. 2-15.

2-5 FREE-FALL ACCELERATION

Learning Objectives

After reading this module, you should be able to . . .

2.16 Identify that if a particle is in free flight (whether upward or downward) and if we can neglect the effects of air on its motion, the particle has a constant

downward acceleration with a magnitude g that we take to be 9.8 m/s^2 .

2.17 Apply the constant-acceleration equations (Table 2-1) to free-fall motion.

Key Ideas

● An important example of straight-line motion with constant acceleration is that of an object rising or falling freely near Earth's surface. The constant acceleration equations describe this motion, but we make two changes in notation:

(1) we refer the motion to the vertical y axis with $+y$ vertically up; (2) we replace a with $-g$, where g is the magnitude of the free-fall acceleration. Near Earth's surface,

$$g = 9.8 \text{ m/s}^2 = 32 \text{ ft/s}^2.$$

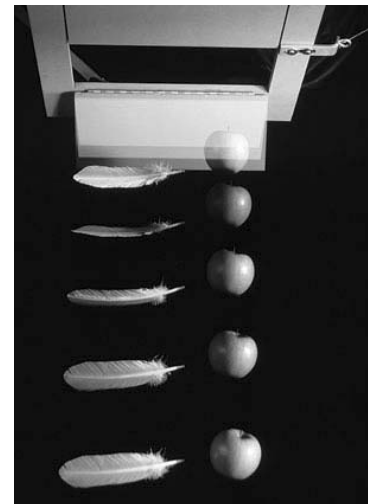
Free-Fall Acceleration

If you tossed an object either up or down and could somehow eliminate the effects of air on its flight, you would find that the object accelerates downward at a certain constant rate. That rate is called the **free-fall acceleration**, and its magnitude is represented by g . The acceleration is independent of the object's characteristics, such as mass, density, or shape; it is the same for all objects.

Two examples of free-fall acceleration are shown in Fig. 2-12, which is a series of stroboscopic photos of a feather and an apple. As these objects fall, they accelerate downward—both at the same rate g . Thus, their speeds increase at the same rate, and they fall together.

The value of g varies slightly with latitude and with elevation. At sea level in Earth's midlatitudes the value is 9.8 m/s^2 (or 32 ft/s^2), which is what you should use as an exact number for the problems in this book unless otherwise noted.

The equations of motion in Table 2-1 for constant acceleration also apply to free fall near Earth's surface; that is, they apply to an object in vertical flight, either up or down, when the effects of the air can be neglected. However, note that for free fall: (1) The directions of motion are now along a vertical y axis instead of the x axis, with the positive direction of y upward. (This is important for later chapters when combined horizontal and vertical motions are examined.) (2) The free-fall acceleration is negative—that is, downward on the y axis, toward Earth's center—and so it has the value $-g$ in the equations.



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Figure 2-12 A feather and an apple free fall in vacuum at the same magnitude of acceleration g . The acceleration increases the distance between successive images. In the absence of air, the feather and apple fall together.



The free-fall acceleration near Earth's surface is $a = -g = -9.8 \text{ m/s}^2$, and the magnitude of the acceleration is $g = 9.8 \text{ m/s}^2$. Do not substitute -9.8 m/s^2 for g .

Suppose you toss a tomato directly upward with an initial (positive) velocity v_0 and then catch it when it returns to the release level. During its *free-fall flight* (from just after its release to just before it is caught), the equations of Table 2-1 apply to its motion. The acceleration is always $a = -g = -9.8 \text{ m/s}^2$, negative and thus downward. The velocity, however, changes, as indicated by Eqs. 2-11 and 2-16: during the ascent, the magnitude of the positive velocity decreases, until it momentarily becomes zero. Because the tomato has then stopped, it is at its maximum height. During the descent, the magnitude of the (now negative) velocity increases.



Checkpoint 5

(a) If you toss a ball straight up, what is the sign of the ball's displacement for the ascent, from the release point to the highest point? (b) What is it for the descent, from the highest point back to the release point? (c) What is the ball's acceleration at its highest point?

Sample Problem 2.05 Time for full up-down flight, baseball toss

In Fig. 2-13, a pitcher tosses a baseball up along a y axis, with an initial speed of 12 m/s .

(a) How long does the ball take to reach its maximum height?

KEY IDEAS

(1) Once the ball leaves the pitcher and before it returns to his hand, its acceleration is the free-fall acceleration $a = -g$. Because this is constant, Table 2-1 applies to the motion.
(2) The velocity v at the maximum height must be 0.

Calculation: Knowing v , a , and the initial velocity $v_0 = 12 \text{ m/s}$, and seeking t , we solve Eq. 2-11, which contains those four variables. This yields

$$t = \frac{v - v_0}{a} = \frac{0 - 12 \text{ m/s}}{-9.8 \text{ m/s}^2} = 1.2 \text{ s.} \quad (\text{Answer})$$

(b) What is the ball's maximum height above its release point?

Calculation: We can take the ball's release point to be $y_0 = 0$. We can then write Eq. 2-16 in y notation, set $y - y_0 = y$ and $v = 0$ (at the maximum height), and solve for y . We get

$$y = \frac{v^2 - v_0^2}{2a} = \frac{0 - (12 \text{ m/s})^2}{2(-9.8 \text{ m/s}^2)} = 7.3 \text{ m.} \quad (\text{Answer})$$

(c) How long does the ball take to reach a point 5.0 m above its release point?

Calculations: We know v_0 , $a = -g$, and displacement $y - y_0 = 5.0 \text{ m}$, and we want t , so we choose Eq. 2-15. Rewriting it for y and setting $y_0 = 0$ give us

$$y = v_0 t - \frac{1}{2} g t^2,$$

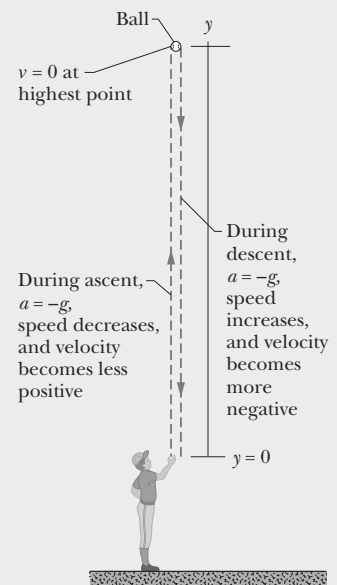


Figure 2-13 A pitcher tosses a baseball straight up into the air. The equations of free fall apply for rising as well as for falling objects, provided any effects from the air can be neglected.

$$\text{or} \quad 5.0 \text{ m} = (12 \text{ m/s})t - \left(\frac{1}{2}\right)(9.8 \text{ m/s}^2)t^2.$$

If we temporarily omit the units (having noted that they are consistent), we can rewrite this as

$$4.9t^2 - 12t + 5.0 = 0.$$

Solving this quadratic equation for t yields

$$t = 0.53 \text{ s} \quad \text{and} \quad t = 1.9 \text{ s.} \quad (\text{Answer})$$

There are two such times! This is not really surprising because the ball passes twice through $y = 5.0 \text{ m}$, once on the way up and once on the way down.



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2-6 GRAPHICAL INTEGRATION IN MOTION ANALYSIS

Learning Objectives

After reading this module, you should be able to . . .

2.18 Determine a particle's change in velocity by graphical integration on a graph of acceleration versus time.

2.19 Determine a particle's change in position by graphical integration on a graph of velocity versus time.

Key Ideas

● On a graph of acceleration a versus time t , the change in the velocity is given by

$$v_1 - v_0 = \int_{t_0}^{t_1} a \, dt.$$

The integral amounts to finding an area on the graph:

$$\int_{t_0}^{t_1} a \, dt = \left(\text{area between acceleration curve and time axis, from } t_0 \text{ to } t_1 \right).$$

● On a graph of velocity v versus time t , the change in the position is given by

$$x_1 - x_0 = \int_{t_0}^{t_1} v \, dt,$$

where the integral can be taken from the graph as

$$\int_{t_0}^{t_1} v \, dt = \left(\text{area between velocity curve and time axis, from } t_0 \text{ to } t_1 \right).$$

Graphical Integration in Motion Analysis

Integrating Acceleration. When we have a graph of an object's acceleration a versus time t , we can integrate on the graph to find the velocity at any given time. Because a is defined as $a = dv/dt$, the Fundamental Theorem of Calculus tells us that

$$v_1 - v_0 = \int_{t_0}^{t_1} a \, dt. \quad (2-27)$$

The right side of the equation is a definite integral (it gives a numerical result rather than a function), v_0 is the velocity at time t_0 , and v_1 is the velocity at later time t_1 . The definite integral can be evaluated from an $a(t)$ graph, such as in Fig. 2-14a. In particular,

$$\int_{t_0}^{t_1} a \, dt = \left(\text{area between acceleration curve and time axis, from } t_0 \text{ to } t_1 \right). \quad (2-28)$$

If a unit of acceleration is 1 m/s^2 and a unit of time is 1 s , then the corresponding unit of area on the graph is

$$(1 \text{ m/s}^2)(1 \text{ s}) = 1 \text{ m/s},$$

which is (properly) a unit of velocity. When the acceleration curve is above the time axis, the area is positive; when the curve is below the time axis, the area is negative.

Integrating Velocity. Similarly, because velocity v is defined in terms of the position x as $v = dx/dt$, then

$$x_1 - x_0 = \int_{t_0}^{t_1} v \, dt, \quad (2-29)$$

where x_0 is the position at time t_0 and x_1 is the position at time t_1 . The definite integral on the right side of Eq. 2-29 can be evaluated from a $v(t)$ graph, like that shown in Fig. 2-14b. In particular,

$$\int_{t_0}^{t_1} v \, dt = \left(\text{area between velocity curve and time axis, from } t_0 \text{ to } t_1 \right). \quad (2-30)$$

If the unit of velocity is 1 m/s and the unit of time is 1 s , then the corresponding unit of area on the graph is

$$(1 \text{ m/s})(1 \text{ s}) = 1 \text{ m},$$

which is (properly) a unit of position and displacement. Whether this area is positive or negative is determined as described for the $a(t)$ curve of Fig. 2-14a.

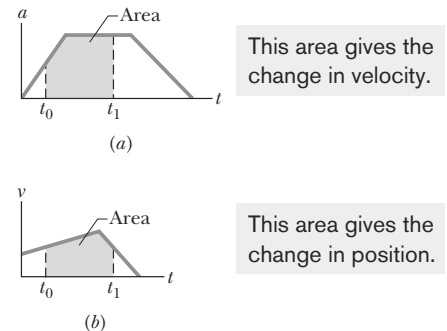


Figure 2-14 The area between a plotted curve and the horizontal time axis, from time t_0 to time t_1 , is indicated for (a) a graph of acceleration a versus t and (b) a graph of velocity v versus t .

Sample Problem 2.06 Graphical integration a versus t , whiplash injury

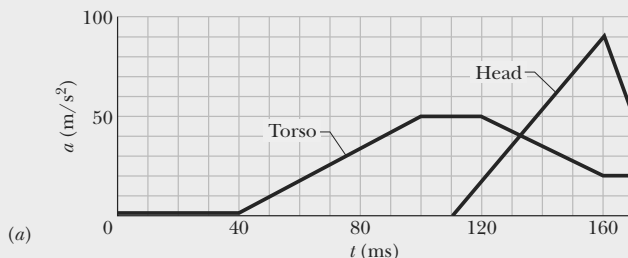
“Whiplash injury” commonly occurs in a rear-end collision where a front car is hit from behind by a second car. In the 1970s, researchers concluded that the injury was due to the occupant’s head being whipped back over the top of the seat as the car was slammed forward. As a result of this finding, head restraints were built into cars, yet neck injuries in rear-end collisions continued to occur.

In a recent test to study neck injury in rear-end collisions, a volunteer was strapped to a seat that was then moved abruptly to simulate a collision by a rear car moving at 10.5 km/h. Figure 2-15a gives the accelerations of the volunteer’s torso and head during the collision, which began at time $t = 0$. The torso acceleration was delayed by 40 ms because during that time interval the seat back had to compress against the volunteer. The head acceleration was delayed by an additional 70 ms. What was the torso speed when the head began to accelerate?

KEY IDEA

We can calculate the torso speed at any time by finding an area on the torso $a(t)$ graph.

Calculations: We know that the initial torso speed is $v_0 = 0$ at time $t_0 = 0$, at the start of the “collision.” We want the torso speed v_1 at time $t_1 = 110$ ms, which is when the head begins to accelerate.



Combining Eqs. 2-27 and 2-28, we can write

$$v_1 - v_0 = \left(\begin{array}{c} \text{area between acceleration curve} \\ \text{and time axis, from } t_0 \text{ to } t_1 \end{array} \right). \quad (2-31)$$

For convenience, let us separate the area into three regions (Fig. 2-15b). From 0 to 40 ms, region A has no area:

$$\text{area}_A = 0.$$

From 40 ms to 100 ms, region B has the shape of a triangle, with area

$$\text{area}_B = \frac{1}{2}(0.060 \text{ s})(50 \text{ m/s}^2) = 1.5 \text{ m/s}.$$

From 100 ms to 110 ms, region C has the shape of a rectangle, with area

$$\text{area}_C = (0.010 \text{ s})(50 \text{ m/s}^2) = 0.50 \text{ m/s}.$$

Substituting these values and $v_0 = 0$ into Eq. 2-31 gives us

$$v_1 - 0 = 0 + 1.5 \text{ m/s} + 0.50 \text{ m/s},$$

or $v_1 = 2.0 \text{ m/s} = 7.2 \text{ km/h}$. (Answer)

Comments: When the head is just starting to move forward, the torso already has a speed of 7.2 km/h. Researchers argue that it is this difference in speeds during the early stage of a rear-end collision that injures the neck. The backward whipping of the head happens later and could, especially if there is no head restraint, increase the injury.

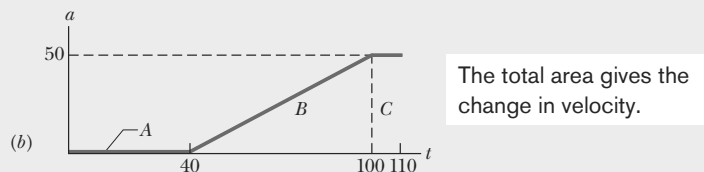


Figure 2-15 (a) The $a(t)$ curve of the torso and head of a volunteer in a simulation of a rear-end collision. (b) Breaking up the region between the plotted curve and the time axis to calculate the area.

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Review & Summary

Position The *position* x of a particle on an x axis locates the particle with respect to the **origin**, or zero point, of the axis. The position is either positive or negative, according to which side of the origin the particle is on, or zero if the particle is at the origin. The **positive direction** on an axis is the direction of increasing positive numbers; the opposite direction is the **negative direction** on the axis.

Displacement The *displacement* Δx of a particle is the change in its position:

$$\Delta x = x_2 - x_1. \quad (2-1)$$

Displacement is a vector quantity. It is positive if the particle has moved in the positive direction of the x axis and negative if the particle has moved in the negative direction.

Average Velocity When a particle has moved from position x_1 to position x_2 during a time interval $\Delta t = t_2 - t_1$, its *average velocity* during that interval is

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}. \quad (2-2)$$

The algebraic sign of v_{avg} indicates the direction of motion (v_{avg} is a vector quantity). Average velocity does not depend on the actual distance a particle moves, but instead depends on its original and final positions.

On a graph of x versus t , the average velocity for a time interval Δt is the slope of the straight line connecting the points on the curve that represent the two ends of the interval.

Average Speed The *average speed* s_{avg} of a particle during a time interval Δt depends on the total distance the particle moves in that time interval:

$$s_{\text{avg}} = \frac{\text{total distance}}{\Delta t}. \quad (2-3)$$

Instantaneous Velocity The *instantaneous velocity* (or simply **velocity**) v of a moving particle is

$$v = \lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}, \quad (2-4)$$

where Δx and Δt are defined by Eq. 2-2. The instantaneous velocity (at a particular time) may be found as the slope (at that particular time) of the graph of x versus t . **Speed** is the magnitude of instantaneous velocity.

Average Acceleration *Average acceleration* is the ratio of a change in velocity Δv to the time interval Δt in which the change occurs:

$$a_{\text{avg}} = \frac{\Delta v}{\Delta t}. \quad (2-7)$$

The algebraic sign indicates the direction of a_{avg} .

Instantaneous Acceleration *Instantaneous acceleration* (or simply **acceleration**) a is the first time derivative of velocity $v(t)$

and the second time derivative of position $x(t)$:

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2}. \quad (2-8, 2-9)$$

On a graph of v versus t , the acceleration a at any time t is the slope of the curve at the point that represents t .

Constant Acceleration The five equations in Table 2-1 describe the motion of a particle with constant acceleration:

$$v = v_0 + at, \quad (2-11)$$

$$x - x_0 = v_0t + \frac{1}{2}at^2, \quad (2-15)$$

$$v^2 = v_0^2 + 2a(x - x_0), \quad (2-16)$$

$$x - x_0 = \frac{1}{2}(v_0 + v)t, \quad (2-17)$$

$$x - x_0 = vt - \frac{1}{2}at^2. \quad (2-18)$$

These are *not* valid when the acceleration is not constant.

Free-Fall Acceleration An important example of straight-line motion with constant acceleration is that of an object rising or falling freely near Earth's surface. The constant acceleration equations describe this motion, but we make two changes in notation: (1) we refer the motion to the vertical y axis with $+y$ vertically *up*; (2) we replace a with $-g$, where g is the magnitude of the free-fall acceleration. Near Earth's surface, $g = 9.8 \text{ m/s}^2 (= 32 \text{ ft/s}^2)$.

Questions

1 Figure 2-16 gives the velocity of a particle moving on an x axis. What are (a) the initial and (b) the final directions of travel? (c) Does the particle stop momentarily? (d) Is the acceleration positive or negative? (e) Is it constant or varying?

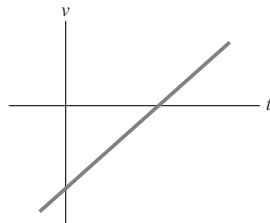


Figure 2-16 Question 1.

2 Figure 2-17 gives the acceleration $a(t)$ of a Chihuahua as it chases a German shepherd along an axis. In which of the time periods indicated does the Chihuahua move at constant speed?

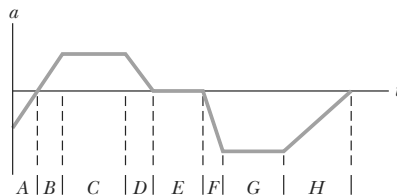


Figure 2-17 Question 2.

3 Figure 2-18 shows four paths along which objects move from a starting point to a final point, all in the same time interval. The paths pass over a grid of equally spaced straight lines. Rank the paths according to (a) the average velocity of the objects and (b) the average speed of the objects, greatest first.

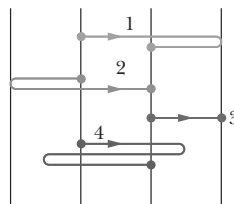


Figure 2-18 Question 3.

4 Figure 2-19 is a graph of a particle's position along an x axis versus time. (a) At time $t = 0$, what

is the sign of the particle's position? Is the particle's velocity positive, negative, or 0 at (b) $t = 1 \text{ s}$, (c) $t = 2 \text{ s}$, and (d) $t = 3 \text{ s}$? (e) How many times does the particle go through the point $x = 0$?

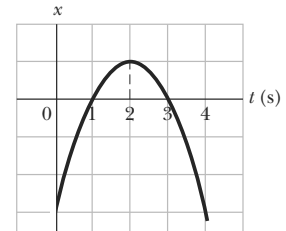


Figure 2-19 Question 4.

5 Figure 2-20 gives the velocity of a particle moving along an axis. Point 1 is at the highest point on the curve; point 4 is at the lowest point; and points 2 and 6 are at the same height. What is the direction of travel at (a) time $t = 0$ and (b) point 4? (c) At which of the six numbered points does the particle reverse its direction of travel? (d) Rank the six points according to the magnitude of the acceleration, greatest first.

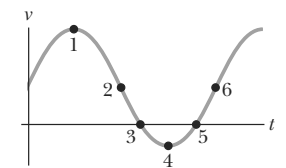


Figure 2-20 Question 5.

6 At $t = 0$, a particle moving along an x axis is at position $x_0 = -20 \text{ m}$. The signs of the particle's initial velocity v_0 (at time t_0) and constant acceleration a are, respectively, for four situations: (1) $+, +$; (2) $+, -$; (3) $-, +$; (4) $-, -$. In which situations will the particle (a) stop momentarily, (b) pass through the origin, and (c) never pass through the origin?

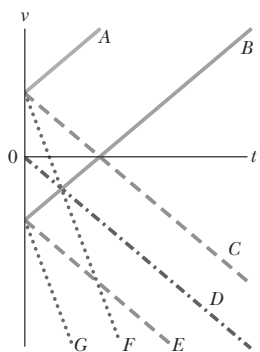


Figure 2-21 Question 7.

7 Hanging over the railing of a bridge, you drop an egg (no initial velocity) as you throw a second egg downward. Which curves in Fig. 2-21

give the velocity $v(t)$ for (a) the dropped egg and (b) the thrown egg? (Curves A and B are parallel; so are C , D , and E ; so are F and G .)

8 The following equations give the velocity $v(t)$ of a particle in four situations: (a) $v = 3$; (b) $v = 4t^2 + 2t - 6$; (c) $v = 3t - 4$; (d) $v = 5t^2 - 3$. To which of these situations do the equations of Table 2-1 apply?

9 In Fig. 2-22, a cream tangerine is thrown directly upward past three evenly spaced windows of equal heights. Rank the windows according to (a) the average speed of the cream tangerine while passing them, (b) the time the cream tangerine takes to pass them, (c) the magnitude of the acceleration of the cream tangerine while passing them, and (d) the change Δv in the speed of the cream tangerine during the passage, greatest first.

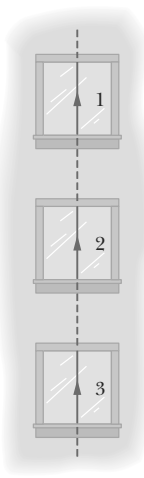


Figure 2-22
Question 9.

10 Suppose that a passenger intent on lunch during his first ride in a hot-air balloon accidentally drops an apple over the side during the balloon's liftoff. At the moment of the

apple's release, the balloon is accelerating upward with a magnitude of 4.0 m/s^2 and has an upward velocity of magnitude 2 m/s . What are the (a) magnitude and (b) direction of the acceleration of the apple just after it is released? (c) Just then, is the apple moving upward or downward, or is it stationary? (d) What is the magnitude of its velocity just then? (e) In the next few moments, does the speed of the apple increase, decrease, or remain constant?

11 Figure 2-23 shows that a particle moving along an x axis undergoes three periods of acceleration. Without written computation, rank the acceleration periods according to the increases they produce in the particle's velocity, greatest first.

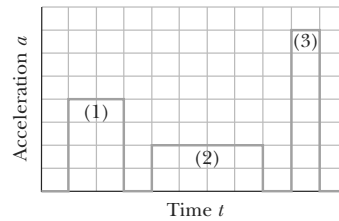


Figure 2-23 Question 11.

Problems



Tutoring problem available (at instructor's discretion) in WileyPLUS and WebAssign



Worked-out solution available in Student Solutions Manual

WWW Worked-out solution is at



Number of dots indicates level of problem difficulty

ILW Interactive solution is at

<http://www.wiley.com/college/halliday>



Additional information available in *The Flying Circus of Physics* and at flyingcircusofphysics.com

Module 2-1 Position, Displacement, and Average Velocity

•1 While driving a car at 90 km/h , how far do you move while your eyes shut for 0.50 s during a hard sneeze?

•2 Compute your average velocity in the following two cases: (a) You walk 73.2 m at a speed of 1.22 m/s and then run 73.2 m at a speed of 3.05 m/s along a straight track. (b) You walk for 1.00 min at a speed of 1.22 m/s and then run for 1.00 min at 3.05 m/s along a straight track. (c) Graph x versus t for both cases and indicate how the average velocity is found on the graph.

•3 SSM WWW An automobile travels on a straight road for 40 km at 30 km/h . It then continues in the same direction for another 40 km at 60 km/h . (a) What is the average velocity of the car during the full 80 km trip? (Assume that it moves in the positive x direction.) (b) What is the average speed? (c) Graph x versus t and indicate how the average velocity is found on the graph.

•4 A car moves uphill at 40 km/h and then back downhill at 60 km/h . What is the average speed for the round trip?

•5 SSM The position of an object moving along an x axis is given by $x = 3t - 4t^2 + t^3$, where x is in meters and t in seconds. Find the position of the object at the following values of t : (a) 1 s , (b) 2 s , (c) 3 s , and (d) 4 s . (e) What is the object's displacement between $t = 0$ and $t = 4 \text{ s}$? (f) What is its average velocity for the time interval from $t = 2 \text{ s}$ to $t = 4 \text{ s}$? (g) Graph x versus t for $0 \leq t \leq 4 \text{ s}$ and indicate how the answer for (f) can be found on the graph.

•6 The 1992 world speed record for a bicycle (human-powered vehicle) was set by Chris Huber. His time through the measured 200 m stretch was a sizzling 6.509 s , at which he commented,

"Cogito ergo zoom!" (I think, therefore I go fast!). In 2001, Sam Whittingham beat Huber's record by 19.0 km/h . What was Whittingham's time through the 200 m ?

••7 Two trains, each having a speed of 30 km/h , are headed at each other on the same straight track. A bird that can fly 60 km/h flies off the front of one train when they are 60 km apart and heads directly for the other train. On reaching the other train, the (crazy) bird flies directly back to the first train, and so forth. What is the total distance the bird travels before the trains collide?

••8 **Panic escape.** Figure 2-24 shows a general situation in which a stream of people attempt to escape through an exit door that turns out to be locked. The people move toward the door at speed $v_s = 3.50 \text{ m/s}$, are each $d = 0.25 \text{ m}$ in depth, and are separated by $L = 1.75 \text{ m}$. The arrangement in Fig. 2-24 occurs at time $t = 0$. (a) At what average rate does the layer of people at the door increase? (b) At what time does the layer's depth reach 5.0 m ? (The answers reveal how quickly such a situation becomes dangerous.)

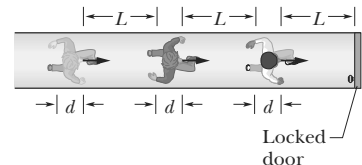


Figure 2-24 Problem 8.

••9 ILW In 1 km races, runner 1 on track 1 (with time $2 \text{ min}, 27.95 \text{ s}$) appears to be faster than runner 2 on track 2 ($2 \text{ min}, 28.15 \text{ s}$). However, length L_2 of track 2 might be slightly greater than length L_1 of track 1. How large can $L_2 - L_1$ be for us still to conclude that runner 1 is faster?

••10 To set a speed record in a measured (straight-line) distance d , a race car must be driven first in one direction (in time t_1) and then in the opposite direction (in time t_2). (a) To eliminate the effects of the wind and obtain the car's speed v_c in a windless situation, should we find the average of d/t_1 and d/t_2 (method 1) or should we divide d by the average of t_1 and t_2 ? (b) What is the fractional difference in the two methods when a steady wind blows along the car's route and the ratio of the wind speed v_w to the car's speed v_c is 0.0240?

••11 You are to drive 300 km to an interview. The interview is at 11:15 A.M. You plan to drive at 100 km/h, so you leave at 8:00 A.M. to allow some extra time. You drive at that speed for the first 100 km, but then construction work forces you to slow to 40 km/h for 40 km. What would be the least speed needed for the rest of the trip to arrive in time for the interview?

••12 **Traffic shock wave.** An abrupt slowdown in concentrated traffic can travel as a pulse, termed a *shock wave*, along the line of cars, either downstream (in the traffic direction) or upstream, or it can be stationary. Figure 2-25 shows a uniformly spaced line of cars moving at speed $v = 25.0$ m/s toward a uniformly spaced line of slow cars moving at speed $v_s = 5.00$ m/s. Assume that each faster car adds length $L = 12.0$ m (car length plus buffer zone) to the line of slow cars when it joins the line, and assume it slows abruptly at the last instant. (a) For what separation distance d between the faster cars does the shock wave remain stationary? If the separation is twice that amount, what are the (b) speed and (c) direction (upstream or downstream) of the shock wave?

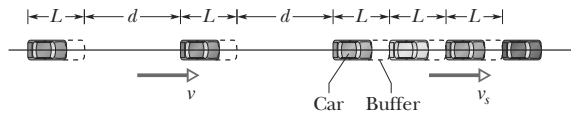


Figure 2-25 Problem 12.

••13 You drive on Interstate 10 from San Antonio to Houston, half the *time* at 55 km/h and the other half at 90 km/h. On the way back you travel half the *distance* at 55 km/h and the other half at 90 km/h. What is your average speed (a) from San Antonio to Houston, (b) from Houston back to San Antonio, and (c) for the entire trip? (d) What is your average velocity for the entire trip? (e) Sketch x versus t for (a), assuming the motion is all in the positive x direction. Indicate how the average velocity can be found on the sketch.

Module 2-2 Instantaneous Velocity and Speed

•14 An electron moving along the x axis has a position given by $x = 16te^{-t}$ m, where t is in seconds. How far is the electron from the origin when it momentarily stops?

•15 (a) If a particle's position is given by $x = 4 - 12t + 3t^2$ (where t is in seconds and x is in meters), what is its velocity at $t = 1$ s? (b) Is it moving in the positive or negative direction of x just then? (c) What is its speed just then? (d) Is the speed increasing or decreasing just then? (Try answering the next two questions without further calculation.) (e) Is there ever an instant when the velocity is zero? If so, give the time t ; if not, answer no. (f) Is there a time after $t = 3$ s when the particle is moving in the negative direction of x ? If so, give the time t ; if not, answer no.

•16 The position function $x(t)$ of a particle moving along an x axis is $x = 4.0 - 6.0t^2$, with x in meters and t in seconds. (a) At what time and (b) where does the particle (momentarily) stop? At what (c) negative time and (d) positive time does the particle pass through the origin? (e) Graph x versus t for the range -5 s to $+5$ s. (f) To shift the curve rightward on the graph, should we include the

term $+20t$ or the term $-20t$ in $x(t)$? (g) Does that inclusion increase or decrease the value of x at which the particle momentarily stops?

••17 The position of a particle moving along the x axis is given in centimeters by $x = 9.75 + 1.50t^3$, where t is in seconds. Calculate (a) the average velocity during the time interval $t = 2.00$ s to $t = 3.00$ s; (b) the instantaneous velocity at $t = 2.00$ s; (c) the instantaneous velocity at $t = 3.00$ s; (d) the instantaneous velocity at $t = 2.50$ s; and (e) the instantaneous velocity when the particle is midway between its positions at $t = 2.00$ s and $t = 3.00$ s. (f) Graph x versus t and indicate your answers graphically.

Module 2-3 Acceleration

•18 The position of a particle moving along an x axis is given by $x = 12t^2 - 2t^3$, where x is in meters and t is in seconds. Determine (a) the position, (b) the velocity, and (c) the acceleration of the particle at $t = 3.0$ s. (d) What is the maximum positive coordinate reached by the particle and (e) at what time is it reached? (f) What is the maximum positive velocity reached by the particle and (g) at what time is it reached? (h) What is the acceleration of the particle at the instant the particle is not moving (other than at $t = 0$)? (i) Determine the average velocity of the particle between $t = 0$ and $t = 3$ s.

•19 At a certain time a particle had a speed of 18 m/s in the positive x direction, and 2.4 s later its speed was 30 m/s in the opposite direction. What is the average acceleration of the particle during this 2.4 s interval?

•20 (a) If the position of a particle is given by $x = 20t - 5t^3$, where x is in meters and t is in seconds, when, if ever, is the particle's velocity zero? (b) When is its acceleration a zero? (c) For what time range (positive or negative) is a negative? (d) Positive? (e) Graph $x(t)$, $v(t)$, and $a(t)$.

••21 From $t = 0$ to $t = 5.00$ min, a man stands still, and from $t = 5.00$ min to $t = 10.0$ min, he walks briskly in a straight line at a constant speed of 2.20 m/s. What are (a) his average velocity v_{avg} and (b) his average acceleration a_{avg} in the time interval 2.00 min to 8.00 min? What are (c) v_{avg} and (d) a_{avg} in the time interval 3.00 min to 9.00 min? (e) Sketch x versus t and v versus t , and indicate how the answers to (a) through (d) can be obtained from the graphs.

••22 The position of a particle moving along the x axis depends on the time according to the equation $x = ct^2 - bt^3$, where x is in meters and t is in seconds. What are the units of (a) constant c and (b) constant b ? Let their numerical values be 3.0 and 2.0, respectively. (c) At what time does the particle reach its maximum positive x position? From $t = 0.0$ s to $t = 4.0$ s, (d) what distance does the particle move and (e) what is its displacement? Find its velocity at times (f) 1.0 s, (g) 2.0 s, (h) 3.0 s, and (i) 4.0 s. Find its acceleration at times (j) 1.0 s, (k) 2.0 s, (l) 3.0 s, and (m) 4.0 s.

Module 2-4 Constant Acceleration

•23 An electron with an initial velocity $v_0 = 1.50 \times 10^5$ m/s enters a region of length $L = 1.00$ cm where it is electrically accelerated (Fig. 2-26). It emerges with $v = 5.70 \times 10^6$ m/s. What is its acceleration, assumed constant?

•24 **Catapulting mushrooms.** Certain mushrooms launch their spores by a catapult mechanism. As water condenses from the air onto a spore that is attached to

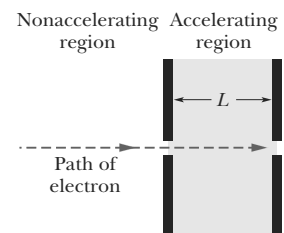


Figure 2-26 Problem 23.

the mushroom, a drop grows on one side of the spore and a film grows on the other side. The spore is bent over by the drop's weight, but when the film reaches the drop, the drop's water suddenly spreads into the film and the spore springs upward so rapidly that it is slung off into the air. Typically, the spore reaches a speed of 1.6 m/s in a 5.0 μm launch; its speed is then reduced to zero in 1.0 mm by the air. Using those data and assuming constant accelerations, find the acceleration in terms of g during (a) the launch and (b) the speed reduction.

•25 An electric vehicle starts from rest and accelerates at a rate of 2.0 m/s² in a straight line until it reaches a speed of 20 m/s. The vehicle then slows at a constant rate of 1.0 m/s² until it stops. (a) How much time elapses from start to stop? (b) How far does the vehicle travel from start to stop?

•26 A muon (an elementary particle) enters a region with a speed of 5.00×10^6 m/s and then is slowed at the rate of 1.25×10^{14} m/s². (a) How far does the muon take to stop? (b) Graph x versus t and v versus t for the muon.

•27 An electron has a constant acceleration of +3.2 m/s². At a certain instant its velocity is +9.6 m/s. What is its velocity (a) 2.5 s earlier and (b) 2.5 s later?

•28 On a dry road, a car with good tires may be able to brake with a constant deceleration of 4.92 m/s². (a) How long does such a car, initially traveling at 24.6 m/s, take to stop? (b) How far does it travel in this time? (c) Graph x versus t and v versus t for the deceleration.

•29 **ILW** A certain elevator cab has a total run of 190 m and a maximum speed of 305 m/min, and it accelerates from rest and then back to rest at 1.22 m/s². (a) How far does the cab move while accelerating to full speed from rest? (b) How long does it take to make the nonstop 190 m run, starting and ending at rest?

•30 The brakes on your car can slow you at a rate of 5.2 m/s². (a) If you are going 137 km/h and suddenly see a state trooper, what is the minimum time in which you can get your car under the 90 km/h speed limit? (The answer reveals the futility of braking to keep your high speed from being detected with a radar or laser gun.) (b) Graph x versus t and v versus t for such a slowing.

•31 **SSM** Suppose a rocket ship in deep space moves with constant acceleration equal to 9.8 m/s², which gives the illusion of normal gravity during the flight. (a) If it starts from rest, how long will it take to acquire a speed one-tenth that of light, which travels at 3.0×10^8 m/s? (b) How far will it travel in so doing?

•32 **✎** A world's land speed record was set by Colonel John P. Stapp when in March 1954 he rode a rocket-propelled sled that moved along a track at 1020 km/h. He and the sled were brought to a stop in 1.4 s. (See Fig. 2-7.) In terms of g , what acceleration did he experience while stopping?

•33 **SSM ILW** A car traveling 56.0 km/h is 24.0 m from a barrier when the driver slams on the brakes. The car hits the barrier 2.00 s later. (a) What is the magnitude of the car's constant acceleration before impact? (b) How fast is the car traveling at impact?

•34 **GO** In Fig. 2-27, a red car and a green car, identical except for the color, move toward each other in adjacent lanes and parallel to an x axis. At time $t = 0$, the red car is at $x_r = 0$ and the green car is at $x_g = 220$ m. If the red car has a constant velocity of 20 km/h, the cars pass each other at $x = 44.5$ m, and if it has a constant velocity of 40 km/h, they pass each other at $x = 76.6$ m. What are (a) the initial velocity and (b) the constant acceleration of the green car?



Figure 2-27 Problems 34 and 35.

•35 Figure 2-27 shows a red car and a green car that move toward each other. Figure 2-28 is a graph of their motion, showing the positions $x_{g0} = 270$ m and $x_{r0} = -35.0$ m at time $t = 0$. The green car has a constant speed of 20.0 m/s and the red car begins from rest. What is the acceleration magnitude of the red car?

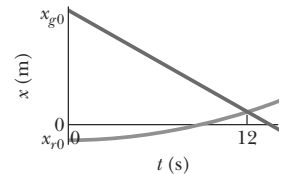


Figure 2-28 Problem 35.

•36 A car moves along an x axis through a distance of 900 m, starting at rest (at $x = 0$) and ending at rest (at $x = 900$ m). Through the first $\frac{1}{4}$ of that distance, its acceleration is +2.25 m/s². Through the rest of that distance, its acceleration is -0.750 m/s². What are (a) its travel time through the 900 m and (b) its maximum speed? (c) Graph position x , velocity v , and acceleration a versus time t for the trip.

•37 Figure 2-29 depicts the motion of a particle moving along an x axis with a constant acceleration. The figure's vertical scaling is set by $x_s = 6.0$ m. What are the (a) magnitude and (b) direction of the particle's acceleration?

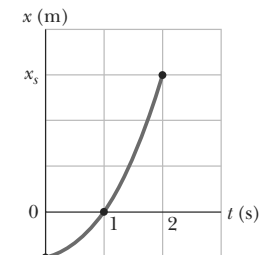


Figure 2-29 Problem 37.

•38 (a) If the maximum acceleration that is tolerable for passengers in a subway train is 1.34 m/s² and subway stations are located 806 m apart, what is the maximum speed a subway train can attain between stations? (b) What is the travel time between stations? (c) If a subway train stops for 20 s at each station, what is the maximum average speed of the train, from one start-up to the next? (d) Graph x , v , and a versus t for the interval from one start-up to the next.

•39 Cars A and B move in the same direction in adjacent lanes. The position x of car A is given in Fig. 2-30, from time $t = 0$ to $t = 7.0$ s. The figure's vertical scaling is set by $x_s = 32.0$ m. At $t = 0$, car B is at $x = 0$, with a velocity of 12 m/s and a negative constant acceleration a_B . (a) What must a_B be such that the cars are (momentarily) side by side (momentarily at the same value of x) at $t = 4.0$ s? (b) For that value of a_B , how many times are the cars side by side? (c) Sketch the position x of car B versus time t on Fig. 2-30. How many times will the cars be side by side if the magnitude of acceleration a_B is (d) more than and (e) less than the answer to part (a)?

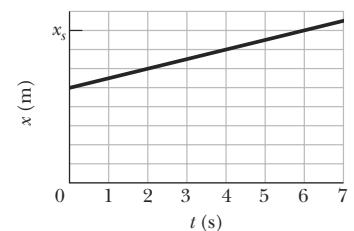


Figure 2-30 Problem 39.

•40 **✎** You are driving toward a traffic signal when it turns yellow. Your speed is the legal speed limit of $v_0 = 55$ km/h; your best deceleration rate has the magnitude $a = 5.18$ m/s². Your best reaction time to begin braking is $T = 0.75$ s. To avoid having the front of your car enter the intersection after the light turns red, should you brake to a stop or continue to move at 55 km/h if the distance to

the intersection and the duration of the yellow light are (a) 40 m and 2.8 s, and (b) 32 m and 1.8 s? Give an answer of brake, continue, either (if either strategy works), or neither (if neither strategy works and the yellow duration is inappropriate).

••41 **GO** As two trains move along a track, their conductors suddenly notice that they are headed toward each other. Figure 2-31 gives their velocities v as functions of time t as the conductors slow the trains. The figure's vertical scaling is set by $v_s = 40.0$ m/s. The slowing processes begin when the trains are 200 m apart. What is their separation when both trains have stopped?

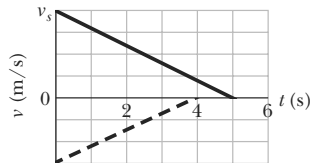


Figure 2-31 Problem 41.

••42 **GO** You are arguing over a cell phone while trailing an unmarked police car by 25 m; both your car and the police car are traveling at 110 km/h. Your argument diverts your attention from the police car for 2.0 s (long enough for you to look at the phone and yell, "I won't do that!"). At the beginning of that 2.0 s, the police officer begins braking suddenly at 5.0 m/s². (a) What is the separation between the two cars when your attention finally returns? Suppose that you take another 0.40 s to realize your danger and begin braking. (b) If you too brake at 5.0 m/s², what is your speed when you hit the police car?

••43 **GO** When a high-speed passenger train traveling at 161 km/h rounds a bend, the engineer is shocked to see that a locomotive has improperly entered onto the track from a siding and is a distance $D = 676$ m ahead (Fig. 2-32). The locomotive is moving at 29.0 km/h. The engineer of the high-speed train immediately applies the brakes. (a) What must be the magnitude of the resulting constant deceleration if a collision is to be just avoided? (b) Assume that the engineer is at $x = 0$ when, at $t = 0$, he first spots the locomotive. Sketch $x(t)$ curves for the locomotive and high-speed train for the cases in which a collision is just avoided and is not quite avoided.

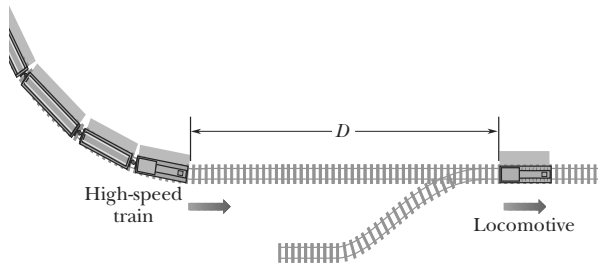


Figure 2-32 Problem 43.

Module 2-5 Free-Fall Acceleration

•44 When startled, an armadillo will leap upward. Suppose it rises 0.544 m in the first 0.200 s. (a) What is its initial speed as it leaves the ground? (b) What is its speed at the height of 0.544 m? (c) How much higher does it go?

•45 **SSM WWW** (a) With what speed must a ball be thrown vertically from ground level to rise to a maximum height of 50 m? (b) How long will it be in the air? (c) Sketch graphs of y , v , and a versus t for the ball. On the first two graphs, indicate the time at which 50 m is reached.

•46 Raindrops fall 1700 m from a cloud to the ground. (a) If they were not slowed by air resistance, how fast would the drops be moving when they struck the ground? (b) Would it be safe to walk outside during a rainstorm?

•47 **SSM** At a construction site a pipe wrench struck the ground with a speed of 24 m/s. (a) From what height was it inadvertently dropped? (b) How long was it falling? (c) Sketch graphs of y , v , and a versus t for the wrench.

•48 A hoodlum throws a stone vertically downward with an initial speed of 12.0 m/s from the roof of a building, 30.0 m above the ground. (a) How long does it take the stone to reach the ground? (b) What is the speed of the stone at impact?

•49 **SSM** A hot-air balloon is ascending at the rate of 12 m/s and is 80 m above the ground when a package is dropped over the side. (a) How long does the package take to reach the ground? (b) With what speed does it hit the ground?

•50 At time $t = 0$, apple 1 is dropped from a bridge onto a roadway beneath the bridge; somewhat later, apple 2 is thrown down from the same height. Figure 2-33 gives the vertical positions y of the apples versus t during the falling, until both apples have hit the roadway. The scaling is set by $t_s = 2.0$ s. With approximately what speed is apple 2 thrown down?

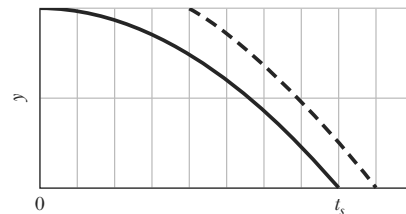


Figure 2-33 Problem 50.

•51 As a runaway scientific balloon ascends at 19.6 m/s, one of its instrument packages breaks free of a harness and free-falls. Figure 2-34 gives the vertical velocity of the package versus time, from before it breaks free to when it reaches the ground. (a) What maximum height above the break-free point does it rise? (b) How high is the break-free point above the ground?

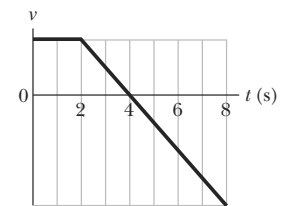


Figure 2-34 Problem 51.

•52 **GO** A bolt is dropped from a bridge under construction, falling 90 m to the valley below the bridge. (a) In how much time does it pass through the last 20% of its fall? What is its speed (b) when it begins that last 20% of its fall and (c) when it reaches the valley beneath the bridge?

•53 **SSM ILW** A key falls from a bridge that is 45 m above the water. It falls directly into a model boat, moving with constant velocity, that is 12 m from the point of impact when the key is released. What is the speed of the boat?

•54 **GO** A stone is dropped into a river from a bridge 43.9 m above the water. Another stone is thrown vertically down 1.00 s after the first is dropped. The stones strike the water at the same time. (a) What is the initial speed of the second stone? (b) Plot velocity versus time on a graph for each stone, taking zero time as the instant the first stone is released.

••55 SSM A ball of moist clay falls 15.0 m to the ground. It is in contact with the ground for 20.0 ms before stopping. (a) What is the magnitude of the average acceleration of the ball during the time it is in contact with the ground? (Treat the ball as a particle.) (b) Is the average acceleration up or down?

••56 GO Figure 2-35 shows the speed v versus height y of a ball tossed directly upward, along a y axis. Distance d is 0.40 m. The speed at height y_A is v_A . The speed at height y_B is $\frac{1}{3}v_A$. What is speed v_A ?

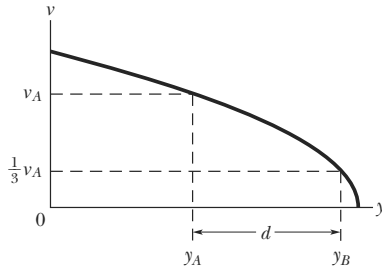


Figure 2-35 Problem 56.

••57 To test the quality of a tennis ball, you drop it onto the floor from a height of 4.00 m. It rebounds to a height of 2.00 m. If the ball is in contact with the floor for 12.0 ms, (a) what is the magnitude of its average acceleration during that contact and (b) is the average acceleration up or down?

••58 An object falls a distance h from rest. If it travels $0.50h$ in the last 1.00 s, find (a) the time and (b) the height of its fall. (c) Explain the physically unacceptable solution of the quadratic equation in t that you obtain.

••59 Water drips from the nozzle of a shower onto the floor 200 cm below. The drops fall at regular (equal) intervals of time, the first drop striking the floor at the instant the fourth drop begins to fall. When the first drop strikes the floor, how far below the nozzle are the (a) second and (b) third drops?

••60 GO A rock is thrown vertically upward from ground level at time $t = 0$. At $t = 1.5$ s it passes the top of a tall tower, and 1.0 s later it reaches its maximum height. What is the height of the tower?

••61 GO A steel ball is dropped from a building's roof and passes a window, taking 0.125 s to fall from the top to the bottom of the window, a distance of 1.20 m. It then falls to a sidewalk and bounces back past the window, moving from bottom to top in 0.125 s. Assume that the upward flight is an exact reverse of the fall. The time the ball spends below the bottom of the window is 2.00 s. How tall is the building?

••62 A basketball player grabbing a rebound jumps 76.0 cm vertically. How much total time (ascent and descent) does the player spend (a) in the top 15.0 cm of this jump and (b) in the bottom 15.0 cm? (The player seems to hang in the air at the top.)

••63 GO A drowsy cat spots a flowerpot that sails first up and then down past an open window. The pot is in view for a total of 0.50 s, and the top-to-bottom height of the window is 2.00 m. How high above the window top does the flowerpot go?

••64 A ball is shot vertically upward from the surface of another planet. A plot of y versus t for the ball is shown in Fig. 2-36, where y is the height of the ball above its starting point and $t = 0$ at the instant the ball is shot. The figure's vertical scaling is set by $y_s = 30.0$ m. What are the magnitudes of (a) the free-fall acceleration on the planet and (b) the initial velocity of the ball?

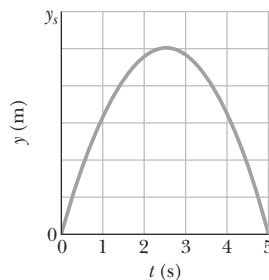


Figure 2-36 Problem 64.

Module 2-6 Graphical Integration in Motion Analysis

••65 Figure 2-15a gives the acceleration of a volunteer's head and torso during a rear-end collision. At maximum head acceleration, what is the speed of (a) the head and (b) the torso?

••66 In a forward punch in karate, the fist begins at rest at the waist and is brought rapidly forward until the arm is fully extended. The speed $v(t)$ of the fist is given in Fig. 2-37 for someone skilled in karate. The vertical scaling is set by $v_s = 8.0$ m/s. How far has the fist moved at (a) time $t = 50$ ms and (b) when the speed of the fist is maximum?

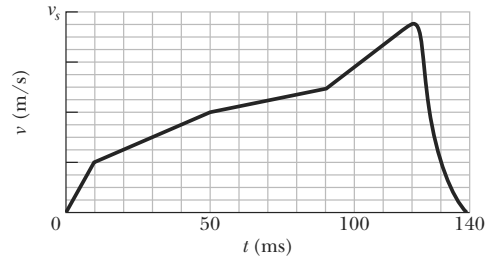


Figure 2-37 Problem 66.

••67 When a soccer ball is kicked toward a player and the player deflects the ball by "heading" it, the acceleration of the head during the collision can be significant. Figure 2-38 gives the measured acceleration

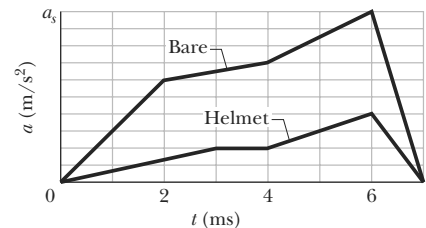


Figure 2-38 Problem 67.

$a(t)$ of a soccer player's head for a bare head and a helmeted head, starting from rest. The scaling on the vertical axis is set by $a_s = 200$ m/s². At time $t = 7.0$ ms, what is the difference in the speed acquired by the bare head and the speed acquired by the helmeted head?

••68 A salamander of the genus *Hydromantes* captures prey by launching its tongue as a projectile: The skeletal part of the tongue is shot forward, unfolding the rest of the tongue, until the outer portion lands on the prey, sticking to it. Figure 2-39 shows the acceleration magnitude a versus time t for the acceleration phase of the launch in a typical situation. The indicated accelerations are $a_2 = 400$ m/s² and $a_1 = 100$ m/s². What is the outward speed of the tongue at the end of the acceleration phase?

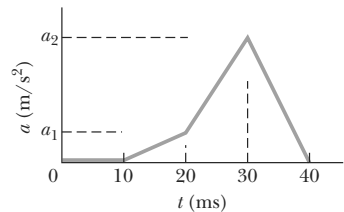


Figure 2-39 Problem 68.

The indicated accelerations are $a_2 = 400$ m/s² and $a_1 = 100$ m/s². What is the outward speed of the tongue at the end of the acceleration phase?

••69 ILW How far does the runner whose velocity–time graph is shown in Fig. 2-40 travel in 16 s? The figure's vertical scaling is set by $v_s = 8.0$ m/s.

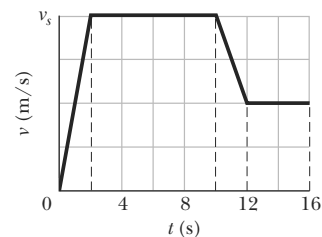


Figure 2-40 Problem 69.

***70 Two particles move along an x axis. The position of particle 1 is given by $x = 6.00t^2 + 3.00t + 2.00$ (in meters and seconds); the acceleration of particle 2 is given by $a = -8.00t$ (in meters per second squared and seconds) and, at $t = 0$, its velocity is 20 m/s. When the velocities of the particles match, what is their velocity?

Additional Problems

71 In an arcade video game, a spot is programmed to move across the screen according to $x = 9.00t - 0.750t^3$, where x is distance in centimeters measured from the left edge of the screen and t is time in seconds. When the spot reaches a screen edge, at either $x = 0$ or $x = 15.0$ cm, t is reset to 0 and the spot starts moving again according to $x(t)$. (a) At what time after starting is the spot instantaneously at rest? (b) At what value of x does this occur? (c) What is the spot's acceleration (including sign) when this occurs? (d) Is it moving right or left just prior to coming to rest? (e) Just after? (f) At what time $t > 0$ does it first reach an edge of the screen?

72 A rock is shot vertically upward from the edge of the top of a tall building. The rock reaches its maximum height above the top of the building 1.60 s after being shot. Then, after barely missing the edge of the building as it falls downward, the rock strikes the ground 6.00 s after it is launched. In SI units: (a) with what upward velocity is the rock shot, (b) what maximum height above the top of the building is reached by the rock, and (c) how tall is the building?

73 GO At the instant the traffic light turns green, an automobile starts with a constant acceleration a of 2.2 m/s^2 . At the same instant a truck, traveling with a constant speed of 9.5 m/s , overtakes and passes the automobile. (a) How far beyond the traffic signal will the automobile overtake the truck? (b) How fast will the automobile be traveling at that instant?

74 A pilot flies horizontally at 1300 km/h , at height $h = 35 \text{ m}$ above initially level ground. However, at time $t = 0$, the pilot begins to fly over ground sloping upward at angle $\theta = 4.3^\circ$ (Fig. 2-41). If the pilot does not change the airplane's heading, at what time t does the plane strike the ground?

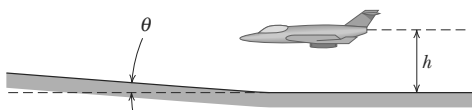


Figure 2-41 Problem 74.

75 GO To stop a car, first you require a certain reaction time to begin braking; then the car slows at a constant rate. Suppose that the total distance moved by your car during these two phases is 56.7 m when its initial speed is 80.5 km/h , and 24.4 m when its initial speed is 48.3 km/h . What are (a) your reaction time and (b) the magnitude of the acceleration?

76 GO Figure 2-42 shows part of a street where traffic flow is to be controlled to allow a platoon of cars to move smoothly along the street. Suppose that the platoon leaders have just

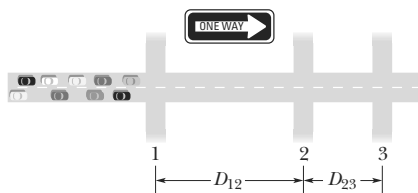


Figure 2-42 Problem 76.

reached intersection 2, where the green appeared when they were distance d from the intersection. They continue to travel at a certain speed v_p (the speed limit) to reach intersection 3, where the green appears when they are distance d from it. The intersections are separated by distances D_{23} and D_{12} . (a) What should be the time delay of the onset of green at intersection 3 relative to that at intersection 2 to keep the platoon moving smoothly?

Suppose, instead, that the platoon had been stopped by a red light at intersection 1. When the green comes on there, the leaders require a certain time t_r to respond to the change and an additional time to accelerate at some rate a to the cruising speed v_p . (b) If the green at intersection 2 is to appear when the leaders are distance d from that intersection, how long after the light at intersection 1 turns green should the light at intersection 2 turn green?

77 SSM A hot rod can accelerate from 0 to 60 km/h in 5.4 s . (a) What is its average acceleration, in m/s^2 , during this time? (b) How far will it travel during the 5.4 s , assuming its acceleration is constant? (c) From rest, how much time would it require to go a distance of 0.25 km if its acceleration could be maintained at the value in (a)?

78 GO A red train traveling at 72 km/h and a green train traveling at 144 km/h are headed toward each other along a straight, level track. When they are 950 m apart, each engineer sees the other's train and applies the brakes. The brakes slow each train at the rate of 1.0 m/s^2 . Is there a collision? If so, answer yes and give the speed of the red train and the speed of the green train at impact, respectively. If not, answer no and give the separation between the trains when they stop.

79 GO At time $t = 0$, a rock climber accidentally allows a piton to fall freely from a high point on the rock wall to the valley below him. Then, after a short delay, his climbing partner, who is 10 m higher on the wall, throws a piton downward. The positions y of the pitons versus t during the falling are given in Fig. 2-43.

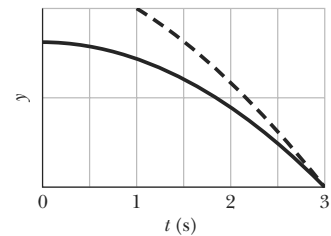


Figure 2-43 Problem 79.

With what speed is the second piton thrown?

80 A train started from rest and moved with constant acceleration. At one time it was traveling 30 m/s , and 160 m farther on it was traveling 50 m/s . Calculate (a) the acceleration, (b) the time required to travel the 160 m mentioned, (c) the time required to attain the speed of 30 m/s , and (d) the distance moved from rest to the time the train had a speed of 30 m/s . (e) Graph x versus t and v versus t for the train, from rest.

81 SSM A particle's acceleration along an x axis is $a = 5.0t$, with t in seconds and a in meters per second squared. At $t = 2.0 \text{ s}$, its velocity is $+17 \text{ m/s}$. What is its velocity at $t = 4.0 \text{ s}$?

82 Figure 2-44 gives the acceleration a versus time t for a particle moving along an x axis. The a -axis scale is set by $a_s = 12.0 \text{ m/s}^2$. At $t = -2.0 \text{ s}$, the particle's velocity is 7.0 m/s . What is its velocity at $t = 6.0 \text{ s}$?

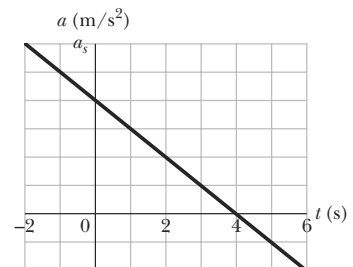


Figure 2-44 Problem 82.

83 Figure 2-45 shows a simple device for measuring your reaction time. It consists of a cardboard strip marked with a scale and two large dots. A friend holds the strip *vertically*, with thumb and forefinger at the dot on the right in Fig. 2-45. You then position your thumb and forefinger at the other dot (on the left in Fig. 2-45), being careful not to touch the strip. Your friend releases the strip, and you try to pinch it as soon as possible after you see it begin to fall. The mark at the place where you pinch the strip gives your reaction time. (a) How far from the lower dot should you place the 50.0 ms mark? How much higher should you place the marks for (b) 100, (c) 150, (d) 200, and (e) 250 ms? (For example, should the 100 ms marker be 2 times as far from the dot as the 50 ms marker? If so, give an answer of 2 times. Can you find any pattern in the answers?)

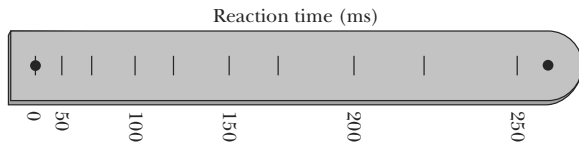


Figure 2-45 Problem 83.

84 A rocket-driven sled running on a straight, level track is used to investigate the effects of large accelerations on humans. One such sled can attain a speed of 1600 km/h in 1.8 s, starting from rest. Find (a) the acceleration (assumed constant) in terms of g and (b) the distance traveled.

85 A mining cart is pulled up a hill at 20 km/h and then pulled back down the hill at 35 km/h through its original level. (The time required for the cart's reversal at the top of its climb is negligible.) What is the average speed of the cart for its round trip, from its original level back to its original level?

86 A motorcyclist who is moving along an x axis directed toward the east has an acceleration given by $a = (6.1 - 1.2t)$ m/s² for $0 \leq t \leq 6.0$ s. At $t = 0$, the velocity and position of the cyclist are 2.7 m/s and 7.3 m. (a) What is the maximum speed achieved by the cyclist? (b) What total distance does the cyclist travel between $t = 0$ and 6.0 s?

87 SSM When the legal speed limit for the New York Thruway was increased from 55 mi/h to 65 mi/h, how much time was saved by a motorist who drove the 700 km between the Buffalo entrance and the New York City exit at the legal speed limit?

88 A car moving with constant acceleration covered the distance between two points 60.0 m apart in 6.00 s. Its speed as it passed the second point was 15.0 m/s. (a) What was the speed at the first point? (b) What was the magnitude of the acceleration? (c) At what prior distance from the first point was the car at rest? (d) Graph x versus t and v versus t for the car, from rest ($t = 0$).

89 SSM A certain juggler usually tosses balls vertically to a height H . To what height must they be tossed if they are to spend twice as much time in the air?

90 A particle starts from the origin at $t = 0$ and moves along the positive x axis. A graph of the velocity of the particle as a function of the time is shown in Fig. 2-46; the v -axis scale is set by $v_s = 4.0$ m/s. (a) What is the coordinate of the particle at $t = 5.0$ s? (b) What is the velocity of the particle at $t = 5.0$ s? (c) What is

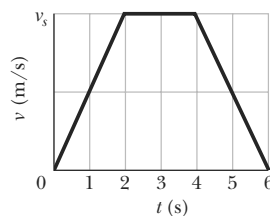


Figure 2-46 Problem 90.

the acceleration of the particle at $t = 5.0$ s? (d) What is the average velocity of the particle between $t = 1.0$ s and $t = 5.0$ s? (e) What is the average acceleration of the particle between $t = 1.0$ s and $t = 5.0$ s?

91 A rock is dropped from a 100-m-high cliff. How long does it take to fall (a) the first 50 m and (b) the second 50 m?

92 Two subway stops are separated by 1100 m. If a subway train accelerates at $+1.2$ m/s² from rest through the first half of the distance and decelerates at -1.2 m/s² through the second half, what are (a) its travel time and (b) its maximum speed? (c) Graph x , v , and a versus t for the trip.

93 A stone is thrown vertically upward. On its way up it passes point A with speed v , and point B , 3.00 m higher than A , with speed $\frac{1}{2}v$. Calculate (a) the speed v and (b) the maximum height reached by the stone above point B .

94 A rock is dropped (from rest) from the top of a 60-m-tall building. How far above the ground is the rock 1.2 s before it reaches the ground?

95 SSM An iceboat has a constant velocity toward the east when a sudden gust of wind causes the iceboat to have a constant acceleration toward the east for a period of 3.0 s. A plot of x versus t is shown in Fig. 2-47, where $t = 0$ is taken to be the instant the wind starts to blow and the positive x axis is toward the east. (a) What is the acceleration of the iceboat during the 3.0 s interval? (b) What is the velocity of the iceboat at the end of the 3.0 s interval? (c) If the acceleration remains constant for an additional 3.0 s, how far does the iceboat travel during this second 3.0 s interval?

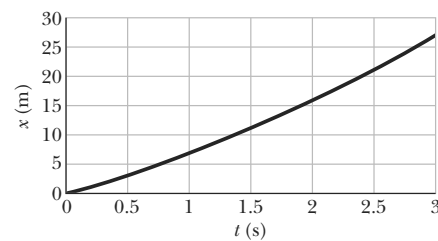


Figure 2-47 Problem 95.

96 A lead ball is dropped in a lake from a diving board 5.20 m above the water. It hits the water with a certain velocity and then sinks to the bottom with this same constant velocity. It reaches the bottom 4.80 s after it is dropped. (a) How deep is the lake? What are the (b) magnitude and (c) direction (up or down) of the average velocity of the ball for the entire fall? Suppose that all the water is drained from the lake. The ball is now thrown from the diving board so that it again reaches the bottom in 4.80 s. What are the (d) magnitude and (e) direction of the initial velocity of the ball?

97 The single cable supporting an unoccupied construction elevator breaks when the elevator is at rest at the top of a 120-m-high building. (a) With what speed does the elevator strike the ground? (b) How long is it falling? (c) What is its speed when it passes the halfway point on the way down? (d) How long has it been falling when it passes the halfway point?

98 Two diamonds begin a free fall from rest from the same height, 1.0 s apart. How long after the first diamond begins to fall will the two diamonds be 10 m apart?

99 A ball is thrown vertically downward from the top of a 36.6-m-tall building. The ball passes the top of a window that is 12.2 m above the ground 2.00 s after being thrown. What is the speed of the ball as it passes the top of the window?

100 A parachutist bails out and freely falls 50 m. Then the parachute opens, and thereafter she decelerates at 2.0 m/s^2 . She reaches the ground with a speed of 3.0 m/s . (a) How long is the parachutist in the air? (b) At what height does the fall begin?

101 A ball is thrown *down* vertically with an initial speed of v_0 from a height of h . (a) What is its speed just before it strikes the ground? (b) How long does the ball take to reach the ground? What would be the answers to (c) part a and (d) part b if the ball were thrown *upward* from the same height and with the same initial speed? Before solving any equations, decide whether the answers to (c) and (d) should be greater than, less than, or the same as in (a) and (b).

102 The sport with the fastest moving ball is jai alai, where measured speeds have reached 303 km/h . If a professional jai alai player faces a ball at that speed and involuntarily blinks, he blacks out the scene for 100 ms . How far does the ball move during the blackout?

103 If a baseball pitcher throws a fastball at a horizontal speed of 160 km/h , how long does the ball take to reach home plate 18.4 m away?

104 A proton moves along the x axis according to the equation $x = 50t + 10t^2$, where x is in meters and t is in seconds. Calculate (a) the average velocity of the proton during the first 3.0 s of its motion, (b) the instantaneous velocity of the proton at $t = 3.0 \text{ s}$, and (c) the instantaneous acceleration of the proton at $t = 3.0 \text{ s}$. (d) Graph x versus t and indicate how the answer to (a) can be obtained from the plot. (e) Indicate the answer to (b) on the graph. (f) Plot v versus t and indicate on it the answer to (c).

105 A motorcycle is moving at 30 m/s when the rider applies the brakes, giving the motorcycle a constant deceleration. During the 3.0 s interval immediately after braking begins, the speed decreases to 15 m/s . What distance does the motorcycle travel from the instant braking begins until the motorcycle stops?

106 A shuffleboard disk is accelerated at a constant rate from rest to a speed of 6.0 m/s over a 1.8 m distance by a player using a cue. At this point the disk loses contact with the cue and slows at a constant rate of 2.5 m/s^2 until it stops. (a) How much time elapses from when the disk begins to accelerate until it stops? (b) What total distance does the disk travel?

107 The head of a rattlesnake can accelerate at 50 m/s^2 in striking a victim. If a car could do as well, how long would it take to reach a speed of 100 km/h from rest?

108 A jumbo jet must reach a speed of 360 km/h on the runway for takeoff. What is the lowest constant acceleration needed for takeoff from a 1.80 km runway?

109 An automobile driver increases the speed at a constant rate from 25 km/h to 55 km/h in 0.50 min . A bicycle rider speeds up at a constant rate from rest to 30 km/h in 0.50 min . What are the magnitudes of (a) the driver's acceleration and (b) the rider's acceleration?


110 On average, an eye blink lasts about 100 ms . How far does a MiG-25 "Foxbat" fighter travel during a pilot's blink if the plane's average velocity is 3400 km/h ?

111 A certain sprinter has a top speed of 11.0 m/s . If the sprinter starts from rest and accelerates at a constant rate, he is able to reach his top speed in a distance of 12.0 m . He is then able to maintain this top speed for the remainder of a 100 m race. (a) What is his time for the 100 m race? (b) In order to improve his time, the sprinter tries to decrease the distance required for him to reach his

top speed. What must this distance be if he is to achieve a time of 10.0 s for the race?

112 The speed of a bullet is measured to be 640 m/s as the bullet emerges from a barrel of length 1.20 m . Assuming constant acceleration, find the time that the bullet spends in the barrel after it is fired.

113 The Zero Gravity Research Facility at the NASA Glenn Research Center includes a 145 m drop tower. This is an evacuated vertical tower through which, among other possibilities, a 1-m -diameter sphere containing an experimental package can be dropped. (a) How long is the sphere in free fall? (b) What is its speed just as it reaches a catching device at the bottom of the tower? (c) When caught, the sphere experiences an average deceleration of $25g$ as its speed is reduced to zero. Through what distance does it travel during the deceleration?

114  A car can be braked to a stop from the autobahn-like speed of 200 km/h in 170 m . Assuming the acceleration is constant, find its magnitude in (a) SI units and (b) in terms of g . (c) How much time T_b is required for the braking? Your reaction time T_r is the time you require to perceive an emergency, move your foot to the brake, and begin the braking. If $T_r = 400 \text{ ms}$, then (d) what is T_b in terms of T_r , and (e) is most of the full time required to stop spent in reacting or braking? Dark sunglasses delay the visual signals sent from the eyes to the visual cortex in the brain, increasing T_r . (f) In the extreme case in which T_r is increased by 100 ms , how much farther does the car travel during your reaction time?

115 In 1889, at Jubbulpore, India, a tug-of-war was finally won after $2 \text{ h } 41 \text{ min}$, with the winning team displacing the center of the rope 3.7 m . In centimeters per minute, what was the magnitude of the average velocity of that center point during the contest?

116 Most important in an investigation of an airplane crash by the U.S. National Transportation Safety Board is the data stored on the airplane's flight-data recorder, commonly called the "black box" in spite of its orange coloring and reflective tape. The recorder is engineered to withstand a crash with an average deceleration of magnitude $3400g$ during a time interval of 6.50 ms . In such a crash, if the recorder and airplane have zero speed at the end of that time interval, what is their speed at the beginning of the interval?

117 From January 26, 1977, to September 18, 1983, George Meegan of Great Britain walked from Ushuaia, at the southern tip of South America, to Prudhoe Bay in Alaska, covering $30\,600 \text{ km}$. In meters per second, what was the magnitude of his average velocity during that time period?

118 The wings on a stonefly do not flap, and thus the insect cannot fly. However, when the insect is on a water surface, it can sail across the surface by lifting its wings into a breeze. Suppose that you time stoneflies as they move at constant speed along a straight path of a certain length. On average, the trips each take 7.1 s with the wings set as sails and 25.0 s with the wings tucked in. (a) What is the ratio of the sailing speed v_s to the nonsailing speed v_{ns} ? (b) In terms of v_s , what is the difference in the times the insects take to travel the first 2.0 m along the path with and without sailing?

119 The position of a particle as it moves along a y axis is given by

$$y = (2.0 \text{ cm}) \sin(\pi t/4),$$

with t in seconds and y in centimeters. (a) What is the average velocity of the particle between $t = 0$ and $t = 2.0 \text{ s}$? (b) What is the instantaneous velocity of the particle at $t = 0, 1.0$, and 2.0 s ? (c) What is the average acceleration of the particle between $t = 0$ and $t = 2.0 \text{ s}$? (d) What is the instantaneous acceleration of the particle at $t = 0, 1.0$, and 2.0 s ?

C H A P T E R 3

Vectors

3-1 VECTORS AND THEIR COMPONENTS

Learning Objectives

After reading this module, you should be able to . . .

- 3.01** Add vectors by drawing them in head-to-tail arrangements, applying the commutative and associative laws.
- 3.02** Subtract a vector from a second one.
- 3.03** Calculate the components of a vector on a given coordinate system, showing them in a drawing.

- 3.04** Given the components of a vector, draw the vector and determine its magnitude and orientation.
- 3.05** Convert angle measures between degrees and radians.

Key Ideas

- Scalars, such as temperature, have magnitude only. They are specified by a number with a unit (10°C) and obey the rules of arithmetic and ordinary algebra. Vectors, such as displacement, have both magnitude and direction (5 m, north) and obey the rules of vector algebra.
- Two vectors \vec{a} and \vec{b} may be added geometrically by drawing them to a common scale and placing them head to tail. The vector connecting the tail of the first to the head of the second is the vector sum \vec{c} . To subtract \vec{b} from \vec{a} , reverse the direction of \vec{b} to get $-\vec{b}$; then add $-\vec{b}$ to \vec{a} . Vector addition is commutative and obeys the associative law.

- The (scalar) components a_x and a_y of any two-dimensional vector \vec{a} along the coordinate axes are found by dropping perpendicular lines from the ends of \vec{a} onto the coordinate axes. The components are given by

$$a_x = a \cos \theta \quad \text{and} \quad a_y = a \sin \theta,$$

where θ is the angle between the positive direction of the x axis and the direction of \vec{a} . The algebraic sign of a component indicates its direction along the associated axis. Given its components, we can find the magnitude and orientation of the vector \vec{a} with

$$a = \sqrt{a_x^2 + a_y^2} \quad \text{and} \quad \tan \theta = \frac{a_y}{a_x}.$$

What Is Physics?

Physics deals with a great many quantities that have both size and direction, and it needs a special mathematical language—the language of vectors—to describe those quantities. This language is also used in engineering, the other sciences, and even in common speech. If you have ever given directions such as “Go five blocks down this street and then hang a left,” you have used the language of vectors. In fact, navigation of any sort is based on vectors, but physics and engineering also need vectors in special ways to explain phenomena involving rotation and magnetic forces, which we get to in later chapters. In this chapter, we focus on the basic language of vectors.

Vectors and Scalars

A particle moving along a straight line can move in only two directions. We can take its motion to be positive in one of these directions and negative in the other. For a particle moving in three dimensions, however, a plus sign or minus sign is no longer enough to indicate a direction. Instead, we must use a *vector*.

A **vector** has magnitude as well as direction, and vectors follow certain (vector) rules of combination, which we examine in this chapter. A **vector quantity** is a quantity that has both a magnitude and a direction and thus can be represented with a vector. Some physical quantities that are vector quantities are displacement, velocity, and acceleration. You will see many more throughout this book, so learning the rules of vector combination now will help you greatly in later chapters.

Not all physical quantities involve a direction. Temperature, pressure, energy, mass, and time, for example, do not “point” in the spatial sense. We call such quantities **scalars**, and we deal with them by the rules of ordinary algebra. A single value, with a sign (as in a temperature of -40°F), specifies a scalar.

The simplest vector quantity is displacement, or change of position. A vector that represents a displacement is called, reasonably, a **displacement vector**. (Similarly, we have velocity vectors and acceleration vectors.) If a particle changes its position by moving from A to B in Fig. 3-1a, we say that it undergoes a displacement from A to B , which we represent with an arrow pointing from A to B . The arrow specifies the vector graphically. To distinguish vector symbols from other kinds of arrows in this book, we use the outline of a triangle as the arrowhead.

In Fig. 3-1a, the arrows from A to B , from A' to B' , and from A'' to B'' have the same magnitude and direction. Thus, they specify identical displacement vectors and represent the same *change of position* for the particle. A vector can be shifted without changing its value *if* its length and direction are not changed.

The displacement vector tells us nothing about the actual path that the particle takes. In Fig. 3-1b, for example, all three paths connecting points A and B correspond to the same displacement vector, that of Fig. 3-1a. Displacement vectors represent only the overall effect of the motion, not the motion itself.

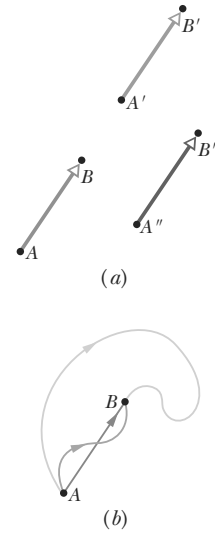


Figure 3-1 (a) All three arrows have the same magnitude and direction and thus represent the same displacement. (b) All three paths connecting the two points correspond to the same displacement vector.

Adding Vectors Geometrically

Suppose that, as in the vector diagram of Fig. 3-2a, a particle moves from A to B and then later from B to C . We can represent its overall displacement (no matter what its actual path) with two successive displacement vectors, AB and BC . The *net* displacement of these two displacements is a single displacement from A to C . We call AC the **vector sum** (or **resultant**) of the vectors AB and BC . This sum is not the usual algebraic sum.

In Fig. 3-2b, we redraw the vectors of Fig. 3-2a and relabel them in the way that we shall use from now on, namely, with an arrow over an italic symbol, as in \vec{a} . If we want to indicate only the magnitude of the vector (a quantity that lacks a sign or direction), we shall use the italic symbol, as in a , b , and s . (You can use just a handwritten symbol.) A symbol with an overhead arrow always implies both properties of a vector, magnitude and direction.

We can represent the relation among the three vectors in Fig. 3-2b with the *vector equation*

$$\vec{s} = \vec{a} + \vec{b}, \quad (3-1)$$

which says that the vector \vec{s} is the vector sum of vectors \vec{a} and \vec{b} . The symbol $+$ in Eq. 3-1 and the words “sum” and “add” have different meanings for vectors than they do in the usual algebra because they involve both magnitude *and* direction.

Figure 3-2 suggests a procedure for adding two-dimensional vectors \vec{a} and \vec{b} geometrically. (1) On paper, sketch vector \vec{a} to some convenient scale and at the proper angle. (2) Sketch vector \vec{b} to the same scale, with its tail at the head of vector \vec{a} , again at the proper angle. (3) The vector sum \vec{s} is the vector that extends from the tail of \vec{a} to the head of \vec{b} .

Properties. Vector addition, defined in this way, has two important properties. First, the order of addition does not matter. Adding \vec{a} to \vec{b} gives the same

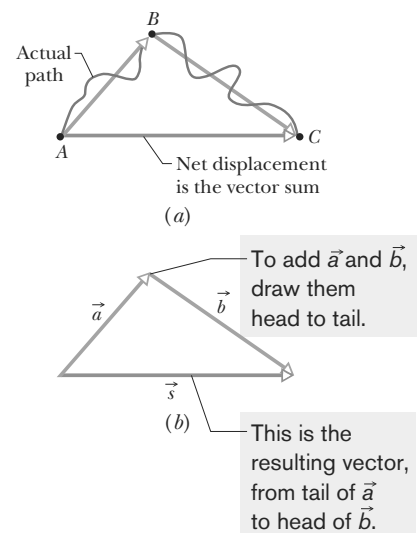
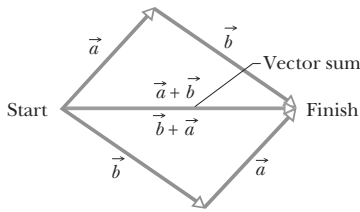


Figure 3-2 (a) AC is the vector sum of the vectors AB and BC . (b) The same vectors relabeled.



You get the same vector result for either order of adding vectors.

Figure 3-3 The two vectors \vec{a} and \vec{b} can be added in either order; see Eq. 3-2.

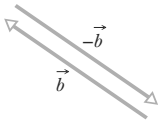


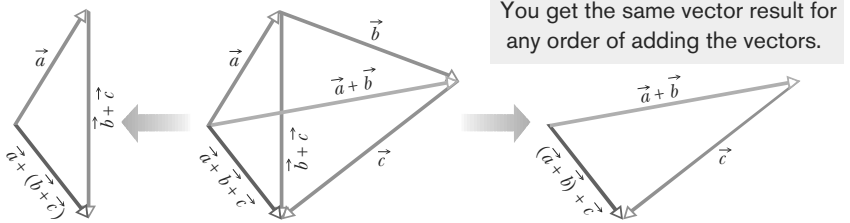
Figure 3-5 The vectors \vec{b} and $-\vec{b}$ have the same magnitude and opposite directions.

result as adding \vec{b} to \vec{a} (Fig. 3-3); that is,

$$\vec{a} + \vec{b} = \vec{b} + \vec{a} \quad (\text{commutative law}). \quad (3-2)$$

Second, when there are more than two vectors, we can group them in any order as we add them. Thus, if we want to add vectors \vec{a} , \vec{b} , and \vec{c} , we can add \vec{a} and \vec{b} first and then add their vector sum to \vec{c} . We can also add \vec{b} and \vec{c} first and then add *that* sum to \vec{a} . We get the same result either way, as shown in Fig. 3-4. That is,

$$(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c}) \quad (\text{associative law}). \quad (3-3)$$



You get the same vector result for any order of adding the vectors.

Figure 3-4 The three vectors \vec{a} , \vec{b} , and \vec{c} can be grouped in any way as they are added; see Eq. 3-3.

The vector $-\vec{b}$ is a vector with the same magnitude as \vec{b} but the opposite direction (see Fig. 3-5). Adding the two vectors in Fig. 3-5 would yield

$$\vec{b} + (-\vec{b}) = 0.$$

Thus, adding $-\vec{b}$ has the effect of subtracting \vec{b} . We use this property to define the difference between two vectors: let $\vec{d} = \vec{a} - \vec{b}$. Then

$$\vec{d} = \vec{a} - \vec{b} = \vec{a} + (-\vec{b}) \quad (\text{vector subtraction}); \quad (3-4)$$

that is, we find the difference vector \vec{d} by adding the vector $-\vec{b}$ to the vector \vec{a} . Figure 3-6 shows how this is done geometrically.

As in the usual algebra, we can move a term that includes a vector symbol from one side of a vector equation to the other, but we must change its sign. For example, if we are given Eq. 3-4 and need to solve for \vec{a} , we can rearrange the equation as

$$\vec{d} + \vec{b} = \vec{a} \quad \text{or} \quad \vec{a} = \vec{d} + \vec{b}.$$

Remember that, although we have used displacement vectors here, the rules for addition and subtraction hold for vectors of all kinds, whether they represent velocities, accelerations, or any other vector quantity. However, we can add only vectors of the same kind. For example, we can add two displacements, or two velocities, but adding a displacement and a velocity makes no sense. In the arithmetic of scalars, that would be like trying to add 21 s and 12 m.

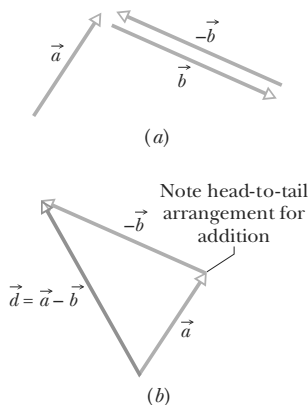


Figure 3-6 (a) Vectors \vec{a} , \vec{b} , and $-\vec{b}$. (b) To subtract vector \vec{b} from vector \vec{a} , add vector $-\vec{b}$ to vector \vec{a} .

Checkpoint 1

The magnitudes of displacements \vec{a} and \vec{b} are 3 m and 4 m, respectively, and $\vec{c} = \vec{a} + \vec{b}$. Considering various orientations of \vec{a} and \vec{b} , what are (a) the maximum possible magnitude for \vec{c} and (b) the minimum possible magnitude?

Components of Vectors

Adding vectors geometrically can be tedious. A neater and easier technique involves algebra but requires that the vectors be placed on a rectangular coordinate system. The x and y axes are usually drawn in the plane of the page, as shown

in Fig. 3-7a. The z axis comes directly out of the page at the origin; we ignore it for now and deal only with two-dimensional vectors.

A **component** of a vector is the projection of the vector on an axis. In Fig. 3-7a, for example, a_x is the component of vector \vec{a} on (or along) the x axis and a_y is the component along the y axis. To find the projection of a vector along an axis, we draw perpendicular lines from the two ends of the vector to the axis, as shown. The projection of a vector on an x axis is its x component, and similarly the projection on the y axis is the y component. The process of finding the components of a vector is called **resolving the vector**.

A component of a vector has the same direction (along an axis) as the vector. In Fig. 3-7, a_x and a_y are both positive because \vec{a} extends in the positive direction of both axes. (Note the small arrowheads on the components, to indicate their direction.) If we were to reverse vector \vec{a} , then both components would be negative and their arrowheads would point toward negative x and y . Resolving vector \vec{b} in Fig. 3-8 yields a positive component b_x and a negative component b_y .

In general, a vector has three components, although for the case of Fig. 3-7a the component along the z axis is zero. As Figs. 3-7a and b show, if you shift a vector without changing its direction, its components do not change.

Finding the Components. We can find the components of \vec{a} in Fig. 3-7a geometrically from the right triangle there:

$$a_x = a \cos \theta \quad \text{and} \quad a_y = a \sin \theta, \quad (3-5)$$

where θ is the angle that the vector \vec{a} makes with the positive direction of the x axis, and a is the magnitude of \vec{a} . Figure 3-7c shows that \vec{a} and its x and y components form a right triangle. It also shows how we can reconstruct a vector from its components: we arrange those components *head to tail*. Then we complete a right triangle with the vector forming the hypotenuse, from the tail of one component to the head of the other component.

Once a vector has been resolved into its components along a set of axes, the components themselves can be used in place of the vector. For example, \vec{a} in Fig. 3-7a is given (completely determined) by a and θ . It can also be given by its components a_x and a_y . Both pairs of values contain the same information. If we know a vector in *component notation* (a_x and a_y) and want it in *magnitude-angle notation* (a and θ), we can use the equations

$$a = \sqrt{a_x^2 + a_y^2} \quad \text{and} \quad \tan \theta = \frac{a_y}{a_x} \quad (3-6)$$

to transform it.

In the more general three-dimensional case, we need a magnitude and two angles (say, a , θ , and ϕ) or three components (a_x , a_y , and a_z) to specify a vector.

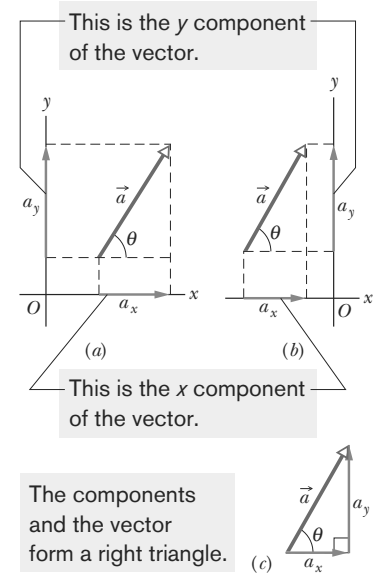


Figure 3-7 (a) The components a_x and a_y of vector \vec{a} . (b) The components are unchanged if the vector is shifted, as long as the magnitude and orientation are maintained. (c) The components form the legs of a right triangle whose hypotenuse is the magnitude of the vector.

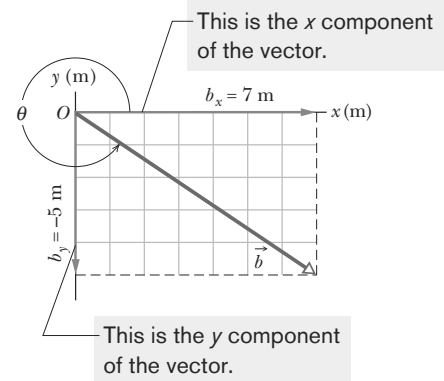
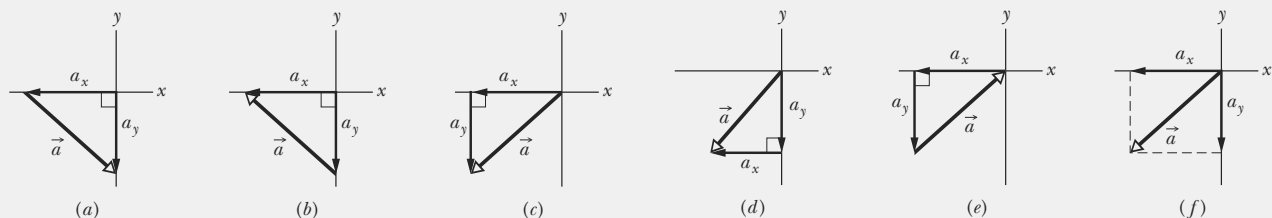


Figure 3-8 The component of \vec{b} on the x axis is positive, and that on the y axis is negative.



Checkpoint 2

In the figure, which of the indicated methods for combining the x and y components of vector \vec{a} are proper to determine that vector?



Sample Problem 3.01 Adding vectors in a drawing, orienteering

In an orienteering class, you have the goal of moving as far (straight-line distance) from base camp as possible by making three straight-line moves. You may use the following displacements in any order: (a) \vec{a} , 2.0 km due east (directly toward the east); (b) \vec{b} , 2.0 km 30° north of east (at an angle of 30° toward the north from due east); (c) \vec{c} , 1.0 km due west. Alternatively, you may substitute either $-\vec{b}$ for \vec{b} or $-\vec{c}$ for \vec{c} . What is the greatest distance you can be from base camp at the end of the third displacement? (We are not concerned about the direction.)

Reasoning: Using a convenient scale, we draw vectors \vec{a} , \vec{b} , \vec{c} , $-\vec{b}$, and $-\vec{c}$ as in Fig. 3-9a. We then mentally slide the vectors over the page, connecting three of them at a time in head-to-tail arrangements to find their vector sum \vec{d} . The tail of the first vector represents base camp. The head of the third vector represents the point at which you stop. The vector sum \vec{d} extends from the tail of the first vector to the head of the third vector. Its magnitude d is your distance from base camp. Our goal here is to maximize that base-camp distance.

We find that distance d is greatest for a head-to-tail arrangement of vectors \vec{a} , \vec{b} , and $-\vec{c}$. They can be in any

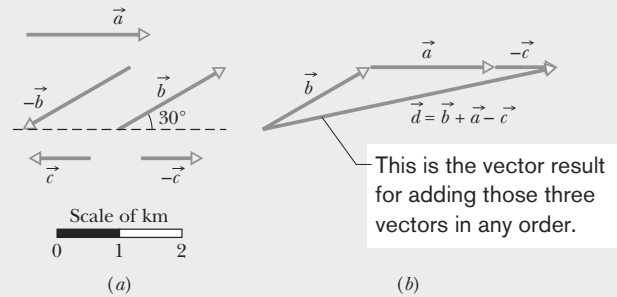


Figure 3-9 (a) Displacement vectors; three are to be used. (b) Your distance from base camp is greatest if you undergo displacements \vec{a} , \vec{b} , and $-\vec{c}$, in any order.

order, because their vector sum is the same for any order. (Recall from Eq. 3-2 that vectors commute.) The order shown in Fig. 3-9b is for the vector sum

$$\vec{d} = \vec{b} + \vec{a} + (-\vec{c}).$$

Using the scale given in Fig. 3-9a, we measure the length d of this vector sum, finding

$$d = 4.8 \text{ m.} \quad (\text{Answer})$$

Sample Problem 3.02 Finding components, airplane flight

A small airplane leaves an airport on an overcast day and is later sighted 215 km away, in a direction making an angle of 22° east of due north. This means that the direction is not due north (directly toward the north) but is rotated 22° toward the east from due north. How far east and north is the airplane from the airport when sighted?

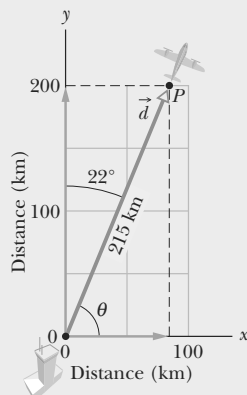


Figure 3-10 A plane takes off from an airport at the origin and is later sighted at P .

KEY IDEA

We are given the magnitude (215 km) and the angle (22° east of due north) of a vector and need to find the components of the vector.

Calculations: We draw an xy coordinate system with the positive direction of x due east and that of y due north (Fig. 3-10). For convenience, the origin is placed at the airport. (We don't have to do this. We could shift and misalign the coordinate system but, given a choice, why make the problem more difficult?) The airplane's displacement \vec{d} points from the origin to where the airplane is sighted.

To find the components of \vec{d} , we use Eq. 3-5 with $\theta = 68^\circ (= 90^\circ - 22^\circ)$:

$$d_x = d \cos \theta = (215 \text{ km})(\cos 68^\circ) = 81 \text{ km} \quad (\text{Answer})$$

$$d_y = d \sin \theta = (215 \text{ km})(\sin 68^\circ) = 199 \text{ km} \approx 2.0 \times 10^2 \text{ km.} \quad (\text{Answer})$$

Thus, the airplane is 81 km east and 2.0×10^2 km north of the airport.



Problem-Solving Tactics Angles, trig functions, and inverse trig functions

Tactic 1: Angles—Degrees and Radians Angles that are measured relative to the positive direction of the x axis are positive if they are measured in the counterclockwise direction and negative if measured clockwise. For example, 210° and -150° are the same angle.

Angles may be measured in degrees or radians (rad). To relate the two measures, recall that a full circle is 360° and 2π rad. To convert, say, 40° to radians, write

$$40^\circ \frac{2\pi \text{ rad}}{360^\circ} = 0.70 \text{ rad.}$$

Tactic 2: Trig Functions You need to know the definitions of the common trigonometric functions—sine, cosine, and tangent—because they are part of the language of science and engineering. They are given in Fig. 3-11 in a form that does not depend on how the triangle is labeled.

You should also be able to sketch how the trig functions vary with angle, as in Fig. 3-12, in order to be able to judge whether a calculator result is reasonable. Even knowing the signs of the functions in the various quadrants can be of help.

Tactic 3: Inverse Trig Functions When the inverse trig functions \sin^{-1} , \cos^{-1} , and \tan^{-1} are taken on a calculator, you must consider the reasonableness of the answer you get, because there is usually another possible answer that the calculator does not give. The range of operation for a calculator in taking each inverse trig function is indicated in Fig. 3-12. As an example, $\sin^{-1} 0.5$ has associated angles of 30° (which is displayed by the calculator, since 30° falls within its range of operation) and 150° . To see both values, draw a horizontal line through 0.5 in Fig. 3-12a and note where it cuts the sine curve. How do you distinguish a correct answer? It is the one that seems more reasonable for the given situation.

Tactic 4: Measuring Vector Angles The equations for $\cos \theta$ and $\sin \theta$ in Eq. 3-5 and for $\tan \theta$ in Eq. 3-6 are valid only if the angle is measured from the positive direction of

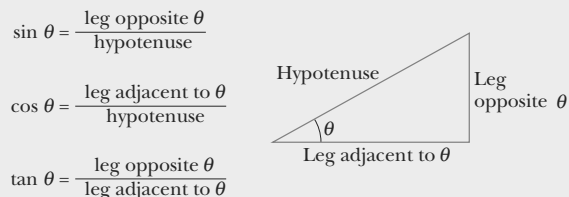
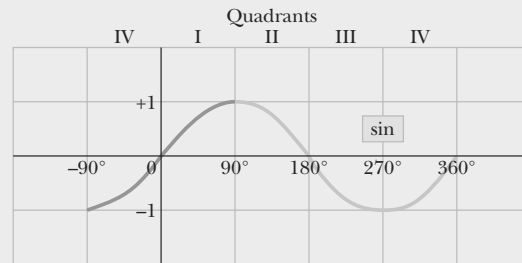
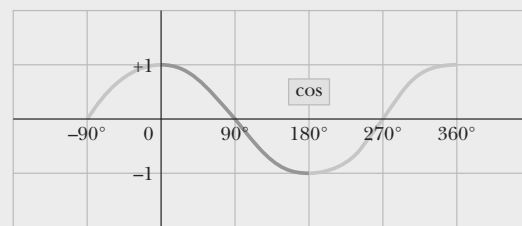


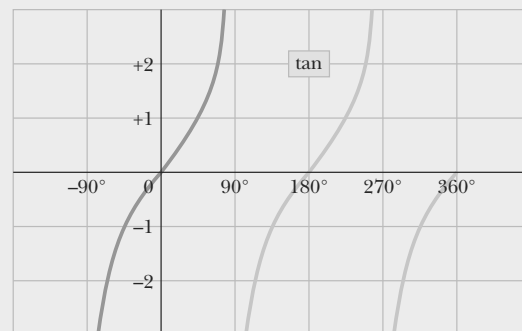
Figure 3-11 A triangle used to define the trigonometric functions. See also Appendix E.



(a)



(b)



(c)

Figure 3-12 Three useful curves to remember. A calculator's range of operation for taking *inverse* trig functions is indicated by the darker portions of the colored curves.

the x axis. If it is measured relative to some other direction, then the trig functions in Eq. 3-5 may have to be interchanged and the ratio in Eq. 3-6 may have to be inverted. A safer method is to convert the angle to one measured from the positive direction of the x axis. In *WileyPLUS*, the system expects you to report an angle of direction like this (and positive if counterclockwise and negative if clockwise).



Additional examples, video, and practice available at *WileyPLUS*



3-2 UNIT VECTORS, ADDING VECTORS BY COMPONENTS

Learning Objectives

After reading this module, you should be able to . . .

3.06 Convert a vector between magnitude-angle and unit-vector notations.

3.07 Add and subtract vectors in magnitude-angle notation and in unit-vector notation.

3.08 Identify that, for a given vector, rotating the coordinate system about the origin can change the vector's components but not the vector itself.

Key Ideas

● Unit vectors \hat{i} , \hat{j} , and \hat{k} have magnitudes of unity and are directed in the positive directions of the x , y , and z axes, respectively, in a right-handed coordinate system. We can write a vector \vec{a} in terms of unit vectors as

$$\vec{a} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k},$$

in which $a_x \hat{i}$, $a_y \hat{j}$, and $a_z \hat{k}$ are the vector components of \vec{a} and a_x , a_y , and a_z are its scalar components.

● To add vectors in component form, we use the rules

$$r_x = a_x + b_x \quad r_y = a_y + b_y \quad r_z = a_z + b_z.$$

Here \vec{a} and \vec{b} are the vectors to be added, and \vec{r} is the vector sum. Note that we add components axis by axis.

The unit vectors point along axes.

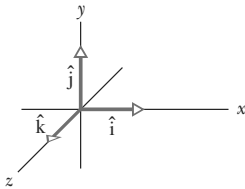


Figure 3.13 Unit vectors \hat{i} , \hat{j} , and \hat{k} define the directions of a right-handed coordinate system.

Unit Vectors

A **unit vector** is a vector that has a magnitude of exactly 1 and points in a particular direction. It lacks both dimension and unit. Its sole purpose is to point—that is, to specify a direction. The unit vectors in the positive directions of the x , y , and z axes are labeled \hat{i} , \hat{j} , and \hat{k} , where the hat $\hat{}$ is used instead of an overhead arrow as for other vectors (Fig. 3-13). The arrangement of axes in Fig. 3-13 is said to be a **right-handed coordinate system**. The system remains right-handed if it is rotated rigidly. We use such coordinate systems exclusively in this book.

Unit vectors are very useful for expressing other vectors; for example, we can express \vec{a} and \vec{b} of Figs. 3-7 and 3-8 as

$$\vec{a} = a_x \hat{i} + a_y \hat{j} \quad (3-7)$$

and

$$\vec{b} = b_x \hat{i} + b_y \hat{j}. \quad (3-8)$$

These two equations are illustrated in Fig. 3-14. The quantities $a_x \hat{i}$ and $a_y \hat{j}$ are vectors, called the **vector components** of \vec{a} . The quantities a_x and a_y are scalars, called the **scalar components** of \vec{a} (or, as before, simply its **components**).

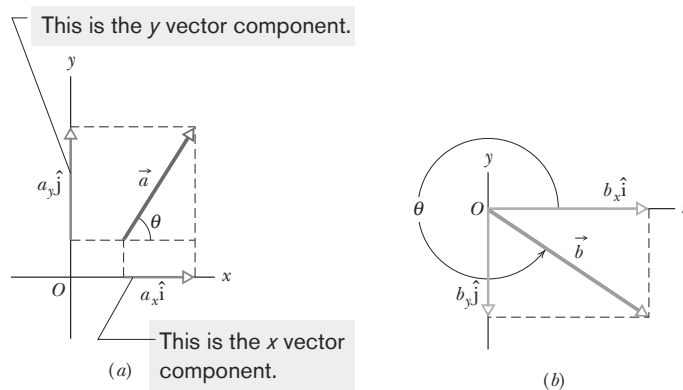


Figure 3-14 (a) The vector components of vector \vec{a} . (b) The vector components of vector \vec{b} .

Adding Vectors by Components

We can add vectors geometrically on a sketch or directly on a vector-capable calculator. A third way is to combine their components axis by axis.

To start, consider the statement

$$\vec{r} = \vec{a} + \vec{b}, \quad (3-9)$$

which says that the vector \vec{r} is the same as the vector $(\vec{a} + \vec{b})$. Thus, each component of \vec{r} must be the same as the corresponding component of $(\vec{a} + \vec{b})$:

$$r_x = a_x + b_x \quad (3-10)$$

$$r_y = a_y + b_y \quad (3-11)$$

$$r_z = a_z + b_z. \quad (3-12)$$

In other words, two vectors must be equal if their corresponding components are equal. Equations 3-9 to 3-12 tell us that to add vectors \vec{a} and \vec{b} , we must (1) resolve the vectors into their scalar components; (2) combine these scalar components, axis by axis, to get the components of the sum \vec{r} ; and (3) combine the components of \vec{r} to get \vec{r} itself. We have a choice in step 3. We can express \vec{r} in unit-vector notation or in magnitude-angle notation.

This procedure for adding vectors by components also applies to vector subtractions. Recall that a subtraction such as $\vec{d} = \vec{a} - \vec{b}$ can be rewritten as an addition $\vec{d} = \vec{a} + (-\vec{b})$. To subtract, we add \vec{a} and $-\vec{b}$ by components, to get

$$d_x = a_x - b_x, \quad d_y = a_y - b_y, \quad \text{and} \quad d_z = a_z - b_z,$$

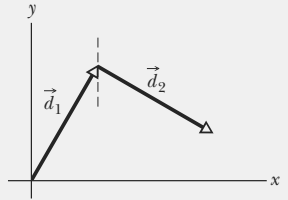
where

$$\vec{d} = d_x \hat{i} + d_y \hat{j} + d_z \hat{k}. \quad (3-13)$$



Checkpoint 3

(a) In the figure here, what are the signs of the x components of \vec{d}_1 and \vec{d}_2 ? (b) What are the signs of the y components of \vec{d}_1 and \vec{d}_2 ? (c) What are the signs of the x and y components of $\vec{d}_1 + \vec{d}_2$?



Vectors and the Laws of Physics

So far, in every figure that includes a coordinate system, the x and y axes are parallel to the edges of the book page. Thus, when a vector \vec{a} is included, its components a_x and a_y are also parallel to the edges (as in Fig. 3-15a). The only reason for that orientation of the axes is that it looks “proper”; there is no deeper reason. We could, instead, rotate the axes (but not the vector \vec{a}) through an angle ϕ as in Fig. 3-15b, in which case the components would have new values, call them a'_x and a'_y . Since there are an infinite number of choices of ϕ , there are an infinite number of different pairs of components for \vec{a} .

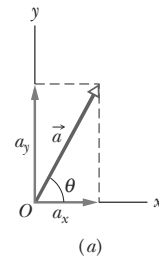
Which then is the “right” pair of components? The answer is that they are all equally valid because each pair (with its axes) just gives us a different way of describing the same vector \vec{a} ; all produce the same magnitude and direction for the vector. In Fig. 3-15 we have

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{a'^2_x + a'^2_y} \quad (3-14)$$

and

$$\theta = \theta' + \phi. \quad (3-15)$$

The point is that we have great freedom in choosing a coordinate system, because the relations among vectors do not depend on the location of the origin or on the orientation of the axes. This is also true of the relations of physics; they are all independent of the choice of coordinate system. Add to that the simplicity and richness of the language of vectors and you can see why the laws of physics are almost always presented in that language: one equation, like Eq. 3-9, can represent three (or even more) relations, like Eqs. 3-10, 3-11, and 3-12.



Rotating the axes changes the components but not the vector.

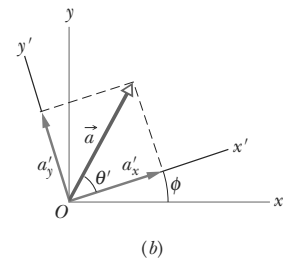


Figure 3-15 (a) The vector \vec{a} and its components. (b) The same vector, with the axes of the coordinate system rotated through an angle ϕ .

Sample Problem 3.03 Searching through a hedge maze

A hedge maze is a maze formed by tall rows of hedge. After entering, you search for the center point and then for the exit. Figure 3-16a shows the entrance to such a maze and the first two choices we make at the junctions we encounter in moving from point i to point c . We undergo three displacements as indicated in the overhead view of Fig. 3-16b:

$$\begin{aligned} d_1 &= 6.00 \text{ m} & \theta_1 &= 40^\circ \\ d_2 &= 8.00 \text{ m} & \theta_2 &= 30^\circ \\ d_3 &= 5.00 \text{ m} & \theta_3 &= 0^\circ, \end{aligned}$$

where the last segment is parallel to the superimposed x axis. When we reach point c , what are the magnitude and angle of our net displacement \vec{d}_{net} from point i ?

KEY IDEAS

(1) To find the net displacement \vec{d}_{net} , we need to sum the three individual displacement vectors:

$$\vec{d}_{\text{net}} = \vec{d}_1 + \vec{d}_2 + \vec{d}_3.$$

(2) To do this, we first evaluate this sum for the x components alone,

$$d_{\text{net},x} = d_{1x} + d_{2x} + d_{3x}, \quad (3-16)$$

and then the y components alone,

$$d_{\text{net},y} = d_{1y} + d_{2y} + d_{3y}. \quad (3-17)$$

(3) Finally, we construct \vec{d}_{net} from its x and y components.

Calculations: To evaluate Eqs. 3-16 and 3-17, we find the x and y components of each displacement. As an example, the components for the first displacement are shown in Fig. 3-16c. We draw similar diagrams for the other two displacements and then we apply the x part of Eq. 3-5 to each displacement, using angles relative to the positive direction of the x axis:

$$d_{1x} = (6.00 \text{ m}) \cos 40^\circ = 4.60 \text{ m}$$

$$d_{2x} = (8.00 \text{ m}) \cos (-60^\circ) = 4.00 \text{ m}$$

$$d_{3x} = (5.00 \text{ m}) \cos 0^\circ = 5.00 \text{ m}.$$

Equation 3-16 then gives us

$$\begin{aligned} d_{\text{net},x} &= +4.60 \text{ m} + 4.00 \text{ m} + 5.00 \text{ m} \\ &= 13.60 \text{ m}. \end{aligned}$$

Similarly, to evaluate Eq. 3-17, we apply the y part of Eq. 3-5 to each displacement:

$$d_{1y} = (6.00 \text{ m}) \sin 40^\circ = 3.86 \text{ m}$$

$$d_{2y} = (8.00 \text{ m}) \sin (-60^\circ) = -6.93 \text{ m}$$

$$d_{3y} = (5.00 \text{ m}) \sin 0^\circ = 0 \text{ m}.$$

Equation 3-17 then gives us

$$\begin{aligned} d_{\text{net},y} &= +3.86 \text{ m} - 6.93 \text{ m} + 0 \text{ m} \\ &= -3.07 \text{ m}. \end{aligned}$$

Next we use these components of \vec{d}_{net} to construct the vector as shown in Fig. 3-16d: the components are in a head-to-tail arrangement and form the legs of a right triangle, and

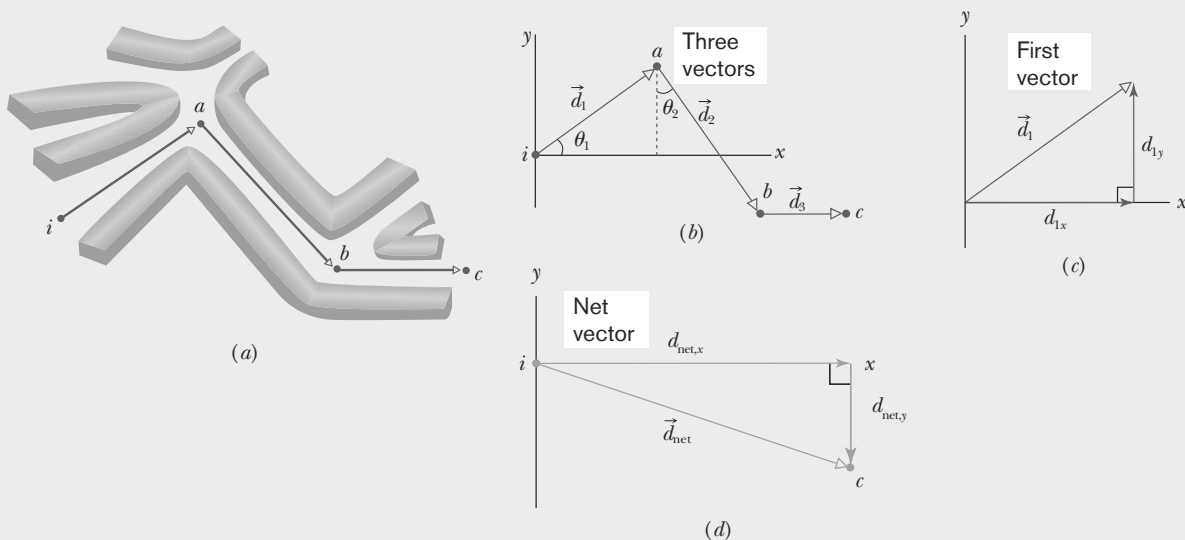


Figure 3-16 (a) Three displacements through a hedge maze. (b) The displacement vectors. (c) The first displacement vector and its components. (d) The net displacement vector and its components.

the vector forms the hypotenuse. We find the magnitude and angle of \vec{d}_{net} with Eq. 3-6. The magnitude is

$$d_{\text{net}} = \sqrt{d_{\text{net},x}^2 + d_{\text{net},y}^2} \quad (3-18)$$

$$= \sqrt{(13.60 \text{ m})^2 + (-3.07 \text{ m})^2} = 13.9 \text{ m.} \quad (\text{Answer})$$

To find the angle (measured from the positive direction of x), we take an inverse tangent:

$$\theta = \tan^{-1}\left(\frac{d_{\text{net},y}}{d_{\text{net},x}}\right) \quad (3-19)$$

$$= \tan^{-1}\left(\frac{-3.07 \text{ m}}{13.60 \text{ m}}\right) = -12.7^\circ. \quad (\text{Answer})$$

The angle is negative because it is measured clockwise from positive x . We must always be alert when we take an inverse

tangent on a calculator. The answer it displays is mathematically correct but it may not be the correct answer for the physical situation. In those cases, we have to add 180° to the displayed answer, to reverse the vector. To check, we always need to draw the vector and its components as we did in Fig. 3-16d. In our physical situation, the figure shows us that $\theta = -12.7^\circ$ is a reasonable answer, whereas $-12.7^\circ + 180^\circ = 167^\circ$ is clearly not.

We can see all this on the graph of tangent versus angle in Fig. 3-12c. In our maze problem, the argument of the inverse tangent is $-3.07/13.60$, or -0.226 . On the graph draw a horizontal line through that value on the vertical axis. The line cuts through the darker plotted branch at -12.7° and also through the lighter branch at 167° . The first cut is what a calculator displays.

Sample Problem 3.04 Adding vectors, unit-vector components

Figure 3-17a shows the following three vectors:

$$\vec{a} = (4.2 \text{ m})\hat{i} - (1.5 \text{ m})\hat{j},$$

$$\vec{b} = (-1.6 \text{ m})\hat{i} + (2.9 \text{ m})\hat{j},$$

and

$$\vec{c} = (-3.7 \text{ m})\hat{j}.$$

What is their vector sum \vec{r} which is also shown?

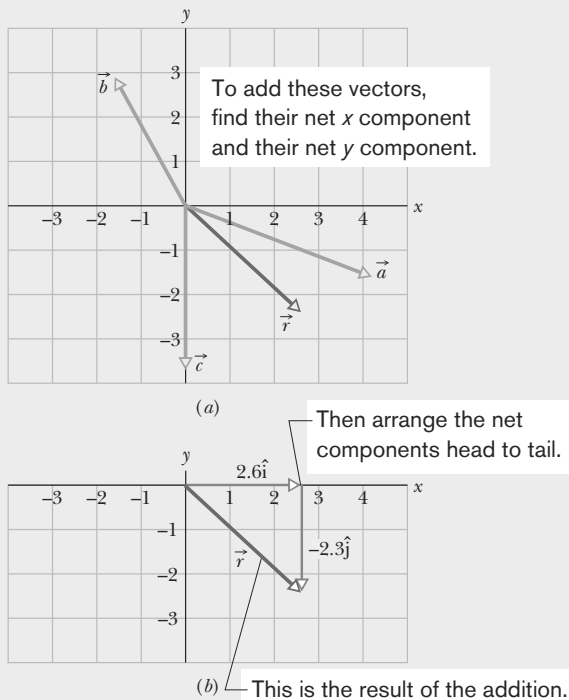


Figure 3-17 Vector \vec{r} is the vector sum of the other three vectors.

KEY IDEA

We can add the three vectors by components, axis by axis, and then combine the components to write the vector sum \vec{r} .

Calculations: For the x axis, we add the x components of \vec{a} , \vec{b} , and \vec{c} , to get the x component of the vector sum \vec{r} :

$$r_x = a_x + b_x + c_x$$

$$= 4.2 \text{ m} - 1.6 \text{ m} + 0 = 2.6 \text{ m.}$$

Similarly, for the y axis,

$$r_y = a_y + b_y + c_y$$

$$= -1.5 \text{ m} + 2.9 \text{ m} - 3.7 \text{ m} = -2.3 \text{ m.}$$

We then combine these components of \vec{r} to write the vector in unit-vector notation:

$$\vec{r} = (2.6 \text{ m})\hat{i} - (2.3 \text{ m})\hat{j}, \quad (\text{Answer})$$

where $(2.6 \text{ m})\hat{i}$ is the vector component of \vec{r} along the x axis and $-(2.3 \text{ m})\hat{j}$ is that along the y axis. Figure 3-17b shows one way to arrange these vector components to form \vec{r} . (Can you sketch the other way?)

We can also answer the question by giving the magnitude and an angle for \vec{r} . From Eq. 3-6, the magnitude is

$$r = \sqrt{(2.6 \text{ m})^2 + (-2.3 \text{ m})^2} \approx 3.5 \text{ m} \quad (\text{Answer})$$

and the angle (measured from the $+x$ direction) is

$$\theta = \tan^{-1}\left(\frac{-2.3 \text{ m}}{2.6 \text{ m}}\right) = -41^\circ, \quad (\text{Answer})$$

where the minus sign means clockwise.



Additional examples, video, and practice available at WileyPLUS



3-3 MULTIPLYING VECTORS

Learning Objectives

After reading this module, you should be able to . . .

3.09 Multiply vectors by scalars.

3.10 Identify that multiplying a vector by a scalar gives a vector, taking the dot (or scalar) product of two vectors gives a scalar, and taking the cross (or vector) product gives a new vector that is perpendicular to the original two.

3.11 Find the dot product of two vectors in magnitude-angle notation and in unit-vector notation.

3.12 Find the angle between two vectors by taking their dot product in both magnitude-angle notation and unit-vector notation.

3.13 Given two vectors, use a dot product to find how much of one vector lies along the other vector.

3.14 Find the cross product of two vectors in magnitude-angle and unit-vector notations.

3.15 Use the right-hand rule to find the direction of the vector that results from a cross product.

3.16 In nested products, where one product is buried inside another, follow the normal algebraic procedure by starting with the innermost product and working outward.

Key Ideas

● The product of a scalar s and a vector \vec{v} is a new vector whose magnitude is sv and whose direction is the same as that of \vec{v} if s is positive, and opposite that of \vec{v} if s is negative. To divide \vec{v} by s , multiply \vec{v} by $1/s$.

● The scalar (or dot) product of two vectors \vec{a} and \vec{b} is written $\vec{a} \cdot \vec{b}$ and is the scalar quantity given by

$$\vec{a} \cdot \vec{b} = ab \cos \phi,$$

in which ϕ is the angle between the directions of \vec{a} and \vec{b} .

A scalar product is the product of the magnitude of one vector and the scalar component of the second vector along the direction of the first vector. In unit-vector notation,

$$\vec{a} \cdot \vec{b} = (a_x \hat{i} + a_y \hat{j} + a_z \hat{k}) \cdot (b_x \hat{i} + b_y \hat{j} + b_z \hat{k}),$$

which may be expanded according to the distributive law. Note that $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$.

● The vector (or cross) product of two vectors \vec{a} and \vec{b} is written $\vec{a} \times \vec{b}$ and is a vector \vec{c} whose magnitude c is given by

$$c = ab \sin \phi,$$

in which ϕ is the smaller of the angles between the directions of \vec{a} and \vec{b} . The direction of \vec{c} is perpendicular to the plane defined by \vec{a} and \vec{b} and is given by a right-hand rule, as shown in Fig. 3-19. Note that $\vec{a} \times \vec{b} = -(\vec{b} \times \vec{a})$. In unit-vector notation,

$$\vec{a} \times \vec{b} = (a_x \hat{i} + a_y \hat{j} + a_z \hat{k}) \times (b_x \hat{i} + b_y \hat{j} + b_z \hat{k}),$$

which we may expand with the distributive law.

● In nested products, where one product is buried inside another, follow the normal algebraic procedure by starting with the innermost product and working outward.

Multiplying Vectors*

There are three ways in which vectors can be multiplied, but none is exactly like the usual algebraic multiplication. As you read this material, keep in mind that a vector-capable calculator will help you multiply vectors only if you understand the basic rules of that multiplication.

Multiplying a Vector by a Scalar

If we multiply a vector \vec{a} by a scalar s , we get a new vector. Its magnitude is the product of the magnitude of \vec{a} and the absolute value of s . Its direction is the direction of \vec{a} if s is positive but the opposite direction if s is negative. To divide \vec{a} by s , we multiply \vec{a} by $1/s$.

Multiplying a Vector by a Vector

There are two ways to multiply a vector by a vector: one way produces a scalar (called the *scalar product*), and the other produces a new vector (called the *vector product*). (Students commonly confuse the two ways.)

*This material will not be employed until later (Chapter 7 for scalar products and Chapter 11 for vector products), and so your instructor may wish to postpone it.

The Scalar Product

The **scalar product** of the vectors \vec{a} and \vec{b} in Fig. 3-18a is written as $\vec{a} \cdot \vec{b}$ and defined to be

$$\vec{a} \cdot \vec{b} = ab \cos \phi, \quad (3-20)$$

where a is the magnitude of \vec{a} , b is the magnitude of \vec{b} , and ϕ is the angle between \vec{a} and \vec{b} (or, more properly, between the directions of \vec{a} and \vec{b}). There are actually two such angles: ϕ and $360^\circ - \phi$. Either can be used in Eq. 3-20, because their cosines are the same.

Note that there are only scalars on the right side of Eq. 3-20 (including the value of $\cos \phi$). Thus $\vec{a} \cdot \vec{b}$ on the left side represents a *scalar* quantity. Because of the notation, $\vec{a} \cdot \vec{b}$ is also known as the **dot product** and is spoken as “a dot b.”

A dot product can be regarded as the product of two quantities: (1) the magnitude of one of the vectors and (2) the scalar component of the second vector along the direction of the first vector. For example, in Fig. 3-18b, \vec{a} has a scalar component $a \cos \phi$ along the direction of \vec{b} ; note that a perpendicular dropped from the head of \vec{a} onto \vec{b} determines that component. Similarly, \vec{b} has a scalar component $b \cos \phi$ along the direction of \vec{a} .



If the angle ϕ between two vectors is 0° , the component of one vector along the other is maximum, and so also is the dot product of the vectors. If, instead, ϕ is 90° , the component of one vector along the other is zero, and so is the dot product.

Equation 3-20 can be rewritten as follows to emphasize the components:

$$\vec{a} \cdot \vec{b} = (a \cos \phi)(b) = (a)(b \cos \phi). \quad (3-21)$$

The commutative law applies to a scalar product, so we can write

$$\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}.$$

When two vectors are in unit-vector notation, we write their dot product as

$$\vec{a} \cdot \vec{b} = (a_x \hat{i} + a_y \hat{j} + a_z \hat{k}) \cdot (b_x \hat{i} + b_y \hat{j} + b_z \hat{k}), \quad (3-22)$$

which we can expand according to the distributive law: Each vector component of the first vector is to be dotted with each vector component of the second vector. By doing so, we can show that

$$\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z. \quad (3-23)$$

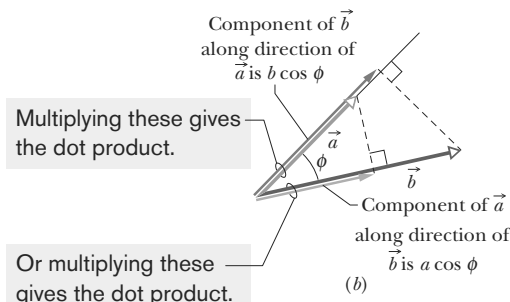
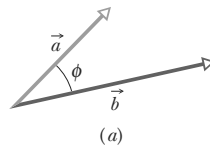


Figure 3-18 (a) Two vectors \vec{a} and \vec{b} , with an angle ϕ between them. (b) Each vector has a component along the direction of the other vector.

**Checkpoint 4**

Vectors \vec{C} and \vec{D} have magnitudes of 3 units and 4 units, respectively. What is the angle between the directions of \vec{C} and \vec{D} if $\vec{C} \cdot \vec{D}$ equals (a) zero, (b) 12 units, and (c) -12 units?

The Vector Product

The **vector product** of \vec{a} and \vec{b} , written $\vec{a} \times \vec{b}$, produces a third vector \vec{c} whose magnitude is

$$c = ab \sin \phi, \quad (3-24)$$

where ϕ is the *smaller* of the two angles between \vec{a} and \vec{b} . (You must use the smaller of the two angles between the vectors because $\sin \phi$ and $\sin(360^\circ - \phi)$ differ in algebraic sign.) Because of the notation, $\vec{a} \times \vec{b}$ is also known as the **cross product**, and in speech it is “a cross b.”



If \vec{a} and \vec{b} are parallel or antiparallel, $\vec{a} \times \vec{b} = 0$. The magnitude of $\vec{a} \times \vec{b}$, which can be written as $|\vec{a} \times \vec{b}|$, is maximum when \vec{a} and \vec{b} are perpendicular to each other.

The direction of \vec{c} is perpendicular to the plane that contains \vec{a} and \vec{b} . Figure 3-19a shows how to determine the direction of $\vec{c} = \vec{a} \times \vec{b}$ with what is known as a **right-hand rule**. Place the vectors \vec{a} and \vec{b} tail to tail without altering their orientations, and imagine a line that is perpendicular to their plane where they meet. Pretend to place your *right* hand around that line in such a way that your fingers would sweep \vec{a} into \vec{b} through the smaller angle between them. Your outstretched thumb points in the direction of \vec{c} .

The order of the vector multiplication is important. In Fig. 3-19b, we are determining the direction of $\vec{c}' = \vec{b} \times \vec{a}$, so the fingers are placed to sweep \vec{b} into \vec{a} through the smaller angle. The thumb ends up in the opposite direction from previously, and so it must be that $\vec{c}' = -\vec{c}$; that is,

$$\vec{b} \times \vec{a} = -(\vec{a} \times \vec{b}). \quad (3-25)$$

In other words, the commutative law does not apply to a vector product.

In unit-vector notation, we write

$$\vec{a} \times \vec{b} = (a_x \hat{i} + a_y \hat{j} + a_z \hat{k}) \times (b_x \hat{i} + b_y \hat{j} + b_z \hat{k}), \quad (3-26)$$

which can be expanded according to the distributive law; that is, each component of the first vector is to be crossed with each component of the second vector. The cross products of unit vectors are given in Appendix E (see “Products of Vectors”). For example, in the expansion of Eq. 3-26, we have

$$a_x \hat{i} \times b_x \hat{i} = a_x b_x (\hat{i} \times \hat{i}) = 0,$$

because the two unit vectors \hat{i} and \hat{i} are parallel and thus have a zero cross product. Similarly, we have

$$a_x \hat{i} \times b_y \hat{j} = a_x b_y (\hat{i} \times \hat{j}) = a_x b_y \hat{k}.$$

In the last step we used Eq. 3-24 to evaluate the magnitude of $\hat{i} \times \hat{j}$ as unity. (These vectors \hat{i} and \hat{j} each have a magnitude of unity, and the angle between them is 90° .) Also, we used the right-hand rule to get the direction of $\hat{i} \times \hat{j}$ as being in the positive direction of the z axis (thus in the direction of \hat{k}).

Continuing to expand Eq. 3-26, you can show that

$$\vec{a} \times \vec{b} = (a_y b_z - b_y a_z)\hat{i} + (a_z b_x - b_z a_x)\hat{j} + (a_x b_y - b_x a_y)\hat{k}. \quad (3-27)$$

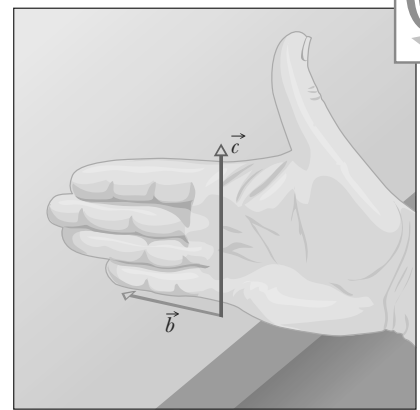
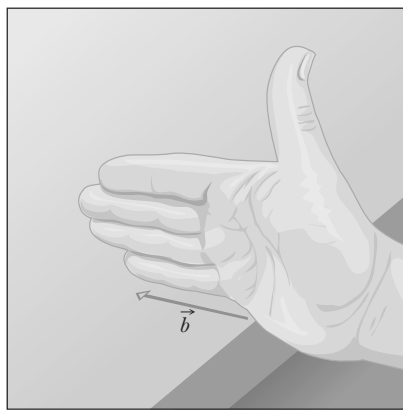
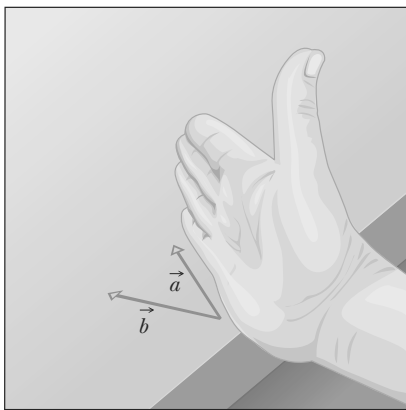
A determinant (Appendix E) or a vector-capable calculator can also be used.

To check whether any xyz coordinate system is a right-handed coordinate system, use the right-hand rule for the cross product $\hat{i} \times \hat{j} = \hat{k}$ with that system. If your fingers sweep \hat{i} (positive direction of x) into \hat{j} (positive direction of y) with the outstretched thumb pointing in the positive direction of z (not the negative direction), then the system is right-handed.

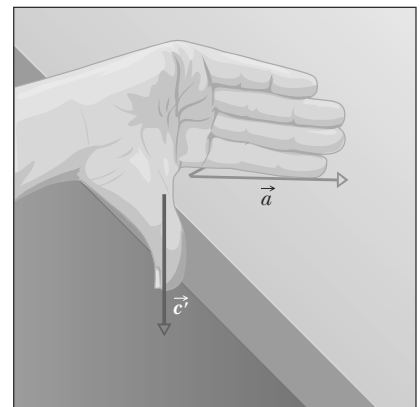
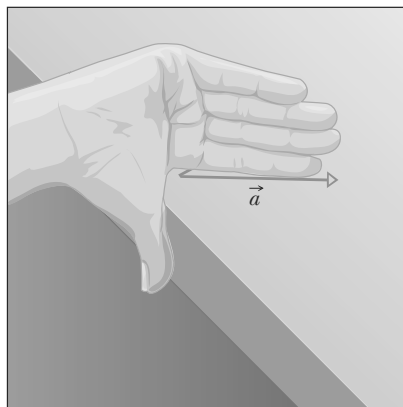
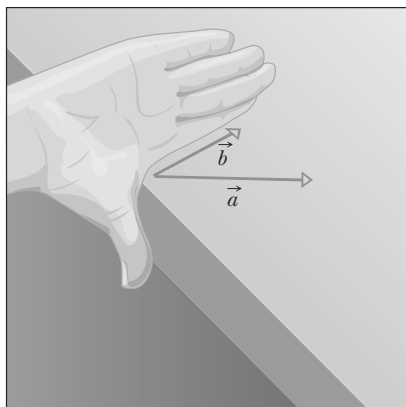


Checkpoint 5

Vectors \vec{C} and \vec{D} have magnitudes of 3 units and 4 units, respectively. What is the angle between the directions of \vec{C} and \vec{D} if the magnitude of the vector product $\vec{C} \times \vec{D}$ is (a) zero and (b) 12 units?



(a)



(b)

Figure 3-19 Illustration of the right-hand rule for vector products. (a) Sweep vector \vec{a} into vector \vec{b} with the fingers of your right hand. Your outstretched thumb shows the direction of vector $\vec{c} = \vec{a} \times \vec{b}$. (b) Showing that $\vec{b} \times \vec{a}$ is the reverse of $\vec{a} \times \vec{b}$.

Sample Problem 3.05 Angle between two vectors using dot products

What is the angle ϕ between $\vec{a} = 3.0\hat{i} - 4.0\hat{j}$ and $\vec{b} = -2.0\hat{i} + 3.0\hat{k}$? (*Caution:* Although many of the following steps can be bypassed with a vector-capable calculator, you will learn more about scalar products if, at least here, you use these steps.)

KEY IDEA

The angle between the directions of two vectors is included in the definition of their scalar product (Eq. 3-20):

$$\vec{a} \cdot \vec{b} = ab \cos \phi. \quad (3-28)$$

Calculations: In Eq. 3-28, a is the magnitude of \vec{a} , or

$$a = \sqrt{3.0^2 + (-4.0)^2} = 5.00, \quad (3-29)$$

and b is the magnitude of \vec{b} , or

$$b = \sqrt{(-2.0)^2 + 3.0^2} = 3.61. \quad (3-30)$$

We can separately evaluate the left side of Eq. 3-28 by writing the vectors in unit-vector notation and using the distributive law:

$$\begin{aligned} \vec{a} \cdot \vec{b} &= (3.0\hat{i} - 4.0\hat{j}) \cdot (-2.0\hat{i} + 3.0\hat{k}) \\ &= (3.0\hat{i}) \cdot (-2.0\hat{i}) + (3.0\hat{i}) \cdot (3.0\hat{k}) \\ &\quad + (-4.0\hat{j}) \cdot (-2.0\hat{i}) + (-4.0\hat{j}) \cdot (3.0\hat{k}). \end{aligned}$$

We next apply Eq. 3-20 to each term in this last expression. The angle between the unit vectors in the first term (\hat{i} and \hat{i}) is 0° , and in the other terms it is 90° . We then have

$$\begin{aligned} \vec{a} \cdot \vec{b} &= -(6.0)(1) + (9.0)(0) + (8.0)(0) - (12)(0) \\ &= -6.0. \end{aligned}$$

Substituting this result and the results of Eqs. 3-29 and 3-30 into Eq. 3-28 yields

$$\begin{aligned} -6.0 &= (5.00)(3.61) \cos \phi, \\ \text{so } \phi &= \cos^{-1} \frac{-6.0}{(5.00)(3.61)} = 109^\circ \approx 110^\circ. \quad (\text{Answer}) \end{aligned}$$

Sample Problem 3.06 Cross product, right-hand rule

In Fig. 3-20, vector \vec{a} lies in the xy plane, has a magnitude of 18 units, and points in a direction 250° from the positive direction of the x axis. Also, vector \vec{b} has a magnitude of 12 units and points in the positive direction of the z axis. What is the vector product $\vec{c} = \vec{a} \times \vec{b}$?

KEY IDEA

When we have two vectors in magnitude-angle notation, we find the magnitude of their cross product with Eq. 3-24 and the direction of their cross product with the right-hand rule of Fig. 3-19.

Calculations: For the magnitude we write

$$c = ab \sin \phi = (18)(12)(\sin 90^\circ) = 216. \quad (\text{Answer})$$

To determine the direction in Fig. 3-20, imagine placing the fingers of your right hand around a line perpendicular to the plane of \vec{a} and \vec{b} (the line on which \vec{c} is shown) such that your fingers sweep \vec{a} into \vec{b} . Your outstretched thumb then

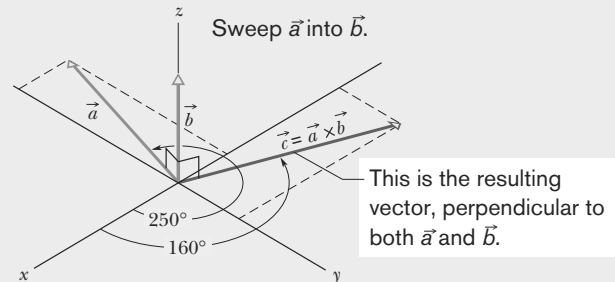


Figure 3-20 Vector \vec{c} (in the xy plane) is the vector (or cross) product of vectors \vec{a} and \vec{b} .

gives the direction of \vec{c} . Thus, as shown in the figure, \vec{c} lies in the xy plane. Because its direction is perpendicular to the direction of \vec{a} (a cross product always gives a perpendicular vector), it is at an angle of

$$250^\circ - 90^\circ = 160^\circ \quad (\text{Answer})$$

from the positive direction of the x axis.

Sample Problem 3.07 Cross product, unit-vector notation

If $\vec{a} = 3\hat{i} - 4\hat{j}$ and $\vec{b} = -2\hat{i} + 3\hat{k}$, what is $\vec{c} = \vec{a} \times \vec{b}$?

KEY IDEA

When two vectors are in unit-vector notation, we can find their cross product by using the distributive law.

Calculations: Here we write

$$\begin{aligned} \vec{c} &= (3\hat{i} - 4\hat{j}) \times (-2\hat{i} + 3\hat{k}) \\ &= 3\hat{i} \times (-2\hat{i}) + 3\hat{i} \times 3\hat{k} + (-4\hat{j}) \times (-2\hat{i}) \\ &\quad + (-4\hat{j}) \times 3\hat{k}. \end{aligned}$$

We next evaluate each term with Eq. 3-24, finding the direction with the right-hand rule. For the first term here, the angle ϕ between the two vectors being crossed is 0. For the other terms, ϕ is 90° . We find

$$\begin{aligned}\vec{c} &= -6(0) + 9(-\hat{j}) + 8(-\hat{k}) - 12\hat{i} \\ &= -12\hat{i} - 9\hat{j} - 8\hat{k}.\end{aligned}\quad (\text{Answer})$$



Additional examples, video, and practice available at WileyPLUS



Review & Summary

Scalars and Vectors *Scalars*, such as temperature, have magnitude only. They are specified by a number with a unit (10°C) and obey the rules of arithmetic and ordinary algebra. *Vectors*, such as displacement, have both magnitude and direction (5 m, north) and obey the rules of vector algebra.

Adding Vectors Geometrically Two vectors \vec{a} and \vec{b} may be added geometrically by drawing them to a common scale and placing them head to tail. The vector connecting the tail of the first to the head of the second is the vector sum \vec{s} . To subtract \vec{b} from \vec{a} , reverse the direction of \vec{b} to get $-\vec{b}$; then add $-\vec{b}$ to \vec{a} . Vector addition is commutative

$$\vec{a} + \vec{b} = \vec{b} + \vec{a} \quad (3-2)$$

and obeys the associative law

$$(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c}). \quad (3-3)$$

Components of a Vector The (scalar) *components* a_x and a_y of any two-dimensional vector \vec{a} along the coordinate axes are found by dropping perpendicular lines from the ends of \vec{a} onto the coordinate axes. The components are given by

$$a_x = a \cos \theta \quad \text{and} \quad a_y = a \sin \theta, \quad (3-5)$$

where θ is the angle between the positive direction of the x axis and the direction of \vec{a} . The algebraic sign of a component indicates its direction along the associated axis. Given its components, we can find the magnitude and orientation (direction) of the vector \vec{a} by using

$$a = \sqrt{a_x^2 + a_y^2} \quad \text{and} \quad \tan \theta = \frac{a_y}{a_x} \quad (3-6)$$

Unit-Vector Notation *Unit vectors* \hat{i} , \hat{j} , and \hat{k} have magnitudes of unity and are directed in the positive directions of the x , y , and z axes, respectively, in a right-handed coordinate system (as defined by the vector products of the unit vectors). We can write a vector \vec{a} in terms of unit vectors as

$$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}, \quad (3-7)$$

in which $a_x\hat{i}$, $a_y\hat{j}$, and $a_z\hat{k}$ are the **vector components** of \vec{a} and a_x , a_y , and a_z are its **scalar components**.

This vector \vec{c} is perpendicular to both \vec{a} and \vec{b} , a fact you can check by showing that $\vec{c} \cdot \vec{a} = 0$ and $\vec{c} \cdot \vec{b} = 0$; that is, there is no component of \vec{c} along the direction of either \vec{a} or \vec{b} .

In general: A cross product gives a perpendicular vector, two perpendicular vectors have a zero dot product, and two vectors along the same axis have a zero cross product.

Adding Vectors in Component Form To add vectors in component form, we use the rules

$$r_x = a_x + b_x \quad r_y = a_y + b_y \quad r_z = a_z + b_z. \quad (3-10 \text{ to } 3-12)$$

Here \vec{a} and \vec{b} are the vectors to be added, and \vec{r} is the vector sum. Note that we add components axis by axis. We can then express the sum in unit-vector notation or magnitude-angle notation.

Product of a Scalar and a Vector The product of a scalar s and a vector \vec{v} is a new vector whose magnitude is sv and whose direction is the same as that of \vec{v} if s is positive, and opposite that of \vec{v} if s is negative. (The negative sign reverses the vector.) To divide \vec{v} by s , multiply \vec{v} by $1/s$.

The Scalar Product The **scalar** (or **dot**) **product** of two vectors \vec{a} and \vec{b} is written $\vec{a} \cdot \vec{b}$ and is the *scalar* quantity given by

$$\vec{a} \cdot \vec{b} = ab \cos \phi, \quad (3-20)$$

in which ϕ is the angle between the directions of \vec{a} and \vec{b} . A scalar product is the product of the magnitude of one vector and the scalar component of the second vector along the direction of the first vector. Note that $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$, which means that the scalar product obeys the commutative law.

In unit-vector notation,

$$\vec{a} \cdot \vec{b} = (a_x\hat{i} + a_y\hat{j} + a_z\hat{k}) \cdot (b_x\hat{i} + b_y\hat{j} + b_z\hat{k}), \quad (3-22)$$

which may be expanded according to the distributive law.

The Vector Product The **vector** (or **cross**) **product** of two vectors \vec{a} and \vec{b} is written $\vec{a} \times \vec{b}$ and is a *vector* \vec{c} whose magnitude c is given by

$$c = ab \sin \phi, \quad (3-24)$$

in which ϕ is the smaller of the angles between the directions of \vec{a} and \vec{b} . The direction of \vec{c} is perpendicular to the plane defined by \vec{a} and \vec{b} and is given by a right-hand rule, as shown in Fig. 3-19. Note that $\vec{a} \times \vec{b} = -(\vec{b} \times \vec{a})$, which means that the vector product does not obey the commutative law.

In unit-vector notation,

$$\vec{a} \times \vec{b} = (a_x\hat{i} + a_y\hat{j} + a_z\hat{k}) \times (b_x\hat{i} + b_y\hat{j} + b_z\hat{k}), \quad (3-26)$$

which we may expand with the distributive law.

Questions

1 Can the sum of the magnitudes of two vectors ever be equal to the magnitude of the sum of the same two vectors? If no, why not? If yes, when?

2 The two vectors shown in Fig. 3-21 lie in an xy plane. What are the signs of the x and y components, respectively, of (a) $\vec{d}_1 + \vec{d}_2$, (b) $\vec{d}_1 - \vec{d}_2$, and (c) $\vec{d}_2 - \vec{d}_1$?

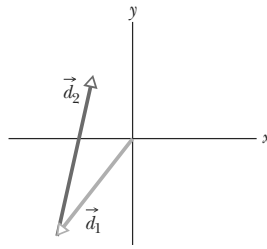


Figure 3-21 Question 2.

3 Being part of the “Gators,” the University of Florida golfing team must play on a putting green with an alligator pit. Figure 3-22 shows an overhead view of one putting challenge of the team; an xy coordinate system is superimposed. Team members must putt from the origin to the hole, which is at xy coordinates (8 m, 12 m), but they can putt the golf ball using only one or more of the following displacements, one or more times:

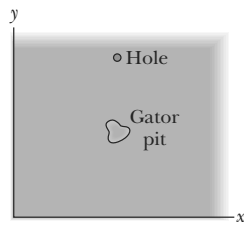


Figure 3-22 Question 3.

$$\vec{d}_1 = (8 \text{ m})\hat{i} + (6 \text{ m})\hat{j}, \quad \vec{d}_2 = (6 \text{ m})\hat{j}, \quad \vec{d}_3 = (8 \text{ m})\hat{i}.$$

The pit is at coordinates (8 m, 6 m). If a team member putts the ball into or through the pit, the member is automatically transferred to Florida State University, the arch rival. What sequence of displacements should a team member use to avoid the pit and the school transfer?

4 Equation 3-2 shows that the addition of two vectors \vec{a} and \vec{b} is commutative. Does that mean subtraction is commutative, so that $\vec{a} - \vec{b} = \vec{b} - \vec{a}$?

5 Which of the arrangements of axes in Fig. 3-23 can be labeled “right-handed coordinate system”? As usual, each axis label indicates the positive side of the axis.

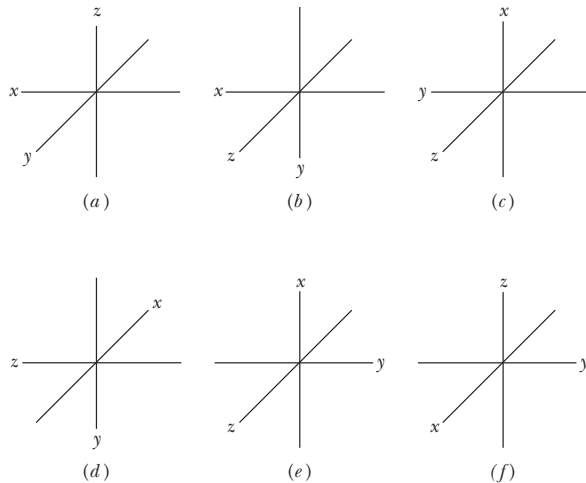


Figure 3-23 Question 5.

6 Describe two vectors \vec{a} and \vec{b} such that

(a) $\vec{a} + \vec{b} = \vec{c}$ and $a + b = c$;

(b) $\vec{a} + \vec{b} = \vec{a} - \vec{b}$;

(c) $\vec{a} + \vec{b} = \vec{c}$ and $a^2 + b^2 = c^2$.

7 If $\vec{d} = \vec{a} + \vec{b} + (-\vec{c})$, does (a) $\vec{a} + (-\vec{d}) = \vec{c} + (-\vec{b})$, (b) $\vec{a} = (-\vec{b}) + \vec{d} + \vec{c}$, and (c) $\vec{c} + (-\vec{d}) = \vec{a} + \vec{b}$?

8 If $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c}$, must \vec{b} equal \vec{c} ?

9 If $\vec{F} = q(\vec{v} \times \vec{B})$ and \vec{v} is perpendicular to \vec{B} , then what is the direction of \vec{B} in the three situations shown in Fig. 3-24 when constant q is (a) positive and (b) negative?

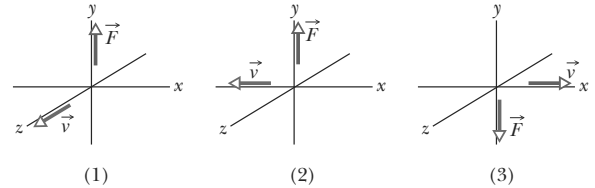


Figure 3-24 Question 9.

10 Figure 3-25 shows vector \vec{A} and four other vectors that have the same magnitude but differ in orientation.

(a) Which of those other four vectors have the same dot product with \vec{A} ? (b) Which have a negative dot product with \vec{A} ?

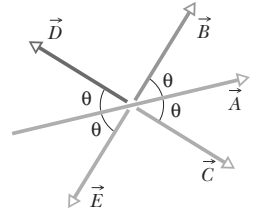


Figure 3-25 Question 10.

11 In a game held within a three-dimensional maze, you must move your game piece from *start*, at xyz coordinates (0, 0, 0), to *finish*, at coordinates (−2 cm, 4 cm, −4 cm). The game piece can undergo only the displacements (in centimeters) given below. If, along the way, the game piece lands at coordinates (−5 cm, −1 cm, −1 cm) or (5 cm, 2 cm, −1 cm), you lose the game. Which displacements and in what sequence will get your game piece to *finish*?

$$\begin{aligned} \vec{p} &= -7\hat{i} + 2\hat{j} - 3\hat{k} & \vec{r} &= 2\hat{i} - 3\hat{j} + 2\hat{k} \\ \vec{q} &= 2\hat{i} - \hat{j} + 4\hat{k} & \vec{s} &= 3\hat{i} + 5\hat{j} - 3\hat{k}. \end{aligned}$$

12 The x and y components of four vectors \vec{a} , \vec{b} , \vec{c} , and \vec{d} are given below. For which vectors will your calculator give you the correct angle θ when you use it to find θ with Eq. 3-6? Answer first by examining Fig. 3-12, and then check your answers with your calculator.

$$\begin{array}{cccc} a_x = 3 & a_y = 3 & c_x = -3 & c_y = -3 \\ b_x = -3 & b_y = 3 & d_x = 3 & d_y = -3. \end{array}$$

13 Which of the following are correct (meaningful) vector expressions? What is wrong with any incorrect expression?

- (a) $\vec{A} \cdot (\vec{B} \cdot \vec{C})$
- (b) $\vec{A} \times (\vec{B} \cdot \vec{C})$
- (c) $\vec{A} \cdot (\vec{B} \times \vec{C})$
- (d) $\vec{A} \times (\vec{B} \times \vec{C})$
- (e) $\vec{A} + (\vec{B} \cdot \vec{C})$
- (f) $\vec{A} + (\vec{B} \times \vec{C})$
- (g) $5 + \vec{A}$
- (h) $5 + (\vec{B} \cdot \vec{C})$
- (i) $5 + (\vec{B} \times \vec{C})$
- (j) $(\vec{A} \cdot \vec{B}) + (\vec{B} \times \vec{C})$

Problems



Tutoring problem available (at instructor's discretion) in WileyPLUS and WebAssign



Worked-out solution available in Student Solutions Manual



Worked-out solution is at



Number of dots indicates level of problem difficulty



Interactive solution is at

<http://www.wiley.com/college/halliday>



Additional information available in *The Flying Circus of Physics* and at flyingcircusofphysics.com

Module 3-1 Vectors and Their Components

•1 **SSM** What are (a) the x component and (b) the y component of a vector \vec{a} in the xy plane if its direction is 250° counterclockwise from the positive direction of the x axis and its magnitude is 7.3 m?

•2 A displacement vector \vec{r} in the xy plane is 15 m long and directed at angle $\theta = 30^\circ$ in Fig. 3-26. Determine (a) the x component and (b) the y component of the vector.

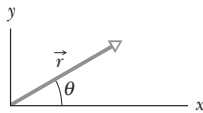


Figure 3-26
Problem 2.

•3 **SSM** The x component of vector \vec{A} is -25.0 m and the y component is $+40.0$ m. (a) What is the magnitude of \vec{A} ? (b) What is the angle between the direction of \vec{A} and the positive direction of x ?

•4 Express the following angles in radians: (a) 20.0° , (b) 50.0° , (c) 100° . Convert the following angles to degrees: (d) 0.330 rad, (e) 2.10 rad, (f) 7.70 rad.

•5 A ship sets out to sail to a point 120 km due north. An unexpected storm blows the ship to a point 100 km due east of its starting point. (a) How far and (b) in what direction must it now sail to reach its original destination?

•6 In Fig. 3-27, a heavy piece of machinery is raised by sliding it a distance $d = 12.5$ m along a plank oriented at angle $\theta = 20.0^\circ$ to the horizontal. How far is it moved (a) vertically and (b) horizontally?

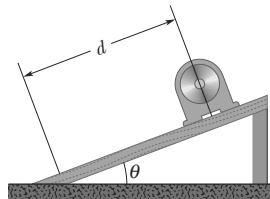


Figure 3-27 Problem 6.

•7 Consider two displacements, one of magnitude 3 m and another of magnitude 4 m. Show how the displacement vectors may be combined to get a resultant displacement of magnitude (a) 7 m, (b) 1 m, and (c) 5 m.

Module 3-2 Unit Vectors, Adding Vectors by Components

•8 A person walks in the following pattern: 3.1 km north, then 2.4 km west, and finally 5.2 km south. (a) Sketch the vector diagram that represents this motion. (b) How far and (c) in what direction would a bird fly in a straight line from the same starting point to the same final point?

•9 Two vectors are given by

$$\vec{a} = (4.0 \text{ m})\hat{i} - (3.0 \text{ m})\hat{j} + (1.0 \text{ m})\hat{k}$$

and $\vec{b} = (-1.0 \text{ m})\hat{i} + (1.0 \text{ m})\hat{j} + (4.0 \text{ m})\hat{k}$.

In unit-vector notation, find (a) $\vec{a} + \vec{b}$, (b) $\vec{a} - \vec{b}$, and (c) a third vector \vec{c} such that $\vec{a} - \vec{b} + \vec{c} = 0$.

•10 Find the (a) x , (b) y , and (c) z components of the sum \vec{r} of the displacements \vec{c} and \vec{d} whose components in meters are $c_x = 7.4$, $c_y = -3.8$, $c_z = -6.1$; $d_x = 4.4$, $d_y = -2.0$, $d_z = 3.3$.

•11 **SSM** (a) In unit-vector notation, what is the sum $\vec{a} + \vec{b}$ if $\vec{a} = (4.0 \text{ m})\hat{i} + (3.0 \text{ m})\hat{j}$ and $\vec{b} = (-13.0 \text{ m})\hat{i} + (7.0 \text{ m})\hat{j}$? What are the (b) magnitude and (c) direction of $\vec{a} + \vec{b}$?

•12 A car is driven east for a distance of 50 km, then north for 30 km, and then in a direction 30° east of north for 25 km. Sketch the vector diagram and determine (a) the magnitude and (b) the angle of the car's total displacement from its starting point.

•13 A person desires to reach a point that is 3.40 km from her present location and in a direction that is 35.0° north of east. However, she must travel along streets that are oriented either north-south or east-west. What is the minimum distance she could travel to reach her destination?

•14 You are to make four straight-line moves over a flat desert floor, starting at the origin of an xy coordinate system and ending at the xy coordinates $(-140 \text{ m}, 30 \text{ m})$. The x component and y component of your moves are the following, respectively, in meters: $(20$ and $60)$, then $(b_x$ and $-70)$, then $(-20$ and $c_y)$, then $(-60$ and $-70)$. What are (a) component b_x and (b) component c_y ? What are (c) the magnitude and (d) the angle (relative to the positive direction of the x axis) of the overall displacement?

•15 **SSM ILW WWW** The two vectors \vec{a} and \vec{b} in Fig. 3-28 have equal magnitudes of 10.0 m and the angles are $\theta_1 = 30^\circ$ and $\theta_2 = 105^\circ$. Find the (a) x and (b) y components of their vector sum \vec{r} , (c) the magnitude of \vec{r} , and (d) the angle \vec{r} makes with the positive direction of the x axis.

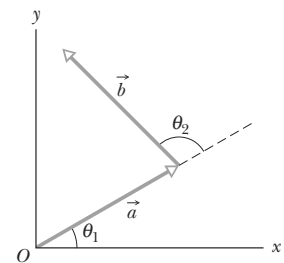


Figure 3-28 Problem 15.

•16 For the displacement vectors $\vec{a} = (3.0 \text{ m})\hat{i} + (4.0 \text{ m})\hat{j}$ and $\vec{b} = (5.0 \text{ m})\hat{i} + (-2.0 \text{ m})\hat{j}$, give $\vec{a} + \vec{b}$ in (a) unit-vector notation, and as (b) a magnitude and (c) an angle (relative to \hat{i}). Now give $\vec{b} - \vec{a}$ in (d) unit-vector notation, and as (e) a magnitude and (f) an angle.

•17 **GO ILW** Three vectors \vec{a} , \vec{b} , and \vec{c} each have a magnitude of 50 m and lie in an xy plane. Their directions relative to the positive direction of the x axis are 30° , 195° , and 315° , respectively. What are (a) the magnitude and (b) the angle of the vector $\vec{a} + \vec{b} + \vec{c}$, and (c) the magnitude and (d) the angle of $\vec{a} - \vec{b} + \vec{c}$? What are the (e) magnitude and (f) angle of a fourth vector \vec{d} such that $(\vec{a} + \vec{b}) - (\vec{c} + \vec{d}) = 0$?

•18 In the sum $\vec{A} + \vec{B} = \vec{C}$, vector \vec{A} has a magnitude of 12.0 m and is angled 40.0° counterclockwise from the $+x$ direction, and vector \vec{C} has a magnitude of 15.0 m and is angled 20.0° counterclockwise from the $-x$ direction. What are (a) the magnitude and (b) the angle (relative to $+x$) of \vec{B} ?

•19 In a game of lawn chess, where pieces are moved between the centers of squares that are each 1.00 m on edge, a knight is moved in the following way: (1) two squares forward, one square rightward; (2) two squares leftward, one square forward; (3) two squares forward, one square leftward. What are (a) the magnitude and (b) the angle (relative to "forward") of the knight's overall displacement for the series of three moves?

••20 🚶 An explorer is caught in a whiteout (in which the snowfall is so thick that the ground cannot be distinguished from the sky) while returning to base camp. He was supposed to travel due north for 5.6 km, but when the snow clears, he discovers that he actually traveled 7.8 km at 50° north of due east. (a) How far and (b) in what direction must he now travel to reach base camp?

••21 🐜 An ant, crazed by the Sun on a hot Texas afternoon, darts over an xy plane scratched in the dirt. The x and y components of four consecutive darts are the following, all in centimeters: $(30.0, 40.0)$, $(b_x, -70.0)$, $(-20.0, c_y)$, $(-80.0, -70.0)$. The overall displacement of the four darts has the xy components $(-140, -20.0)$. What are (a) b_x and (b) c_y ? What are the (c) magnitude and (d) angle (relative to the positive direction of the x axis) of the overall displacement?

••22 (a) What is the sum of the following four vectors in unit-vector notation? For that sum, what are (b) the magnitude, (c) the angle in degrees, and (d) the angle in radians?

$$\vec{E}: 6.00 \text{ m at } +0.900 \text{ rad} \quad \vec{F}: 5.00 \text{ m at } -75.0^\circ$$

$$\vec{G}: 4.00 \text{ m at } +1.20 \text{ rad} \quad \vec{H}: 6.00 \text{ m at } -210^\circ$$

••23 If \vec{B} is added to $\vec{C} = 3.0\hat{i} + 4.0\hat{j}$, the result is a vector in the positive direction of the y axis, with a magnitude equal to that of \vec{C} . What is the magnitude of \vec{B} ?

••24 🐜 Vector \vec{A} , which is directed along an x axis, is to be added to vector \vec{B} , which has a magnitude of 7.0 m. The sum is a third vector that is directed along the y axis, with a magnitude that is 3.0 times that of \vec{A} . What is that magnitude of \vec{A} ?

••25 🐪 Oasis B is 25 km due east of oasis A . Starting from oasis A , a camel walks 24 km in a direction 15° south of east and then walks 8.0 km due north. How far is the camel then from oasis B ?

••26 What is the sum of the following four vectors in (a) unit-vector notation, and as (b) a magnitude and (c) an angle?

$$\vec{A} = (2.00 \text{ m})\hat{i} + (3.00 \text{ m})\hat{j} \quad \vec{B}: 4.00 \text{ m, at } +65.0^\circ$$

$$\vec{C} = (-4.00 \text{ m})\hat{i} + (-6.00 \text{ m})\hat{j} \quad \vec{D}: 5.00 \text{ m, at } -235^\circ$$

••27 🐜 If $\vec{d}_1 + \vec{d}_2 = 5\vec{d}_3$, $\vec{d}_1 - \vec{d}_2 = 3\vec{d}_3$, and $\vec{d}_3 = 2\hat{i} + 4\hat{j}$, then what are, in unit-vector notation, (a) \vec{d}_1 and (b) \vec{d}_2 ?

••28 Two beetles run across flat sand, starting at the same point. Beetle 1 runs 0.50 m due east, then 0.80 m at 30° north of due east. Beetle 2 also makes two runs; the first is 1.6 m at 40° east of due north. What must be (a) the magnitude and (b) the direction of its second run if it is to end up at the new location of beetle 1?

••29 🐜 Typical backyard ants often create a network of chemical trails for guidance. Extending outward from the nest, a trail branches (*bifurcates*) repeatedly, with 60° between the branches. If a roaming ant chances upon a trail, it can tell the way to the nest at any branch point: If it is moving away from the nest, it has two choices of path requiring a small turn in its travel direction, either 30° leftward or 30° rightward. If it is moving toward the nest, it has only one such choice. Figure 3-29 shows a typical ant trail, with lettered straight sections of 2.0 cm length and symmetric bifurcation of 60° . Path v is parallel to the y axis. What are the (a) magnitude and (b) angle (relative to the positive direction of the superimposed x axis) of

an ant's displacement from the nest (find it in the figure) if the ant enters the trail at point A ? What are the (c) magnitude and (d) angle if it enters at point B ?

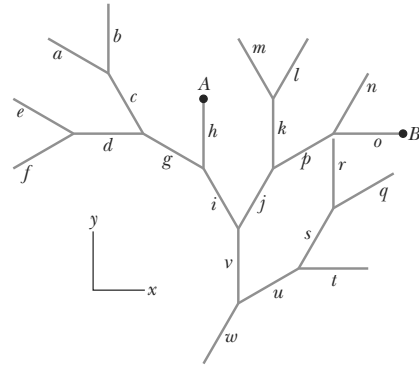


Figure 3-29 Problem 29.

••30 🐜 Here are two vectors:

$$\vec{a} = (4.0 \text{ m})\hat{i} - (3.0 \text{ m})\hat{j} \quad \text{and} \quad \vec{b} = (6.0 \text{ m})\hat{i} + (8.0 \text{ m})\hat{j}.$$

What are (a) the magnitude and (b) the angle (relative to \hat{i}) of \vec{a} ? What are (c) the magnitude and (d) the angle of \vec{b} ? What are (e) the magnitude and (f) the angle of $\vec{a} + \vec{b}$; (g) the magnitude and (h) the angle of $\vec{b} - \vec{a}$; and (i) the magnitude and (j) the angle of $\vec{a} - \vec{b}$? (k) What is the angle between the directions of $\vec{b} - \vec{a}$ and $\vec{a} - \vec{b}$?

••31 In Fig. 3-30, a vector \vec{a} with a magnitude of 17.0 m is directed at angle $\theta = 56.0^\circ$ counterclockwise from the $+x$ axis. What are the components (a) a_x and (b) a_y of the vector? A second coordinate system is inclined by angle $\theta' = 18.0^\circ$ with respect to the first. What are the components (c) a'_x and (d) a'_y in this primed coordinate system?

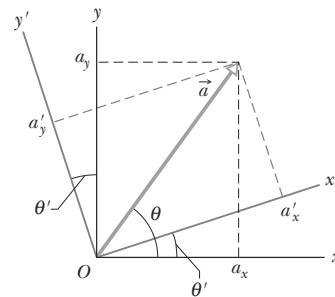


Figure 3-30 Problem 31.

••32 In Fig. 3-31, a cube of edge length a sits with one corner at the origin of an xyz coordinate system. A *body diagonal* is a line that extends from one corner to another through the center. In unit-vector notation, what is the body diagonal that extends from the corner at (a) coordinates $(0, 0, 0)$, (b) coordinates $(a, 0, 0)$, (c) coordinates $(0, a, 0)$, and (d) coordinates $(a, a, 0)$? (e) Determine the

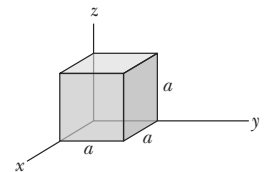


Figure 3-31 Problem 32.

angles that the body diagonals make with the adjacent edges. (f) Determine the length of the body diagonals in terms of a .

Module 3-3 Multiplying Vectors

•33 For the vectors in Fig. 3-32, with $a = 4$, $b = 3$, and $c = 5$, what are (a) the magnitude and (b) the direction of $\vec{a} \times \vec{b}$, (c) the magnitude and (d) the direction of $\vec{a} \times \vec{c}$, and (e) the magnitude and (f) the direction of $\vec{b} \times \vec{c}$? (The z axis is not shown.)

•34 Two vectors are presented as $\vec{a} = 3.0\hat{i} + 5.0\hat{j}$ and $\vec{b} = 2.0\hat{i} + 4.0\hat{j}$. Find (a) $\vec{a} \times \vec{b}$, (b) $\vec{a} \cdot \vec{b}$, (c) $(\vec{a} + \vec{b}) \cdot \vec{b}$, and (d) the component of \vec{a} along the direction of \vec{b} . (Hint: For (d), consider Eq. 3-20 and Fig. 3-18.)

•35 Two vectors, \vec{r} and \vec{s} , lie in the xy plane. Their magnitudes are 4.50 and 7.30 units, respectively, and their directions are 320° and 85.0° , respectively, as measured counterclockwise from the positive x axis. What are the values of (a) $\vec{r} \cdot \vec{s}$ and (b) $\vec{r} \times \vec{s}$?

•36 If $\vec{d}_1 = 3\hat{i} - 2\hat{j} + 4\hat{k}$ and $\vec{d}_2 = -5\hat{i} + 2\hat{j} - \hat{k}$, then what is $(\vec{d}_1 + \vec{d}_2) \cdot (\vec{d}_1 \times 4\vec{d}_2)$?

•37 Three vectors are given by $\vec{a} = 3.0\hat{i} + 3.0\hat{j} - 2.0\hat{k}$, $\vec{b} = -1.0\hat{i} - 4.0\hat{j} + 2.0\hat{k}$, and $\vec{c} = 2.0\hat{i} + 2.0\hat{j} + 1.0\hat{k}$. Find (a) $\vec{a} \cdot (\vec{b} \times \vec{c})$, (b) $\vec{a} \cdot (\vec{b} + \vec{c})$, and (c) $\vec{a} \times (\vec{b} + \vec{c})$.

•38 GO For the following three vectors, what is $3\vec{C} \cdot (2\vec{A} \times \vec{B})$?

$$\vec{A} = 2.00\hat{i} + 3.00\hat{j} - 4.00\hat{k}$$

$$\vec{B} = -3.00\hat{i} + 4.00\hat{j} + 2.00\hat{k} \quad \vec{C} = 7.00\hat{i} - 8.00\hat{j}$$

•39 Vector \vec{A} has a magnitude of 6.00 units, vector \vec{B} has a magnitude of 7.00 units, and $\vec{A} \cdot \vec{B}$ has a value of 14.0. What is the angle between the directions of \vec{A} and \vec{B} ?

•40 GO Displacement \vec{d}_1 is in the yz plane 63.0° from the positive direction of the y axis, has a positive z component, and has a magnitude of 4.50 m. Displacement \vec{d}_2 is in the xz plane 30.0° from the positive direction of the x axis, has a positive z component, and has a magnitude 1.40 m. What are (a) $\vec{d}_1 \cdot \vec{d}_2$, (b) $\vec{d}_1 \times \vec{d}_2$, and (c) the angle between \vec{d}_1 and \vec{d}_2 ?

•41 SSM ILW WWW Use the definition of scalar product, $\vec{a} \cdot \vec{b} = ab \cos \theta$, and the fact that $\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z$ to calculate the angle between the two vectors given by $\vec{a} = 3.0\hat{i} + 3.0\hat{j} + 3.0\hat{k}$ and $\vec{b} = 2.0\hat{i} + 1.0\hat{j} + 3.0\hat{k}$.

•42 In a meeting of mimes, mime 1 goes through a displacement $\vec{d}_1 = (4.0 \text{ m})\hat{i} + (5.0 \text{ m})\hat{j}$ and mime 2 goes through a displacement $\vec{d}_2 = (-3.0 \text{ m})\hat{i} + (4.0 \text{ m})\hat{j}$. What are (a) $\vec{d}_1 \times \vec{d}_2$, (b) $\vec{d}_1 \cdot \vec{d}_2$, (c) $(\vec{d}_1 + \vec{d}_2) \cdot \vec{d}_2$, and (d) the component of \vec{d}_1 along the direction of \vec{d}_2 ? (Hint: For (d), see Eq. 3-20 and Fig. 3-18.)

•43 SSM ILW The three vectors in Fig. 3-33 have magnitudes $a = 3.00 \text{ m}$, $b = 4.00 \text{ m}$, and $c = 10.0 \text{ m}$ and angle $\theta = 30.0^\circ$. What are (a) the x component and (b) the y component of \vec{a} ; (c) the x component and (d) the y component of \vec{b} ; and (e) the x component and (f) the y component of \vec{c} ? If $\vec{c} = p\vec{a} + q\vec{b}$, what are the values of (g) p and (h) q ?

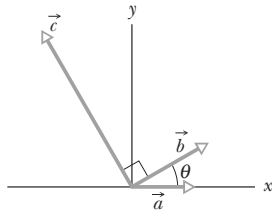


Figure 3-33 Problem 43.

ponent of \vec{b} ; and (e) the x component and (f) the y component of \vec{c} ? If $\vec{c} = p\vec{a} + q\vec{b}$, what are the values of (g) p and (h) q ?

•44 GO In the product $\vec{F} = q\vec{v} \times \vec{B}$, take $q = 2$,

$$\vec{v} = 2.0\hat{i} + 4.0\hat{j} + 6.0\hat{k} \quad \text{and} \quad \vec{F} = 4.0\hat{i} - 20\hat{j} + 12\hat{k}.$$

What then is \vec{B} in unit-vector notation if $B_x = B_y$?

Additional Problems

45 Vectors \vec{A} and \vec{B} lie in an xy plane. \vec{A} has magnitude 8.00 and angle 130° ; \vec{B} has components $B_x = -7.72$ and $B_y = -9.20$. (a) What is $5\vec{A} \cdot \vec{B}$? What is $4\vec{A} \times 3\vec{B}$ in (b) unit-vector notation and (c) magnitude-angle notation with spherical coordinates (see Fig. 3-34)? (d) What is the angle between the directions of \vec{A} and $4\vec{A} \times 3\vec{B}$? (Hint: Think a bit before you resort to a calculation.) What is $\vec{A} + 3.00\hat{k}$ in (e) unit-vector notation and (f) magnitude-angle notation with spherical coordinates?

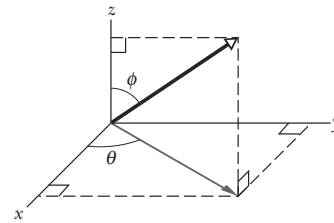


Figure 3-34 Problem 45.

46 GO Vector \vec{a} has a magnitude of 5.0 m and is directed east. Vector \vec{b} has a magnitude of 4.0 m and is directed 35° west of due north. What are (a) the magnitude and (b) the direction of $\vec{a} + \vec{b}$? What are (c) the magnitude and (d) the direction of $\vec{b} - \vec{a}$? (e) Draw a vector diagram for each combination.

47 Vectors \vec{A} and \vec{B} lie in an xy plane. \vec{A} has magnitude 8.00 and angle 130° ; \vec{B} has components $B_x = -7.72$ and $B_y = -9.20$. What are the angles between the negative direction of the y axis and (a) the direction of \vec{A} , (b) the direction of the product $\vec{A} \times \vec{B}$, and (c) the direction of $\vec{A} \times (\vec{B} + 3.00\hat{k})$?

48 GO Two vectors \vec{a} and \vec{b} have the components, in meters, $a_x = 3.2$, $a_y = 1.6$, $b_x = 0.50$, $b_y = 4.5$. (a) Find the angle between the directions of \vec{a} and \vec{b} . There are two vectors in the xy plane that are perpendicular to \vec{a} and have a magnitude of 5.0 m. One, vector \vec{c} , has a positive x component and the other, vector \vec{d} , a negative x component. What are (b) the x component and (c) the y component of vector \vec{c} , and (d) the x component and (e) the y component of vector \vec{d} ?

49 SSM A sailboat sets out from the U.S. side of Lake Erie for a point on the Canadian side, 90.0 km due north. The sailor, however, ends up 50.0 km due east of the starting point. (a) How far and (b) in what direction must the sailor now sail to reach the original destination?

50 Vector \vec{d}_1 is in the negative direction of a y axis, and vector \vec{d}_2 is in the positive direction of an x axis. What are the directions of (a) $\vec{d}_2/4$ and (b) $\vec{d}_1/(-4)$? What are the magnitudes of products (c) $\vec{d}_1 \cdot \vec{d}_2$ and (d) $\vec{d}_1 \cdot (\vec{d}_2/4)$? What is the direction of the vector resulting from (e) $\vec{d}_1 \times \vec{d}_2$ and (f) $\vec{d}_2 \times \vec{d}_1$? What is the magnitude of the vector product in (g) part (e) and (h) part (f)? What are the (i) magnitude and (j) direction of $\vec{d}_1 \times (\vec{d}_2/4)$?

51 Rock *faults* are ruptures along which opposite faces of rock have slid past each other. In Fig. 3-35, points A and B coincided before the rock in the foreground slid down to the right. The net displacement \vec{AB} is along the plane of the fault. The horizontal component of \vec{AB} is the *strike-slip* \vec{AC} . The component of \vec{AB} that is directed down the plane of the fault is the *dip-slip* \vec{AD} . (a) What is the magnitude of the net displacement \vec{AB} if the strike-slip is 22.0 m and the dip-slip is 17.0 m? (b) If the plane of the fault is inclined at angle $\phi = 52.0^\circ$ to the horizontal, what is the vertical component of \vec{AB} ?

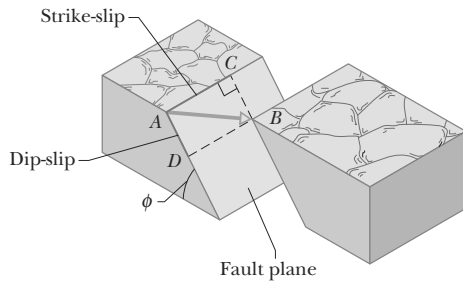


Figure 3-35 Problem 51.

52 Here are three displacements, each measured in meters: $\vec{d}_1 = 4.0\hat{i} + 5.0\hat{j} - 6.0\hat{k}$, $\vec{d}_2 = -1.0\hat{i} + 2.0\hat{j} + 3.0\hat{k}$, and $\vec{d}_3 = 4.0\hat{i} + 3.0\hat{j} + 2.0\hat{k}$. (a) What is $\vec{r} = \vec{d}_1 - \vec{d}_2 + \vec{d}_3$? (b) What is the angle between \vec{r} and the positive z axis? (c) What is the component of \vec{d}_1 along the direction of \vec{d}_2 ? (d) What is the component of \vec{d}_1 that is perpendicular to the direction of \vec{d}_2 and in the plane of \vec{d}_1 and \vec{d}_2 ? (Hint: For (c), consider Eq. 3-20 and Fig. 3-18; for (d), consider Eq. 3-24.)

53 SSM A vector \vec{a} of magnitude 10 units and another vector \vec{b} of magnitude 6.0 units differ in directions by 60° . Find (a) the scalar product of the two vectors and (b) the magnitude of the vector product $\vec{a} \times \vec{b}$.

54 For the vectors in Fig. 3-32, with $a = 4$, $b = 3$, and $c = 5$, calculate (a) $\vec{a} \cdot \vec{b}$, (b) $\vec{a} \cdot \vec{c}$, and (c) $\vec{b} \cdot \vec{c}$.

55 A particle undergoes three successive displacements in a plane, as follows: \vec{d}_1 , 4.00 m southwest; then \vec{d}_2 , 5.00 m east; and finally \vec{d}_3 , 6.00 m in a direction 60.0° north of east. Choose a coordinate system with the y axis pointing north and the x axis pointing east. What are (a) the x component and (b) the y component of \vec{d}_1 ? What are (c) the x component and (d) the y component of \vec{d}_2 ? What are (e) the x component and (f) the y component of \vec{d}_3 ? Next, consider the *net* displacement of the particle for the three successive displacements. What are (g) the x component, (h) the y component, (i) the magnitude, and (j) the direction of the net displacement? If the particle is to return directly to the starting point, (k) how far and (l) in what direction should it move?

56 Find the sum of the following four vectors in (a) unit-vector notation, and as (b) a magnitude and (c) an angle relative to $+x$.

\vec{P} : 10.0 m, at 25.0° counterclockwise from $+x$

\vec{Q} : 12.0 m, at 10.0° counterclockwise from $+y$

\vec{R} : 8.00 m, at 20.0° clockwise from $-y$

\vec{S} : 9.00 m, at 40.0° counterclockwise from $-y$

57 SSM If \vec{B} is added to \vec{A} , the result is $6.0\hat{i} + 1.0\hat{j}$. If \vec{B} is subtracted from \vec{A} , the result is $-4.0\hat{i} + 7.0\hat{j}$. What is the magnitude of \vec{A} ?

58 A vector \vec{d} has a magnitude of 2.5 m and points north. What are (a) the magnitude and (b) the direction of $4.0\vec{d}$? What are (c) the magnitude and (d) the direction of $-3.0\vec{d}$?

59 \vec{A} has the magnitude 12.0 m and is angled 60.0° counterclockwise from the positive direction of the x axis of an xy coordinate system. Also, $\vec{B} = (12.0\text{ m})\hat{i} + (8.00\text{ m})\hat{j}$ on that same coordinate system. We now rotate the system counterclockwise about the origin by 20.0° to form an $x'y'$ system. On this new system, what are (a) \vec{A} and (b) \vec{B} , both in unit-vector notation?

60 If $\vec{a} - \vec{b} = 2\vec{c}$, $\vec{a} + \vec{b} = 4\vec{c}$, and $\vec{c} = 3\hat{i} + 4\hat{j}$, then what are (a) \vec{a} and (b) \vec{b} ?

61 (a) In unit-vector notation, what is $\vec{r} = \vec{a} - \vec{b} + \vec{c}$ if $\vec{a} = 5.0\hat{i} + 4.0\hat{j} - 6.0\hat{k}$, $\vec{b} = -2.0\hat{i} + 2.0\hat{j} + 3.0\hat{k}$, and $\vec{c} = 4.0\hat{i} + 3.0\hat{j} + 2.0\hat{k}$? (b) Calculate the angle between \vec{r} and the positive z axis. (c) What is the component of \vec{a} along the direction of \vec{b} ? (d) What is the component of \vec{a} perpendicular to the direction of \vec{b} but in the plane of \vec{a} and \vec{b} ? (Hint: For (c), see Eq. 3-20 and Fig. 3-18; for (d), see Eq. 3-24.)

62 A golfer takes three putts to get the ball into the hole. The first putt displaces the ball 3.66 m north, the second 1.83 m southeast, and the third 0.91 m southwest. What are (a) the magnitude and (b) the direction of the displacement needed to get the ball into the hole on the first putt?

63 Here are three vectors in meters:

$$\vec{d}_1 = -3.0\hat{i} + 3.0\hat{j} + 2.0\hat{k}$$

$$\vec{d}_2 = -2.0\hat{i} - 4.0\hat{j} + 2.0\hat{k}$$

$$\vec{d}_3 = 2.0\hat{i} + 3.0\hat{j} + 1.0\hat{k}$$

What results from (a) $\vec{d}_1 \cdot (\vec{d}_2 + \vec{d}_3)$, (b) $\vec{d}_1 \cdot (\vec{d}_2 \times \vec{d}_3)$, and (c) $\vec{d}_1 \times (\vec{d}_2 + \vec{d}_3)$?

64 SSM WWW A room has dimensions 3.00 m (height) \times 3.70 m \times 4.30 m. A fly starting at one corner flies around, ending up at the diagonally opposite corner. (a) What is the magnitude of its displacement? (b) Could the length of its path be less than this magnitude? (c) Greater? (d) Equal? (e) Choose a suitable coordinate system and express the components of the displacement vector in that system in unit-vector notation. (f) If the fly walks, what is the length of the shortest path? (Hint: This can be answered without calculus. The room is like a box. Unfold its walls to flatten them into a plane.)

65 A protester carries his sign of protest, starting from the origin of an xyz coordinate system, with the xy plane horizontal. He moves 40 m in the negative direction of the x axis, then 20 m along a perpendicular path to his left, and then 25 m up a water tower. (a) In unit-vector notation, what is the displacement of the sign from start to end? (b) The sign then falls to the foot of the tower. What is the magnitude of the displacement of the sign from start to this new end?

66 Consider \vec{a} in the positive direction of x , \vec{b} in the positive direction of y , and a scalar d . What is the direction of \vec{b}/d if d is (a) positive and (b) negative? What is the magnitude of (c) $\vec{a} \cdot \vec{b}$ and (d) $\vec{a} \cdot \vec{b}/d$? What is the direction of the vector resulting from (e) $\vec{a} \times \vec{b}$ and (f) $\vec{b} \times \vec{a}$? (g) What is the magnitude of the vector product in (e)? (h) What is the magnitude of the vector product in (f)? What are (i) the magnitude and (j) the direction of $\vec{a} \times \vec{b}/d$ if d is positive?

67 Let \hat{i} be directed to the east, \hat{j} be directed to the north, and \hat{k} be directed upward. What are the values of products (a) $\hat{i} \cdot \hat{k}$, (b) $(-\hat{k}) \cdot (-\hat{j})$, and (c) $\hat{j} \cdot (-\hat{j})$? What are the directions (such as east or down) of products (d) $\hat{k} \times \hat{j}$, (e) $(-\hat{i}) \times (-\hat{j})$, and (f) $(-\hat{k}) \times (-\hat{j})$?

68 A bank in downtown Boston is robbed (see the map in Fig. 3-36). To elude police, the robbers escape by helicopter, making three successive flights described by the following displacements: 32 km, 45° south of east; 53 km, 26° north of west; 26 km, 18° east of south. At the end of the third flight they are captured. In what town are they apprehended?

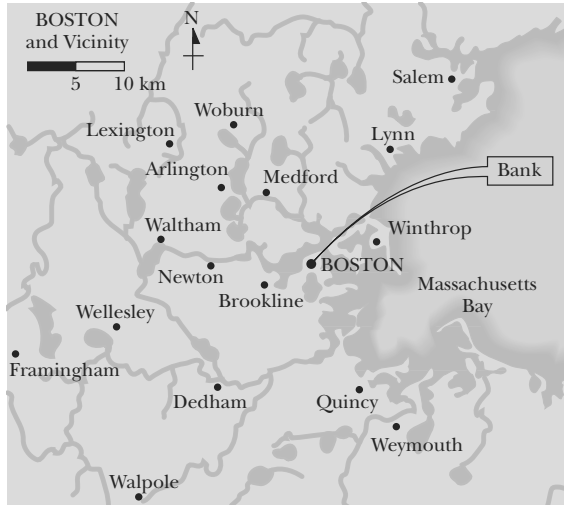


Figure 3-36 Problem 68.

69 A wheel with a radius of 45.0 cm rolls without slipping along a horizontal floor (Fig. 3-37). At time t_1 , the dot P painted on the rim of the wheel is at the point of contact between the wheel and the floor. At a later time t_2 , the wheel has rolled through one-half of a revolution. What are (a) the magnitude and (b) the angle (relative to the floor) of the displacement of P ?

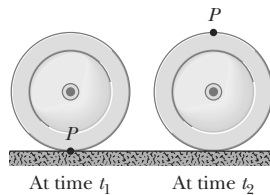


Figure 3-37 Problem 69.

70 A woman walks 250 m in the direction 30° east of north, then 175 m directly east. Find (a) the magnitude and (b) the angle of her final displacement from the starting point. (c) Find the distance she walks. (d) Which is greater, that distance or the magnitude of her displacement?

71 A vector \vec{a} has a magnitude 3.0 m and is directed south. What are (a) the magnitude and (b) the direction of the vector $5.0\vec{a}$? What are (c) the magnitude and (d) the direction of the vector $-2.0\vec{a}$?

72 A fire ant, searching for hot sauce in a picnic area, goes through three displacements along level ground: \vec{d}_1 for 0.40 m southwest (that is, at 45° from directly south and from directly west), \vec{d}_2 for 0.50 m due east, \vec{d}_3 for 0.60 m at 60° north of east. Let the positive x direction be east and the positive y direction be north. What are (a) the x component and (b) the y component of \vec{d}_1 ? Next, what are (c) the x component and (d) the y component of \vec{d}_2 ? Also, what are (e) the x component and (f) the y component of \vec{d}_3 ?

What are (g) the x component, (h) the y component, (i) the magnitude, and (j) the direction of the ant's net displacement? If the ant is to return directly to the starting point, (k) how far and (l) in what direction should it move?

73 Two vectors are given by $\vec{a} = 3.0\hat{i} + 5.0\hat{j}$ and $\vec{b} = 2.0\hat{i} + 4.0\hat{j}$. Find (a) $\vec{a} \times \vec{b}$, (b) $\vec{a} \cdot \vec{b}$, (c) $(\vec{a} + \vec{b}) \cdot \vec{b}$, and (d) the component of \vec{a} along the direction of \vec{b} .

74 Vector \vec{a} lies in the yz plane 63.0° from the positive direction of the y axis, has a positive z component, and has magnitude 3.20 units. Vector \vec{b} lies in the xz plane 48.0° from the positive direction of the x axis, has a positive z component, and has magnitude 1.40 units. Find (a) $\vec{a} \cdot \vec{b}$, (b) $\vec{a} \times \vec{b}$, and (c) the angle between \vec{a} and \vec{b} .

75 Find (a) "north cross west," (b) "down dot south," (c) "east cross up," (d) "west dot west," and (e) "south cross south." Let each "vector" have unit magnitude.

76 A vector \vec{B} , with a magnitude of 8.0 m, is added to a vector \vec{A} , which lies along an x axis. The sum of these two vectors is a third vector that lies along the y axis and has a magnitude that is twice the magnitude of \vec{A} . What is the magnitude of \vec{A} ?

77 A man goes for a walk, starting from the origin of an xyz coordinate system, with the xy plane horizontal and the x axis eastward. Carrying a bad penny, he walks 1300 m east, 2200 m north, and then drops the penny from a cliff 410 m high. (a) In unit-vector notation, what is the displacement of the penny from start to its landing point? (b) When the man returns to the origin, what is the magnitude of his displacement for the return trip?

78 What is the magnitude of $\vec{a} \times (\vec{b} \times \vec{a})$ if $a = 3.90$, $b = 2.70$, and the angle between the two vectors is 63.0° ?

79 In Fig. 3-38, the magnitude of \vec{a} is 4.3, the magnitude of \vec{b} is 5.4, and $\phi = 46^\circ$. Find the area of the triangle contained between the two vectors and the thin diagonal line.

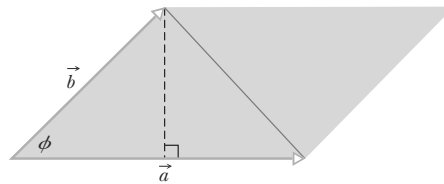


Figure 3-38 Problem 79.

C H A P T E R 4

Motion in Two and Three Dimensions

4-1 POSITION AND DISPLACEMENT

Learning Objectives

After reading this module, you should be able to . . .

4.01 Draw two-dimensional and three-dimensional position vectors for a particle, indicating the components along the axes of a coordinate system.

4.02 On a coordinate system, determine the direction and

magnitude of a particle's position vector from its components, and vice versa.

4.03 Apply the relationship between a particle's displacement vector and its initial and final position vectors.

Key Ideas

● The location of a particle relative to the origin of a coordinate system is given by a position vector \vec{r} , which in unit-vector notation is

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}.$$

Here $x\hat{i}$, $y\hat{j}$, and $z\hat{k}$ are the vector components of position vector \vec{r} , and x , y , and z are its scalar components (as well as the coordinates of the particle).

● A position vector is described either by a magnitude and

one or two angles for orientation, or by its vector or scalar components.


● If a particle moves so that its position vector changes from \vec{r}_1 to \vec{r}_2 , the particle's displacement $\Delta\vec{r}$ is

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1.$$

The displacement can also be written as

$$\begin{aligned}\Delta\vec{r} &= (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k} \\ &= \Delta x\hat{i} + \Delta y\hat{j} + \Delta z\hat{k}.\end{aligned}$$

What Is Physics?

In this chapter we continue looking at the aspect of physics that analyzes motion, but now the motion can be in two or three dimensions. For example, medical researchers and aeronautical engineers might concentrate on the physics of the two- and three-dimensional turns taken by fighter pilots in dog-fights because a modern high-performance jet can take a tight turn so quickly that the pilot immediately loses consciousness. A sports engineer might focus on the physics of basketball. For example, in a *free throw* (where a player gets an uncontested shot at the basket from about 4.3 m), a player might employ the *overhand push shot*, in which the ball is pushed away from about shoulder height and then released. Or the player might use an *underhand loop shot*, in which the ball is brought upward from about the belt-line level and released. The first technique is the overwhelming choice among professional players, but the legendary Rick Barry set the record for free-throw shooting with the underhand technique. 

Motion in three dimensions is not easy to understand. For example, you are probably good at driving a car along a freeway (one-dimensional motion) but would probably have a difficult time in landing an airplane on a runway (three-dimensional motion) without a lot of training.

In our study of two- and three-dimensional motion, we start with position and displacement.

Position and Displacement

One general way of locating a particle (or particle-like object) is with a **position vector** \vec{r} , which is a vector that extends from a reference point (usually the origin) to the particle. In the unit-vector notation of Module 3-2, \vec{r} can be written

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}, \quad (4-1)$$

where $x\hat{i}$, $y\hat{j}$, and $z\hat{k}$ are the vector components of \vec{r} and the coefficients x , y , and z are its scalar components.

The coefficients x , y , and z give the particle's location along the coordinate axes and relative to the origin; that is, the particle has the rectangular coordinates (x, y, z) . For instance, Fig. 4-1 shows a particle with position vector

$$\vec{r} = (-3\text{ m})\hat{i} + (2\text{ m})\hat{j} + (5\text{ m})\hat{k}$$

and rectangular coordinates $(-3\text{ m}, 2\text{ m}, 5\text{ m})$. Along the x axis the particle is 3 m from the origin, in the $-\hat{i}$ direction. Along the y axis it is 2 m from the origin, in the $+\hat{j}$ direction. Along the z axis it is 5 m from the origin, in the $+\hat{k}$ direction.

As a particle moves, its position vector changes in such a way that the vector always extends to the particle from the reference point (the origin). If the position vector changes—say, from \vec{r}_1 to \vec{r}_2 during a certain time interval—then the particle's **displacement** $\Delta\vec{r}$ during that time interval is

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1. \quad (4-2)$$

Using the unit-vector notation of Eq. 4-1, we can rewrite this displacement as

$$\Delta\vec{r} = (x_2\hat{i} + y_2\hat{j} + z_2\hat{k}) - (x_1\hat{i} + y_1\hat{j} + z_1\hat{k})$$

or as
$$\Delta\vec{r} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}, \quad (4-3)$$

where coordinates (x_1, y_1, z_1) correspond to position vector \vec{r}_1 and coordinates (x_2, y_2, z_2) correspond to position vector \vec{r}_2 . We can also rewrite the displacement by substituting Δx for $(x_2 - x_1)$, Δy for $(y_2 - y_1)$, and Δz for $(z_2 - z_1)$:

$$\Delta\vec{r} = \Delta x\hat{i} + \Delta y\hat{j} + \Delta z\hat{k}. \quad (4-4)$$

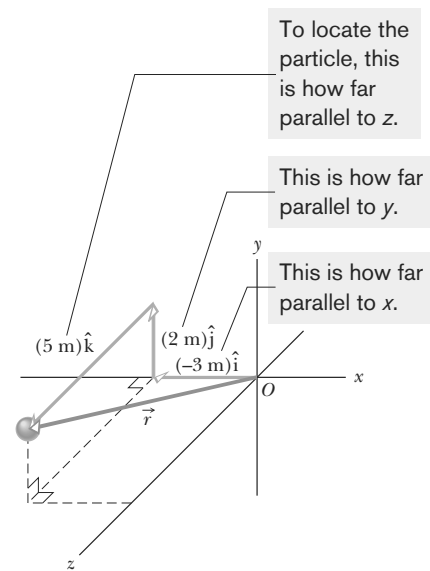


Figure 4-1 The position vector \vec{r} for a particle is the vector sum of its vector components.

Sample Problem 4.01 Two-dimensional position vector, rabbit run

A rabbit runs across a parking lot on which a set of coordinate axes has, strangely enough, been drawn. The coordinates (meters) of the rabbit's position as functions of time t (seconds) are given by

$$x = -0.31t^2 + 7.2t + 28 \quad (4-5)$$

and
$$y = 0.22t^2 - 9.1t + 30. \quad (4-6)$$

(a) At $t = 15$ s, what is the rabbit's position vector \vec{r} in unit-vector notation and in magnitude-angle notation?

KEY IDEA

The x and y coordinates of the rabbit's position, as given by Eqs. 4-5 and 4-6, are the scalar components of the rabbit's

position vector \vec{r} . Let's evaluate those coordinates at the given time, and then we can use Eq. 3-6 to evaluate the magnitude and orientation of the position vector.

Calculations: We can write

$$\vec{r}(t) = x(t)\hat{i} + y(t)\hat{j}. \quad (4-7)$$

(We write $\vec{r}(t)$ rather than \vec{r} because the components are functions of t , and thus \vec{r} is also.)

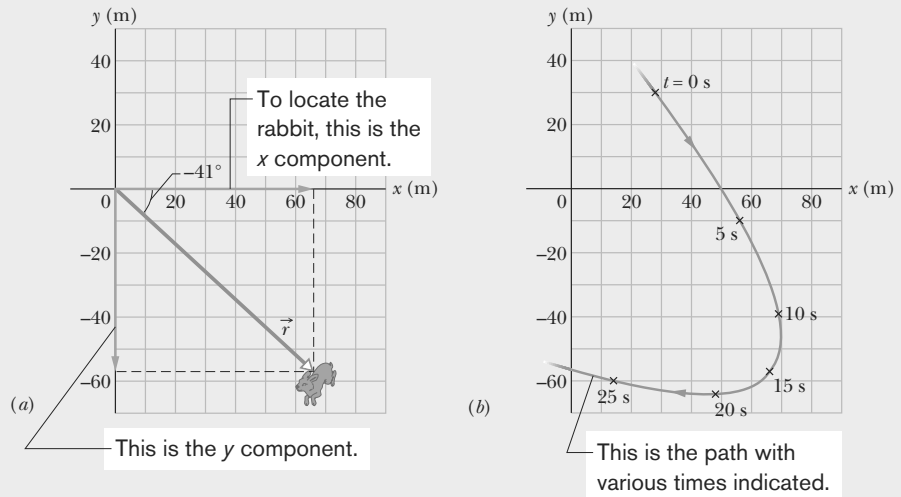
At $t = 15$ s, the scalar components are

$$x = (-0.31)(15)^2 + (7.2)(15) + 28 = 66\text{ m}$$

and
$$y = (0.22)(15)^2 - (9.1)(15) + 30 = -57\text{ m},$$

so
$$\vec{r} = (66\text{ m})\hat{i} - (57\text{ m})\hat{j}, \quad (\text{Answer})$$

Figure 4-2 (a) A rabbit's position vector \vec{r} at time $t = 15$ s. The scalar components of \vec{r} are shown along the axes. (b) The rabbit's path and its position at six values of t .



which is drawn in Fig. 4-2a. To get the magnitude and angle of \vec{r} , notice that the components form the legs of a right triangle and r is the hypotenuse. So, we use Eq. 3-6:

$$r = \sqrt{x^2 + y^2} = \sqrt{(66 \text{ m})^2 + (-57 \text{ m})^2} = 87 \text{ m}, \quad (\text{Answer})$$

$$\text{and} \quad \theta = \tan^{-1} \frac{y}{x} = \tan^{-1} \left(\frac{-57 \text{ m}}{66 \text{ m}} \right) = -41^\circ. \quad (\text{Answer})$$

Check: Although $\theta = 139^\circ$ has the same tangent as -41° , the components of position vector \vec{r} indicate that the desired angle is $139^\circ - 180^\circ = -41^\circ$.

(b) Graph the rabbit's path for $t = 0$ to $t = 25$ s.

Graphing: We have located the rabbit at one instant, but to see its path we need a graph. So we repeat part (a) for several values of t and then plot the results. Figure 4-2b shows the plots for six values of t and the path connecting them.



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4-2 AVERAGE VELOCITY AND INSTANTANEOUS VELOCITY

Learning Objectives

After reading this module, you should be able to . . .

- 4.04** Identify that velocity is a vector quantity and thus has both magnitude and direction and also has components.
- 4.05** Draw two-dimensional and three-dimensional velocity vectors for a particle, indicating the components along the axes of the coordinate system.

- 4.06** In magnitude-angle and unit-vector notations, relate a particle's initial and final position vectors, the time interval between those positions, and the particle's average velocity vector.
- 4.07** Given a particle's position vector as a function of time, determine its (instantaneous) velocity vector.

Key Ideas

- If a particle undergoes a displacement $\Delta \vec{r}$ in time interval Δt , its average velocity \vec{v}_{avg} for that time interval is

$$\vec{v}_{\text{avg}} = \frac{\Delta \vec{r}}{\Delta t}.$$

- As Δt is shrunk to 0, \vec{v}_{avg} reaches a limit called either the velocity or the instantaneous velocity \vec{v} :

$$\vec{v} = \frac{d\vec{r}}{dt},$$

which can be rewritten in unit-vector notation as

$$\vec{v} = v_x \hat{i} + v_y \hat{j} + v_z \hat{k},$$

where $v_x = dx/dt$, $v_y = dy/dt$, and $v_z = dz/dt$.

- The instantaneous velocity \vec{v} of a particle is always directed along the tangent to the particle's path at the particle's position.

Average Velocity and Instantaneous Velocity

If a particle moves from one point to another, we might need to know how fast it moves. Just as in Chapter 2, we can define two quantities that deal with “how fast”: *average velocity* and *instantaneous velocity*. However, here we must consider these quantities as vectors and use vector notation.

If a particle moves through a displacement $\Delta \vec{r}$ in a time interval Δt , then its **average velocity** \vec{v}_{avg} is

$$\text{average velocity} = \frac{\text{displacement}}{\text{time interval}},$$

$$\text{or} \quad \vec{v}_{\text{avg}} = \frac{\Delta \vec{r}}{\Delta t}. \quad (4-8)$$

This tells us that the direction of \vec{v}_{avg} (the vector on the left side of Eq. 4-8) must be the same as that of the displacement $\Delta \vec{r}$ (the vector on the right side). Using Eq. 4-4, we can write Eq. 4-8 in vector components as

$$\vec{v}_{\text{avg}} = \frac{\Delta x \hat{i} + \Delta y \hat{j} + \Delta z \hat{k}}{\Delta t} = \frac{\Delta x}{\Delta t} \hat{i} + \frac{\Delta y}{\Delta t} \hat{j} + \frac{\Delta z}{\Delta t} \hat{k}. \quad (4-9)$$

For example, if a particle moves through displacement $(12 \text{ m})\hat{i} + (3.0 \text{ m})\hat{k}$ in 2.0 s, then its average velocity during that move is

$$\vec{v}_{\text{avg}} = \frac{\Delta \vec{r}}{\Delta t} = \frac{(12 \text{ m})\hat{i} + (3.0 \text{ m})\hat{k}}{2.0 \text{ s}} = (6.0 \text{ m/s})\hat{i} + (1.5 \text{ m/s})\hat{k}.$$

That is, the average velocity (a vector quantity) has a component of 6.0 m/s along the x axis and a component of 1.5 m/s along the z axis.

When we speak of the **velocity** of a particle, we usually mean the particle's **instantaneous velocity** \vec{v} at some instant. This \vec{v} is the value that \vec{v}_{avg} approaches in the limit as we shrink the time interval Δt to 0 about that instant. Using the language of calculus, we may write \vec{v} as the derivative

$$\vec{v} = \frac{d\vec{r}}{dt}. \quad (4-10)$$

Figure 4-3 shows the path of a particle that is restricted to the xy plane. As the particle travels to the right along the curve, its position vector sweeps to the right. During time interval Δt , the position vector changes from \vec{r}_1 to \vec{r}_2 and the particle's displacement is $\Delta \vec{r}$.

To find the instantaneous velocity of the particle at, say, instant t_1 (when the particle is at position 1), we shrink interval Δt to 0 about t_1 . Three things happen as we do so. (1) Position vector \vec{r}_2 in Fig. 4-3 moves toward \vec{r}_1 so that $\Delta \vec{r}$ shrinks

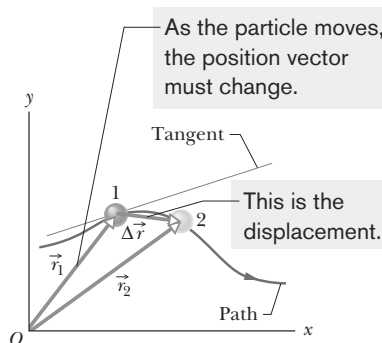


Figure 4-3 The displacement $\Delta \vec{r}$ of a particle during a time interval Δt , from position 1 with position vector \vec{r}_1 at time t_1 to position 2 with position vector \vec{r}_2 at time t_2 . The tangent to the particle's path at position 1 is shown.

toward zero. (2) The direction of $\Delta\vec{r}/\Delta t$ (and thus of \vec{v}_{avg}) approaches the direction of the line tangent to the particle's path at position 1. (3) The average velocity \vec{v}_{avg} approaches the instantaneous velocity \vec{v} at t_1 .

In the limit as $\Delta t \rightarrow 0$, we have $\vec{v}_{\text{avg}} \rightarrow \vec{v}$ and, most important here, \vec{v}_{avg} takes on the direction of the tangent line. Thus, \vec{v} has that direction as well:



The direction of the instantaneous velocity \vec{v} of a particle is always tangent to the particle's path at the particle's position.

The result is the same in three dimensions: \vec{v} is always tangent to the particle's path. To write Eq. 4-10 in unit-vector form, we substitute for \vec{r} from Eq. 4-1:

$$\vec{v} = \frac{d}{dt} (x\hat{i} + y\hat{j} + z\hat{k}) = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}.$$

This equation can be simplified somewhat by writing it as

$$\vec{v} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}, \quad (4-11)$$

where the scalar components of \vec{v} are

$$v_x = \frac{dx}{dt}, \quad v_y = \frac{dy}{dt}, \quad \text{and} \quad v_z = \frac{dz}{dt}. \quad (4-12)$$

For example, dx/dt is the scalar component of \vec{v} along the x axis. Thus, we can find the scalar components of \vec{v} by differentiating the scalar components of \vec{r} .

Figure 4-4 shows a velocity vector \vec{v} and its scalar x and y components. Note that \vec{v} is tangent to the particle's path at the particle's position. **Caution:** When a position vector is drawn, as in Figs. 4-1 through 4-3, it is an arrow that extends from one point (a "here") to another point (a "there"). However, when a velocity vector is drawn, as in Fig. 4-4, it does *not* extend from one point to another. Rather, it shows the instantaneous direction of travel of a particle at the tail, and its length (representing the velocity magnitude) can be drawn to any scale.

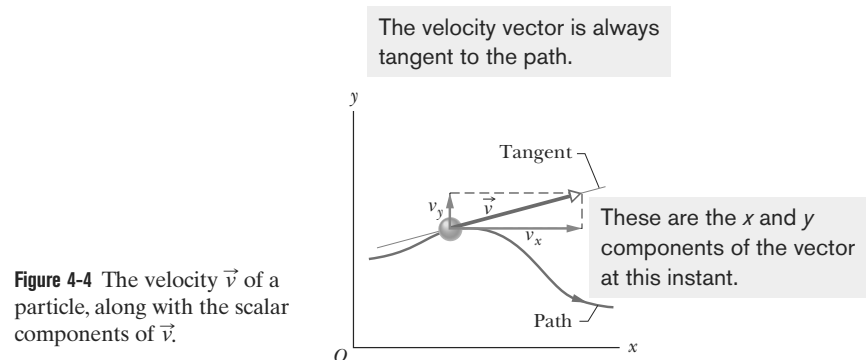
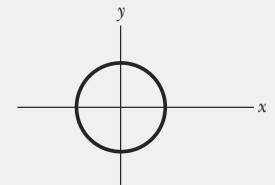


Figure 4-4 The velocity \vec{v} of a particle, along with the scalar components of \vec{v} .



Checkpoint 1

The figure shows a circular path taken by a particle. If the instantaneous velocity of the particle is $\vec{v} = (2 \text{ m/s})\hat{i} - (2 \text{ m/s})\hat{j}$, through which quadrant is the particle moving at that instant if it is traveling (a) clockwise and (b) counterclockwise around the circle? For both cases, draw \vec{v} on the figure.





Sample Problem 4.02 Two-dimensional velocity, rabbit run

For the rabbit in the preceding sample problem, find the velocity \vec{v} at time $t = 15$ s.

KEY IDEA

We can find \vec{v} by taking derivatives of the components of the rabbit's position vector.

Calculations: Applying the v_x part of Eq. 4-12 to Eq. 4-5, we find the x component of \vec{v} to be

$$\begin{aligned} v_x &= \frac{dx}{dt} = \frac{d}{dt}(-0.31t^2 + 7.2t + 28) \\ &= -0.62t + 7.2. \end{aligned} \quad (4-13)$$

At $t = 15$ s, this gives $v_x = -2.1$ m/s. Similarly, applying the v_y part of Eq. 4-12 to Eq. 4-6, we find

$$\begin{aligned} v_y &= \frac{dy}{dt} = \frac{d}{dt}(0.22t^2 - 9.1t + 30) \\ &= 0.44t - 9.1. \end{aligned} \quad (4-14)$$

At $t = 15$ s, this gives $v_y = -2.5$ m/s. Equation 4-11 then yields

$$\vec{v} = (-2.1 \text{ m/s})\hat{i} + (-2.5 \text{ m/s})\hat{j}, \quad (\text{Answer})$$

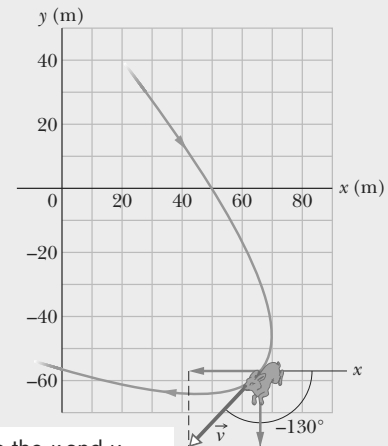
which is shown in Fig. 4-5, tangent to the rabbit's path and in the direction the rabbit is running at $t = 15$ s.

To get the magnitude and angle of \vec{v} , either we use a vector-capable calculator or we follow Eq. 3-6 to write

$$\begin{aligned} v &= \sqrt{v_x^2 + v_y^2} = \sqrt{(-2.1 \text{ m/s})^2 + (-2.5 \text{ m/s})^2} \\ &= 3.3 \text{ m/s} \end{aligned} \quad (\text{Answer})$$

$$\begin{aligned} \text{and } \theta &= \tan^{-1} \frac{v_y}{v_x} = \tan^{-1} \left(\frac{-2.5 \text{ m/s}}{-2.1 \text{ m/s}} \right) \\ &= \tan^{-1} 1.19 = -130^\circ. \end{aligned} \quad (\text{Answer})$$

Check: Is the angle -130° or $-130^\circ + 180^\circ = 50^\circ$?



These are the x and y components of the vector at this instant.

Figure 4-5 The rabbit's velocity \vec{v} at $t = 15$ s.

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4-3 AVERAGE ACCELERATION AND INSTANTANEOUS ACCELERATION

Learning Objectives

After reading this module, you should be able to . . .

- 4.08** Identify that acceleration is a vector quantity and thus has both magnitude and direction and also has components.
- 4.09** Draw two-dimensional and three-dimensional acceleration vectors for a particle, indicating the components.
- 4.10** Given the initial and final velocity vectors of a particle and the time interval between those velocities, determine

the average acceleration vector in magnitude-angle and unit-vector notations.

- 4.11** Given a particle's velocity vector as a function of time, determine its (instantaneous) acceleration vector.
- 4.12** For each dimension of motion, apply the constant-acceleration equations (Chapter 2) to relate acceleration, velocity, position, and time.

Key Ideas

- If a particle's velocity changes from \vec{v}_1 to \vec{v}_2 in time interval Δt , its average acceleration during Δt is

$$\vec{a}_{\text{avg}} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t} = \frac{\Delta \vec{v}}{\Delta t}.$$

- As Δt is shrunk to 0, \vec{a}_{avg} reaches a limiting value called

either the acceleration or the instantaneous acceleration \vec{a} :

$$\vec{a} = \frac{d\vec{v}}{dt}.$$

- In unit-vector notation,

$$\vec{a} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k},$$

where $a_x = dv_x/dt$, $a_y = dv_y/dt$, and $a_z = dv_z/dt$.

Average Acceleration and Instantaneous Acceleration

When a particle's velocity changes from \vec{v}_1 to \vec{v}_2 in a time interval Δt , its **average acceleration** \vec{a}_{avg} during Δt is

$$\text{average acceleration} = \frac{\text{change in velocity}}{\text{time interval}},$$

$$\text{or} \quad \vec{a}_{\text{avg}} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t} = \frac{\Delta \vec{v}}{\Delta t}. \quad (4-15)$$

If we shrink Δt to zero about some instant, then in the limit \vec{a}_{avg} approaches the **instantaneous acceleration** (or **acceleration**) \vec{a} at that instant; that is,

$$\vec{a} = \frac{d\vec{v}}{dt}. \quad (4-16)$$

If the velocity changes in *either* magnitude *or* direction (or both), the particle must have an acceleration.

We can write Eq. 4-16 in unit-vector form by substituting Eq. 4-11 for \vec{v} to obtain

$$\begin{aligned} \vec{a} &= \frac{d}{dt} (v_x \hat{i} + v_y \hat{j} + v_z \hat{k}) \\ &= \frac{dv_x}{dt} \hat{i} + \frac{dv_y}{dt} \hat{j} + \frac{dv_z}{dt} \hat{k}. \end{aligned}$$

We can rewrite this as

$$\vec{a} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k}, \quad (4-17)$$

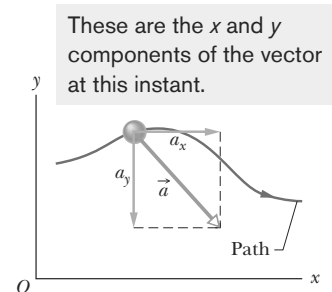
where the scalar components of \vec{a} are

$$a_x = \frac{dv_x}{dt}, \quad a_y = \frac{dv_y}{dt}, \quad \text{and} \quad a_z = \frac{dv_z}{dt}. \quad (4-18)$$

To find the scalar components of \vec{a} , we differentiate the scalar components of \vec{v} .

Figure 4-6 shows an acceleration vector \vec{a} and its scalar components for a particle moving in two dimensions. *Caution:* When an acceleration vector is drawn, as in Fig. 4-6, it does *not* extend from one position to another. Rather, it shows the direction of acceleration for a particle located at its tail, and its length (representing the acceleration magnitude) can be drawn to any scale.

Figure 4-6 The acceleration \vec{a} of a particle and the scalar components of \vec{a} .



**Checkpoint 2**

Here are four descriptions of the position (in meters) of a puck as it moves in an xy plane:

(1) $x = -3t^2 + 4t - 2$ and $y = 6t^2 - 4t$ (3) $\vec{r} = 2t^2\hat{i} - (4t + 3)\hat{j}$

(2) $x = -3t^3 - 4t$ and $y = -5t^2 + 6$ (4) $\vec{r} = (4t^3 - 2t)\hat{i} + 3\hat{j}$

Are the x and y acceleration components constant? Is acceleration \vec{a} constant?

Sample Problem 4.03 Two-dimensional acceleration, rabbit run

For the rabbit in the preceding two sample problems, find the acceleration \vec{a} at time $t = 15$ s.

KEY IDEA

We can find \vec{a} by taking derivatives of the rabbit's velocity components.

Calculations: Applying the a_x part of Eq. 4-18 to Eq. 4-13, we find the x component of \vec{a} to be

$$a_x = \frac{dv_x}{dt} = \frac{d}{dt}(-0.62t + 7.2) = -0.62 \text{ m/s}^2.$$

Similarly, applying the a_y part of Eq. 4-18 to Eq. 4-14 yields the y component as

$$a_y = \frac{dv_y}{dt} = \frac{d}{dt}(0.44t - 9.1) = 0.44 \text{ m/s}^2.$$

We see that the acceleration does not vary with time (it is a constant) because the time variable t does not appear in the expression for either acceleration component. Equation 4-17 then yields

$$\vec{a} = (-0.62 \text{ m/s}^2)\hat{i} + (0.44 \text{ m/s}^2)\hat{j}, \quad (\text{Answer})$$

which is superimposed on the rabbit's path in Fig. 4-7.

To get the magnitude and angle of \vec{a} , either we use a vector-capable calculator or we follow Eq. 3-6. For the magnitude we have

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(-0.62 \text{ m/s}^2)^2 + (0.44 \text{ m/s}^2)^2} = 0.76 \text{ m/s}^2. \quad (\text{Answer})$$

For the angle we have

$$\theta = \tan^{-1} \frac{a_y}{a_x} = \tan^{-1} \left(\frac{0.44 \text{ m/s}^2}{-0.62 \text{ m/s}^2} \right) = -35^\circ.$$

However, this angle, which is the one displayed on a calculator, indicates that \vec{a} is directed to the right and downward in Fig. 4-7. Yet, we know from the components that \vec{a} must be directed to the left and upward. To find the other angle that

has the same tangent as -35° but is not displayed on a calculator, we add 180° :

$$-35^\circ + 180^\circ = 145^\circ. \quad (\text{Answer})$$

This is consistent with the components of \vec{a} because it gives a vector that is to the left and upward. Note that \vec{a} has the same magnitude and direction throughout the rabbit's run because the acceleration is constant. That means that we could draw the very same vector at any other point along the rabbit's path (just shift the vector to put its tail at some other point on the path without changing the length or orientation).

This has been the second sample problem in which we needed to take the derivative of a vector that is written in unit-vector notation. One common error is to neglect the unit vectors themselves, with a result of only a set of numbers and symbols. Keep in mind that a derivative of a vector is always another vector.

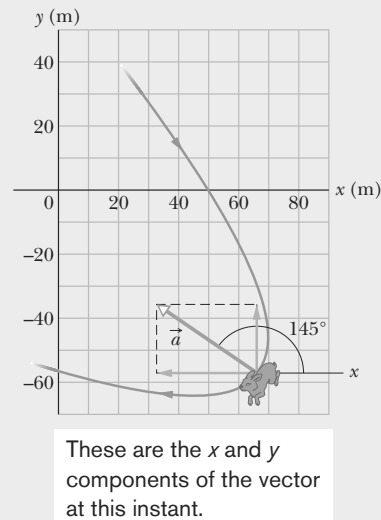


Figure 4-7 The acceleration \vec{a} of the rabbit at $t = 15$ s. The rabbit happens to have this same acceleration at all points on its path.



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4-4 PROJECTILE MOTION

Learning Objectives

After reading this module, you should be able to . . .

4.13 On a sketch of the path taken in projectile motion, explain the magnitudes and directions of the velocity and acceleration components during the flight.

4.14 Given the launch velocity in either magnitude-angle or unit-vector notation, calculate the particle's position, displacement, and velocity at a given instant during the flight.

4.15 Given data for an instant during the flight, calculate the launch velocity.

Key Ideas

- In projectile motion, a particle is launched into the air with a speed v_0 and at an angle θ_0 (as measured from a horizontal x axis). During flight, its horizontal acceleration is zero and its vertical acceleration is $-g$ (downward on a vertical y axis).

- The equations of motion for the particle (while in flight) can be written as

$$\begin{aligned}x - x_0 &= (v_0 \cos \theta_0)t, \\y - y_0 &= (v_0 \sin \theta_0)t - \frac{1}{2}gt^2, \\v_y &= v_0 \sin \theta_0 - gt, \\v_y^2 &= (v_0 \sin \theta_0)^2 - 2g(y - y_0).\end{aligned}$$

- The trajectory (path) of a particle in projectile motion is parabolic and is given by

$$y = (\tan \theta_0)x - \frac{gx^2}{2(v_0 \cos \theta_0)^2},$$

if x_0 and y_0 are zero.

- The particle's horizontal range R , which is the horizontal distance from the launch point to the point at which the particle returns to the launch height, is

$$R = \frac{v_0^2}{g} \sin 2\theta_0.$$

Projectile Motion

We next consider a special case of two-dimensional motion: A particle moves in a vertical plane with some initial velocity \vec{v}_0 but its acceleration is always the free-fall acceleration \vec{g} , which is downward. Such a particle is called a **projectile** (meaning that it is projected or launched), and its motion is called **projectile motion**. A projectile might be a tennis ball (Fig. 4-8) or baseball in flight, but it is not a duck in flight. Many sports involve the study of the projectile motion of a ball. For example, the racquetball player who discovered the Z-shot in the 1970s easily won his games because of the ball's perplexing flight to the rear of the court.

Our goal here is to analyze projectile motion using the tools for two-dimensional motion described in Module 4-1 through 4-3 and making the assumption that air has no effect on the projectile. Figure 4-9, which we shall analyze soon, shows the path followed by a projectile when the air has no effect. The projectile is launched with an initial velocity \vec{v}_0 that can be written as

$$\vec{v}_0 = v_{0x}\hat{i} + v_{0y}\hat{j}. \quad (4-19)$$

The components v_{0x} and v_{0y} can then be found if we know the angle θ_0 between \vec{v}_0 and the positive x direction:

$$v_{0x} = v_0 \cos \theta_0 \quad \text{and} \quad v_{0y} = v_0 \sin \theta_0. \quad (4-20)$$

During its two-dimensional motion, the projectile's position vector \vec{r} and velocity vector \vec{v} change continuously, but its acceleration vector \vec{a} is constant and *always* directed vertically downward. The projectile has *no* horizontal acceleration.

Projectile motion, like that in Figs. 4-8 and 4-9, looks complicated, but we have the following simplifying feature (known from experiment):



In projectile motion, the horizontal motion and the vertical motion are independent of each other; that is, neither motion affects the other.



Richard Megna/Fundamental Photographs

Figure 4-8 A stroboscopic photograph of a yellow tennis ball bouncing off a hard surface. Between impacts, the ball has projectile motion.

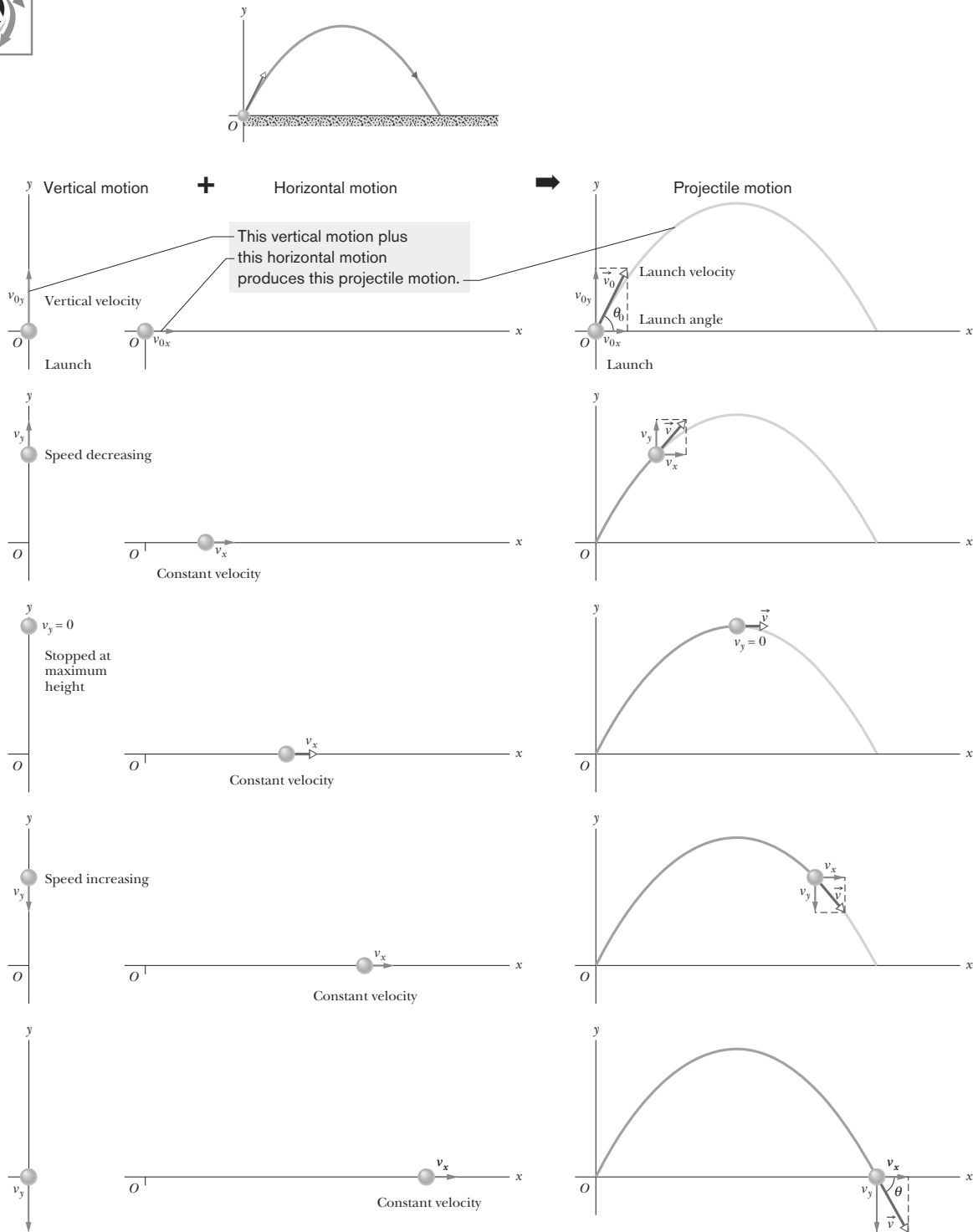
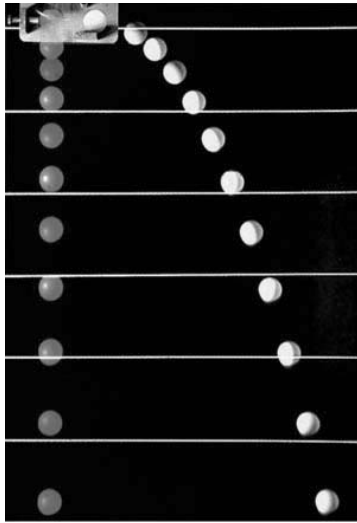


Figure 4-9 The *projectile motion* of an object launched into the air at the origin of a coordinate system and with launch velocity \vec{v}_0 at angle θ_0 . The motion is a combination of vertical motion (constant acceleration) and horizontal motion (constant velocity), as shown by the velocity components.



Richard Megna/Fundamental Photographs

Figure 4-10 One ball is released from rest at the same instant that another ball is shot horizontally to the right. Their vertical motions are identical.

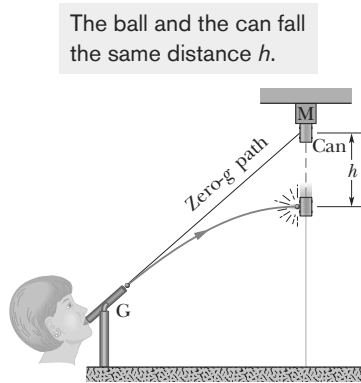


Figure 4-11 The projectile ball always hits the falling can. Each falls a distance h from where it would be were there no free-fall acceleration.

This feature allows us to break up a problem involving two-dimensional motion into two separate and easier one-dimensional problems, one for the horizontal motion (with *zero acceleration*) and one for the vertical motion (with *constant downward acceleration*). Here are two experiments that show that the horizontal motion and the vertical motion are independent.

Two Golf Balls

Figure 4-10 is a stroboscopic photograph of two golf balls, one simply released and the other shot horizontally by a spring. The golf balls have the same vertical motion, both falling through the same vertical distance in the same interval of time. *The fact that one ball is moving horizontally while it is falling has no effect on its vertical motion*; that is, the horizontal and vertical motions are independent of each other.

A Great Student Rouser

In Fig. 4-11, a blowgun G using a ball as a projectile is aimed directly at a can suspended from a magnet M. Just as the ball leaves the blowgun, the can is released. If g (the magnitude of the free-fall acceleration) were zero, the ball would follow the straight-line path shown in Fig. 4-11 and the can would float in place after the magnet released it. The ball would certainly hit the can. However, g is *not* zero, but the ball *still* hits the can! As Fig. 4-11 shows, during the time of flight of the ball, both ball and can fall the same distance h from their zero- g locations. The harder the demonstrator blows, the greater is the ball's initial speed, the shorter the flight time, and the smaller the value of h .



Checkpoint 3

At a certain instant, a fly ball has velocity $\vec{v} = 25\hat{i} - 4.9\hat{j}$ (the x axis is horizontal, the y axis is upward, and \vec{v} is in meters per second). Has the ball passed its highest point?

The Horizontal Motion

Now we are ready to analyze projectile motion, horizontally and vertically. We start with the horizontal motion. Because there is *no acceleration* in the horizontal direction, the horizontal component v_x of the projectile's velocity remains unchanged from its initial value v_{0x} throughout the motion, as demonstrated in Fig. 4-12. At any time t , the projectile's horizontal displacement $x - x_0$ from an initial position x_0 is given by Eq. 2-15 with $a = 0$, which we write as

$$x - x_0 = v_{0x}t.$$

Because $v_{0x} = v_0 \cos \theta_0$, this becomes

$$x - x_0 = (v_0 \cos \theta_0)t. \quad (4-21)$$

The Vertical Motion

The vertical motion is the motion we discussed in Module 2-5 for a particle in free fall. Most important is that the acceleration is constant. Thus, the equations of Table 2-1 apply, provided we substitute $-g$ for a and switch to y notation. Then, for example, Eq. 2-15 becomes

$$\begin{aligned} y - y_0 &= v_{0y}t - \frac{1}{2}gt^2 \\ &= (v_0 \sin \theta_0)t - \frac{1}{2}gt^2, \end{aligned} \quad (4-22)$$

where the initial vertical velocity component v_{0y} is replaced with the equivalent $v_0 \sin \theta_0$. Similarly, Eqs. 2-11 and 2-16 become

$$v_y = v_0 \sin \theta_0 - gt \quad (4-23)$$

and

$$v_y^2 = (v_0 \sin \theta_0)^2 - 2g(y - y_0). \quad (4-24)$$

As is illustrated in Fig. 4-9 and Eq. 4-23, the vertical velocity component behaves just as for a ball thrown vertically upward. It is directed upward initially, and its magnitude steadily decreases to zero, *which marks the maximum height of the path*. The vertical velocity component then reverses direction, and its magnitude becomes larger with time.

The Equation of the Path

We can find the equation of the projectile's path (its **trajectory**) by eliminating time t between Eqs. 4-21 and 4-22. Solving Eq. 4-21 for t and substituting into Eq. 4-22, we obtain, after a little rearrangement,

$$y = (\tan \theta_0)x - \frac{gx^2}{2(v_0 \cos \theta_0)^2} \quad (\text{trajectory}). \quad (4-25)$$

This is the equation of the path shown in Fig. 4-9. In deriving it, for simplicity we let $x_0 = 0$ and $y_0 = 0$ in Eqs. 4-21 and 4-22, respectively. Because g , θ_0 , and v_0 are constants, Eq. 4-25 is of the form $y = ax + bx^2$, in which a and b are constants. This is the equation of a parabola, so the path is *parabolic*.

The Horizontal Range

The *horizontal range* R of the projectile is the *horizontal* distance the projectile has traveled when it returns to its initial height (the height at which it is launched). To find range R , let us put $x - x_0 = R$ in Eq. 4-21 and $y - y_0 = 0$ in Eq. 4-22, obtaining

$$R = (v_0 \cos \theta_0)t$$

and

$$0 = (v_0 \sin \theta_0)t - \frac{1}{2}gt^2.$$

Eliminating t between these two equations yields

$$R = \frac{2v_0^2}{g} \sin \theta_0 \cos \theta_0.$$


Using the identity $\sin 2\theta_0 = 2 \sin \theta_0 \cos \theta_0$ (see Appendix E), we obtain

$$R = \frac{v_0^2}{g} \sin 2\theta_0. \quad (4-26)$$

This equation does *not* give the horizontal distance traveled by a projectile when the final height is not the launch height. Note that R in Eq. 4-26 has its maximum value when $\sin 2\theta_0 = 1$, which corresponds to $2\theta_0 = 90^\circ$ or $\theta_0 = 45^\circ$.



The horizontal range R is maximum for a launch angle of 45° .

However, when the launch and landing heights differ, as in many sports, a launch angle of 45° does not yield the maximum horizontal distance. 

The Effects of the Air

We have assumed that the air through which the projectile moves has no effect on its motion. However, in many situations, the disagreement between our calculations and the actual motion of the projectile can be large because the air resists (opposes) the motion. Figure 4-13, for example, shows two paths for a fly ball that leaves the bat at an angle of 60° with the horizontal and an initial speed of 44.7 m/s . Path I (the baseball player's fly ball) is a calculated path that approximates normal conditions of play, in air. Path II (the physics professor's fly ball) is the path the ball would follow in a vacuum.



Jamie Budge

Figure 4-12 The vertical component of this skateboarder's velocity is changing but not the horizontal component, which matches the skateboard's velocity. As a result, the skateboard stays underneath him, allowing him to land on it.

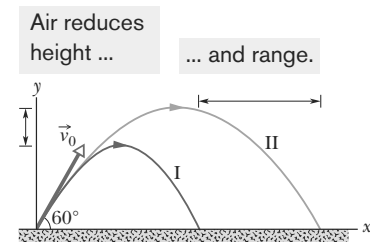


Figure 4-13 (I) The path of a fly ball calculated by taking air resistance into account. (II) The path the ball would follow in a vacuum, calculated by the methods of this chapter. See Table 4-1 for corresponding data. (Based on "The Trajectory of a Fly Ball," by Peter J. Brancazio, *The Physics Teacher*, January 1985.)

Table 4-1 Two Fly Balls^a

	Path I (Air)	Path II (Vacuum)
Range	98.5 m	177 m
Maximum height	53.0 m	76.8 m
Time of flight	6.6 s	7.9 s

^aSee Fig. 4-13. The launch angle is 60° and the launch speed is 44.7 m/s .

**Checkpoint 4**

A fly ball is hit to the outfield. During its flight (ignore the effects of the air), what happens to its (a) horizontal and (b) vertical components of velocity? What are the (c) horizontal and (d) vertical components of its acceleration during ascent, during descent, and at the topmost point of its flight?

Sample Problem 4.04 Projectile dropped from airplane

In Fig. 4-14, a rescue plane flies at 198 km/h ($= 55.0$ m/s) and constant height $h = 500$ m toward a point directly over a victim, where a rescue capsule is to land.

(a) What should be the angle ϕ of the pilot's line of sight to the victim when the capsule release is made?

KEY IDEAS

Once released, the capsule is a projectile, so its horizontal and vertical motions can be considered separately (we need not consider the actual curved path of the capsule).

Calculations: In Fig. 4-14, we see that ϕ is given by

$$\phi = \tan^{-1} \frac{x}{h}, \quad (4-27)$$

where x is the horizontal coordinate of the victim (and of the capsule when it hits the water) and $h = 500$ m. We should be able to find x with Eq. 4-21:

$$x - x_0 = (v_0 \cos \theta_0)t. \quad (4-28)$$

Here we know that $x_0 = 0$ because the origin is placed at the point of release. Because the capsule is *released* and not shot from the plane, its initial velocity \vec{v}_0 is equal to the plane's velocity. Thus, we know also that the initial velocity has magnitude $v_0 = 55.0$ m/s and angle $\theta_0 = 0^\circ$ (measured relative to the positive direction of the x axis). However, we do not know the time t the capsule takes to move from the plane to the victim.

To find t , we next consider the *vertical* motion and specifically Eq. 4-22:

$$y - y_0 = (v_0 \sin \theta_0)t - \frac{1}{2}gt^2. \quad (4-29)$$

Here the vertical displacement $y - y_0$ of the capsule is -500 m (the negative value indicates that the capsule moves *downward*). So,

$$-500 \text{ m} = (55.0 \text{ m/s})(\sin 0^\circ)t - \frac{1}{2}(9.8 \text{ m/s}^2)t^2. \quad (4-30)$$

Solving for t , we find $t = 10.1$ s. Using that value in Eq. 4-28 yields

$$x - 0 = (55.0 \text{ m/s})(\cos 0^\circ)(10.1 \text{ s}), \quad (4-31)$$

or $x = 555.5$ m.

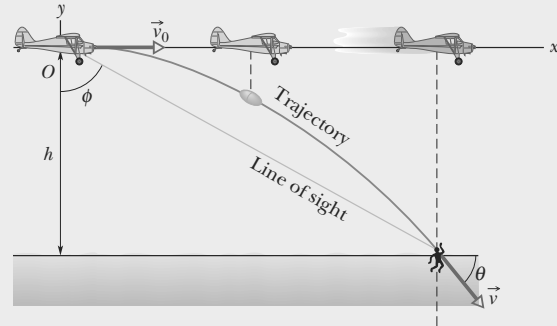


Figure 4-14 A plane drops a rescue capsule while moving at constant velocity in level flight. While falling, the capsule remains under the plane.

Then Eq. 4-27 gives us

$$\phi = \tan^{-1} \frac{555.5 \text{ m}}{500 \text{ m}} = 48.0^\circ. \quad (\text{Answer})$$

(b) As the capsule reaches the water, what is its velocity \vec{v} ?

KEY IDEAS

(1) The horizontal and vertical components of the capsule's velocity are independent. (2) Component v_x does not change from its initial value $v_{0x} = v_0 \cos \theta_0$ because there is no horizontal acceleration. (3) Component v_y changes from its initial value $v_{0y} = v_0 \sin \theta_0$ because there is a vertical acceleration.

Calculations: When the capsule reaches the water,

$$v_x = v_0 \cos \theta_0 = (55.0 \text{ m/s})(\cos 0^\circ) = 55.0 \text{ m/s}.$$

Using Eq. 4-23 and the capsule's time of fall $t = 10.1$ s, we also find that when the capsule reaches the water,

$$\begin{aligned} v_y &= v_0 \sin \theta_0 - gt \\ &= (55.0 \text{ m/s})(\sin 0^\circ) - (9.8 \text{ m/s}^2)(10.1 \text{ s}) \\ &= -99.0 \text{ m/s}. \end{aligned}$$

Thus, at the water

$$\vec{v} = (55.0 \text{ m/s})\hat{i} - (99.0 \text{ m/s})\hat{j}. \quad (\text{Answer})$$

From Eq. 3-6, the magnitude and the angle of \vec{v} are

$$v = 113 \text{ m/s} \quad \text{and} \quad \theta = -60.9^\circ. \quad (\text{Answer})$$



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Sample Problem 4.05 Launched into the air from a water slide

One of the most dramatic videos on the web (but entirely fictitious) supposedly shows a man sliding along a long water slide and then being launched into the air to land in a water pool. Let's attach some reasonable numbers to such a flight to calculate the velocity with which the man would have hit the water. Figure 4-15a indicates the launch and landing sites with its origin conveniently located at the launch site. From the video we take the horizontal flight distance as $D = 20.0$ m, the flight time as $t = 2.50$ s, and the launch angle as $\theta_0 = 40.0^\circ$. Find the magnitude of the velocity at launch and at landing.

KEY IDEAS

(1) For projectile motion, we can apply the equations for constant acceleration along the horizontal and vertical axes *separately*. (2) Throughout the flight, the vertical acceleration is $a_y = -g = -9.8$ m/s² and the horizontal acceleration is $a_x = 0$.

Calculations: In most projectile problems, the initial challenge is to figure out where to start. There is nothing wrong with trying out various equations, to see if we can somehow get to the velocities. But here is a clue. Because we are going to apply the constant-acceleration equations separately to the x and y motions, we should find the horizontal and vertical components of the velocities at launch and at landing. For each site, we can then combine the velocity components to get the velocity.

Because we know the horizontal displacement $D = 20.0$ m, let's start with the horizontal motion. Since $a_x = 0$,

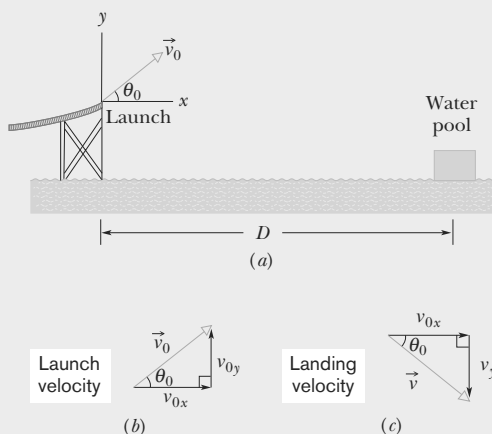


Figure 4-15 (a) Launch from a water slide, to land in a water pool. The velocity at (b) launch and (c) landing.

we know that the horizontal velocity component v_x is constant during the flight and thus is always equal to the horizontal component v_{0x} at launch. We can relate that component, the displacement $x - x_0$, and the flight time $t = 2.50$ s with Eq. 2-15:

$$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2. \quad (4-32)$$

Substituting $a_x = 0$, this becomes Eq. 4-21. With $x - x_0 = D$, we then write

$$20 \text{ m} = v_{0x}(2.50 \text{ s}) + \frac{1}{2}(0)(2.50 \text{ s})^2$$

$$v_{0x} = 8.00 \text{ m/s.}$$

That is a component of the launch velocity, but we need the magnitude of the full vector, as shown in Fig. 4-15b, where the components form the legs of a right triangle and the full vector forms the hypotenuse. We can then apply a trig definition to find the magnitude of the full velocity at launch:

$$\cos \theta_0 = \frac{v_{0x}}{v_0},$$

and so

$$v_0 = \frac{v_{0x}}{\cos \theta_0} = \frac{8.00 \text{ m/s}}{\cos 40^\circ}$$

$$= 10.44 \text{ m/s} \approx 10.4 \text{ m/s.} \quad (\text{Answer})$$

Now let's go after the magnitude v of the landing velocity. We already know the horizontal component, which does not change from its initial value of 8.00 m/s. To find the vertical component v_y and because we know the elapsed time $t = 2.50$ s and the vertical acceleration $a_y = -9.8$ m/s², let's rewrite Eq. 2-11 as

$$v_y = v_{0y} + a_y t$$

and then (from Fig. 4-15b) as

$$v_y = v_0 \sin \theta_0 + a_y t. \quad (4-33)$$

Substituting $a_y = -g$, this becomes Eq. 4-23. We can then write

$$v_y = (10.44 \text{ m/s}) \sin (40.0^\circ) - (9.8 \text{ m/s}^2)(2.50 \text{ s})$$

$$= -17.78 \text{ m/s.}$$

Now that we know both components of the landing velocity, we use Eq. 3-6 to find the velocity magnitude:

$$v = \sqrt{v_x^2 + v_y^2}$$

$$= \sqrt{(8.00 \text{ m/s})^2 + (-17.78 \text{ m/s})^2}$$

$$= 19.49 \text{ m/s}^2 \approx 19.5 \text{ m/s.} \quad (\text{Answer})$$



Additional examples, video, and practice available at WileyPLUS



4-5 UNIFORM CIRCULAR MOTION

Learning Objectives

After reading this module, you should be able to . . .

4.16 Sketch the path taken in uniform circular motion and explain the velocity and acceleration vectors (magnitude and direction) during the motion.

4.17 Apply the relationships between the radius of the circular path, the period, the particle's speed, and the particle's acceleration magnitude.

Key Ideas

● If a particle travels along a circle or circular arc of radius r at constant speed v , it is said to be in uniform circular motion and has an acceleration \vec{a} of constant magnitude

$$a = \frac{v^2}{r}.$$

The direction of \vec{a} is toward the center of the circle or circular

arc, and \vec{a} is said to be centripetal. The time for the particle to complete a circle is

$$T = \frac{2\pi r}{v}.$$

T is called the period of revolution, or simply the period, of the motion.

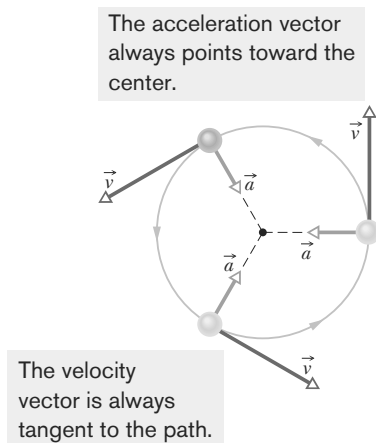


Figure 4-16 Velocity and acceleration vectors for uniform circular motion.

Uniform Circular Motion

A particle is in **uniform circular motion** if it travels around a circle or a circular arc at constant (*uniform*) speed. Although the speed does not vary, *the particle is accelerating* because the velocity changes in direction.

Figure 4-16 shows the relationship between the velocity and acceleration vectors at various stages during uniform circular motion. Both vectors have constant magnitude, but their directions change continuously. The velocity is always directed tangent to the circle in the direction of motion. The acceleration is always directed *radially inward*. Because of this, the acceleration associated with uniform circular motion is called a **centripetal** (meaning “center seeking”) **acceleration**. As we prove next, the magnitude of this acceleration \vec{a} is

$$a = \frac{v^2}{r} \quad (\text{centripetal acceleration}), \quad (4-34)$$

where r is the radius of the circle and v is the speed of the particle.

In addition, during this acceleration at constant speed, the particle travels the circumference of the circle (a distance of $2\pi r$) in time

$$T = \frac{2\pi r}{v} \quad (\text{period}). \quad (4-35)$$

T is called the *period of revolution*, or simply the *period*, of the motion. It is, in general, the time for a particle to go around a closed path exactly once.

Proof of Eq. 4-34

To find the magnitude and direction of the acceleration for uniform circular motion, we consider Fig. 4-17. In Fig. 4-17*a*, particle p moves at constant speed v around a circle of radius r . At the instant shown, p has coordinates x_p and y_p .

Recall from Module 4-2 that the velocity \vec{v} of a moving particle is always tangent to the particle's path at the particle's position. In Fig. 4-17*a*, that means \vec{v} is perpendicular to a radius r drawn to the particle's position. Then the angle θ that \vec{v} makes with a vertical at p equals the angle θ that radius r makes with the x axis.

The scalar components of \vec{v} are shown in Fig. 4-17b. With them, we can write the velocity \vec{v} as

$$\vec{v} = v_x \hat{i} + v_y \hat{j} = (-v \sin \theta) \hat{i} + (v \cos \theta) \hat{j}. \quad (4-36)$$

Now, using the right triangle in Fig. 4-17a, we can replace $\sin \theta$ with y_p/r and $\cos \theta$ with x_p/r to write

$$\vec{v} = \left(-\frac{vy_p}{r} \right) \hat{i} + \left(\frac{vx_p}{r} \right) \hat{j}. \quad (4-37)$$

To find the acceleration \vec{a} of particle p , we must take the time derivative of this equation. Noting that speed v and radius r do not change with time, we obtain

$$\vec{a} = \frac{d\vec{v}}{dt} = \left(-\frac{v}{r} \frac{dy_p}{dt} \right) \hat{i} + \left(\frac{v}{r} \frac{dx_p}{dt} \right) \hat{j}. \quad (4-38)$$

Now note that the rate dy_p/dt at which y_p changes is equal to the velocity component v_y . Similarly, $dx_p/dt = v_x$, and, again from Fig. 4-17b, we see that $v_x = -v \sin \theta$ and $v_y = v \cos \theta$. Making these substitutions in Eq. 4-38, we find

$$\vec{a} = \left(-\frac{v^2}{r} \cos \theta \right) \hat{i} + \left(-\frac{v^2}{r} \sin \theta \right) \hat{j}. \quad (4-39)$$

This vector and its components are shown in Fig. 4-17c. Following Eq. 3-6, we find

$$a = \sqrt{a_x^2 + a_y^2} = \frac{v^2}{r} \sqrt{(\cos \theta)^2 + (\sin \theta)^2} = \frac{v^2}{r} \sqrt{1} = \frac{v^2}{r},$$

as we wanted to prove. To orient \vec{a} , we find the angle ϕ shown in Fig. 4-17c:

$$\tan \phi = \frac{a_y}{a_x} = \frac{-(v^2/r) \sin \theta}{-(v^2/r) \cos \theta} = \tan \theta.$$

Thus, $\phi = \theta$, which means that \vec{a} is directed along the radius r of Fig. 4-17a, toward the circle's center, as we wanted to prove.



Checkpoint 5

An object moves at constant speed along a circular path in a horizontal xy plane, with the center at the origin. When the object is at $x = -2$ m, its velocity is $-(4 \text{ m/s})\hat{j}$. Give the object's (a) velocity and (b) acceleration at $y = 2$ m.

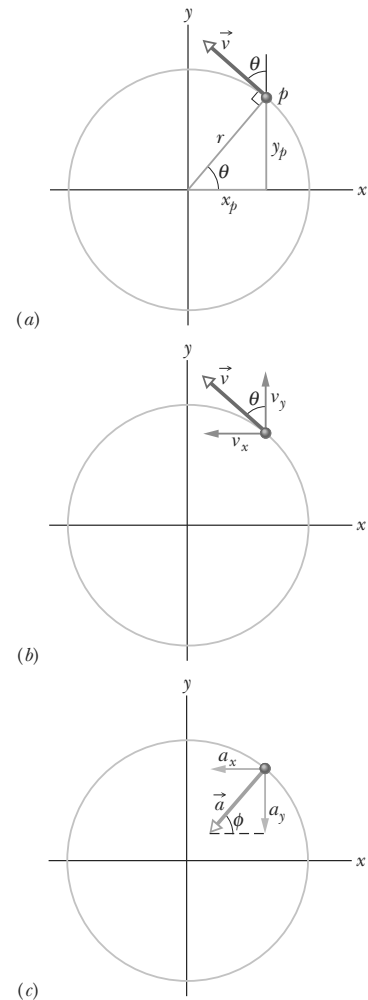


Figure 4-17 Particle p moves in counter-clockwise uniform circular motion. (a) Its position and velocity \vec{v} at a certain instant. (b) Velocity \vec{v} . (c) Acceleration \vec{a} .

Sample Problem 4.06 Top gun pilots in turns

“Top gun” pilots have long worried about taking a turn too tightly. As a pilot’s body undergoes centripetal acceleration, with the head toward the center of curvature, the blood pressure in the brain decreases, leading to loss of brain function.

There are several warning signs. When the centripetal acceleration is $2g$ or $3g$, the pilot feels heavy. At about $4g$, the pilot’s vision switches to black and white and narrows to “tunnel vision.” If that acceleration is sustained or increased, vision ceases and, soon after, the pilot is unconscious—a condition known as g -LOC for “ g -induced loss of consciousness.”

What is the magnitude of the acceleration, in g units, of a pilot whose aircraft enters a horizontal circular turn with a velocity of $\vec{v}_i = (400\hat{i} + 500\hat{j}) \text{ m/s}$ and 24.0 s later leaves the turn with a velocity of $\vec{v}_f = (-400\hat{i} - 500\hat{j}) \text{ m/s}$?

KEY IDEAS

We assume the turn is made with uniform circular motion. Then the pilot’s acceleration is centripetal and has magnitude a given by Eq. 4-34 ($a = v^2/R$), where R is the circle’s radius. Also, the time required to complete a full circle is the period given by Eq. 4-35 ($T = 2\pi R/v$).

Calculations: Because we do not know radius R , let’s solve Eq. 4-35 for R and substitute into Eq. 4-34. We find

$$a = \frac{2\pi v}{T}.$$

To get the constant speed v , let’s substitute the components of the initial velocity into Eq. 3-6:

$$v = \sqrt{(400 \text{ m/s})^2 + (500 \text{ m/s})^2} = 640.31 \text{ m/s}.$$

To find the period T of the motion, first note that the final velocity is the reverse of the initial velocity. This means the aircraft leaves on the opposite side of the circle from the initial point and must have completed half a circle in the given

24.0 s. Thus a full circle would have taken $T = 48.0$ s. Substituting these values into our equation for a , we find

$$a = \frac{2\pi(640.31 \text{ m/s})}{48.0 \text{ s}} = 83.81 \text{ m/s}^2 \approx 8.6g. \quad (\text{Answer})$$



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4-6 RELATIVE MOTION IN ONE DIMENSION

Learning Objective

After reading this module, you should be able to . . .

4.18 Apply the relationship between a particle's position, velocity, and acceleration as measured from two reference

frames that move relative to each other at constant velocity and along a single axis.

Key Idea

● When two frames of reference A and B are moving relative to each other at constant velocity, the velocity of a particle P as measured by an observer in frame A usually differs from that measured from frame B . The two measured velocities are related by

$$\vec{v}_{PA} = \vec{v}_{PB} + \vec{v}_{BA},$$

where \vec{v}_{BA} is the velocity of B with respect to A . Both observers measure the same acceleration for the particle:

$$\vec{a}_{PA} = \vec{a}_{PB}.$$

Relative Motion in One Dimension

Suppose you see a duck flying north at 30 km/h. To another duck flying alongside, the first duck seems to be stationary. In other words, the velocity of a particle depends on the **reference frame** of whoever is observing or measuring the velocity. For our purposes, a reference frame is the physical object to which we attach our coordinate system. In everyday life, that object is the ground. For example, the speed listed on a speeding ticket is always measured relative to the ground. The speed relative to the police officer would be different if the officer were moving while making the speed measurement.

Suppose that Alex (at the origin of frame A in Fig. 4-18) is parked by the side of a highway, watching car P (the “particle”) speed past. Barbara (at the origin of frame B) is driving along the highway at constant speed and is also watching car P . Suppose that they both measure the position of the car at a given moment. From Fig. 4-18 we see that

$$x_{PA} = x_{PB} + x_{BA}. \quad (4-40)$$

The equation is read: “The coordinate x_{PA} of P as measured by A is equal to the coordinate x_{PB} of P as measured by B plus the coordinate x_{BA} of B as measured by A .” Note how this reading is supported by the sequence of the subscripts.

Taking the time derivative of Eq. 4-40, we obtain

$$\frac{d}{dt}(x_{PA}) = \frac{d}{dt}(x_{PB}) + \frac{d}{dt}(x_{BA}).$$

Thus, the velocity components are related by

$$v_{PA} = v_{PB} + v_{BA}. \quad (4-41)$$

This equation is read: “The velocity v_{PA} of P as measured by A is equal to the

Frame B moves past frame A while both observe P .

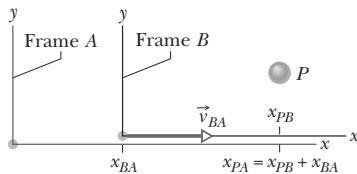


Figure 4-18 Alex (frame A) and Barbara (frame B) watch car P , as both B and P move at different velocities along the common x axis of the two frames. At the instant shown, x_{BA} is the coordinate of B in the A frame. Also, P is at coordinate x_{PB} in the B frame and coordinate $x_{PA} = x_{PB} + x_{BA}$ in the A frame.

velocity v_{PB} of P as measured by B plus the velocity v_{BA} of B as measured by A ." The term v_{BA} is the velocity of frame B relative to frame A .

Here we consider only frames that move at constant velocity relative to each other. In our example, this means that Barbara (frame B) drives always at constant velocity v_{BA} relative to Alex (frame A). Car P (the moving particle), however, can change speed and direction (that is, it can accelerate).

To relate an acceleration of P as measured by Barbara and by Alex, we take the time derivative of Eq. 4-41:

$$\frac{d}{dt}(v_{PA}) = \frac{d}{dt}(v_{PB}) + \frac{d}{dt}(v_{BA}).$$

Because v_{BA} is constant, the last term is zero and we have

$$a_{PA} = a_{PB}. \quad (4-42)$$

In other words,



Observers on different frames of reference that move at constant velocity relative to each other will measure the same acceleration for a moving particle.

Sample Problem 4.07 Relative motion, one dimensional, Alex and Barbara

In Fig. 4-18, suppose that Barbara's velocity relative to Alex is a constant $v_{BA} = 52 \text{ km/h}$ and car P is moving in the negative direction of the x axis.

(a) If Alex measures a constant $v_{PA} = -78 \text{ km/h}$ for car P , what velocity v_{PB} will Barbara measure?

KEY IDEAS

We can attach a frame of reference A to Alex and a frame of reference B to Barbara. Because the frames move at constant velocity relative to each other along one axis, we can use Eq. 4-41 ($v_{PA} = v_{PB} + v_{BA}$) to relate v_{PB} to v_{PA} and v_{BA} .

Calculation: We find

$$-78 \text{ km/h} = v_{PB} + 52 \text{ km/h}.$$

Thus, $v_{PB} = -130 \text{ km/h}$. (Answer)

Comment: If car P were connected to Barbara's car by a cord wound on a spool, the cord would be unwinding at a speed of 130 km/h as the two cars separated.

(b) If car P brakes to a stop relative to Alex (and thus relative to the ground) in time $t = 10 \text{ s}$ at constant acceleration, what is its acceleration a_{PA} relative to Alex?

KEY IDEAS

To calculate the acceleration of car P relative to Alex, we must use the car's velocities *relative to Alex*. Because the acceleration is constant, we can use Eq. 2-11 ($v = v_0 + at$)

to relate the acceleration to the initial and final velocities of P .

Calculation: The initial velocity of P relative to Alex is $v_{PA} = -78 \text{ km/h}$ and the final velocity is 0 . Thus, the acceleration relative to Alex is

$$a_{PA} = \frac{v - v_0}{t} = \frac{0 - (-78 \text{ km/h})}{10 \text{ s}} \frac{1 \text{ m/s}}{3.6 \text{ km/h}} = 2.2 \text{ m/s}^2. \quad (\text{Answer})$$

(c) What is the acceleration a_{PB} of car P relative to Barbara during the braking?

KEY IDEA

To calculate the acceleration of car P relative to Barbara, we must use the car's velocities *relative to Barbara*.

Calculation: We know the initial velocity of P relative to Barbara from part (a) ($v_{PB} = -130 \text{ km/h}$). The final velocity of P relative to Barbara is -52 km/h (because this is the velocity of the stopped car relative to the moving Barbara). Thus,

$$a_{PB} = \frac{v - v_0}{t} = \frac{-52 \text{ km/h} - (-130 \text{ km/h})}{10 \text{ s}} \frac{1 \text{ m/s}}{3.6 \text{ km/h}} = 2.2 \text{ m/s}^2. \quad (\text{Answer})$$

Comment: We should have foreseen this result: Because Alex and Barbara have a constant relative velocity, they must measure the same acceleration for the car.



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4-7 RELATIVE MOTION IN TWO DIMENSIONS

Learning Objective

After reading this module, you should be able to . . .

4.19 Apply the relationship between a particle's position, velocity, and acceleration as measured from two reference

frames that move relative to each other at constant velocity and in two dimensions.

Key Idea

● When two frames of reference A and B are moving relative to each other at constant velocity, the velocity of a particle P as measured by an observer in frame A usually differs from that measured from frame B . The two measured velocities are related by

$$\vec{v}_{PA} = \vec{v}_{PB} + \vec{v}_{BA},$$

where \vec{v}_{BA} is the velocity of B with respect to A . Both observers measure the same acceleration for the particle:

$$\vec{a}_{PA} = \vec{a}_{PB}.$$

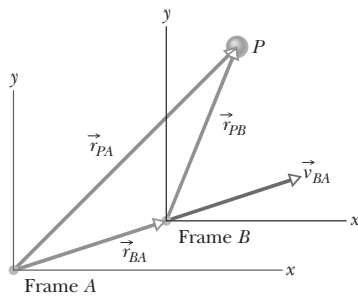


Figure 4-19 Frame B has the constant two-dimensional velocity \vec{v}_{BA} relative to frame A . The position vector of B relative to A is \vec{r}_{BA} . The position vectors of particle P are \vec{r}_{PA} relative to A and \vec{r}_{PB} relative to B .

Relative Motion in Two Dimensions

Our two observers are again watching a moving particle P from the origins of reference frames A and B , while B moves at a constant velocity \vec{v}_{BA} relative to A . (The corresponding axes of these two frames remain parallel.) Figure 4-19 shows a certain instant during the motion. At that instant, the position vector of the origin of B relative to the origin of A is \vec{r}_{BA} . Also, the position vectors of particle P are \vec{r}_{PA} relative to the origin of A and \vec{r}_{PB} relative to the origin of B . From the arrangement of heads and tails of those three position vectors, we can relate the vectors with

$$\vec{r}_{PA} = \vec{r}_{PB} + \vec{r}_{BA}. \quad (4-43)$$

By taking the time derivative of this equation, we can relate the velocities \vec{v}_{PA} and \vec{v}_{PB} of particle P relative to our observers:

$$\vec{v}_{PA} = \vec{v}_{PB} + \vec{v}_{BA}. \quad (4-44)$$

By taking the time derivative of this relation, we can relate the accelerations \vec{a}_{PA} and \vec{a}_{PB} of the particle P relative to our observers. However, note that because \vec{v}_{BA} is constant, its time derivative is zero. Thus, we get

$$\vec{a}_{PA} = \vec{a}_{PB}. \quad (4-45)$$

As for one-dimensional motion, we have the following rule: Observers on different frames of reference that move at constant velocity relative to each other will measure the *same* acceleration for a moving particle.

Sample Problem 4.08 Relative motion, two dimensional, airplanes

In Fig. 4-20a, a plane moves due east while the pilot points the plane somewhat south of east, toward a steady wind that blows to the northeast. The plane has velocity \vec{v}_{PW} relative to the wind, with an airspeed (speed relative to the wind) of 215 km/h, directed at angle θ south of east. The wind has velocity \vec{v}_{WG} relative to the ground with speed 65.0 km/h, directed 20.0° east of north. What is the magnitude of the velocity \vec{v}_{PG} of the plane relative to the ground, and what is θ ?

KEY IDEAS

The situation is like the one in Fig. 4-19. Here the moving particle P is the plane, frame A is attached to the ground (call it G), and frame B is “attached” to the wind (call it W). We need a vector diagram like Fig. 4-19 but with three velocity vectors.

Calculations: First we construct a sentence that relates the three vectors shown in Fig. 4-20b:

velocity of plane relative to ground = velocity of plane relative to wind + velocity of wind relative to ground.
(PG) (PW) (WG)

This relation is written in vector notation as

$$\vec{v}_{PG} = \vec{v}_{PW} + \vec{v}_{WG}. \quad (4-46)$$

We need to resolve the vectors into components on the coordinate system of Fig. 4-20b and then solve Eq. 4-46 axis by axis. For the y components, we find

$$v_{PG,y} = v_{PW,y} + v_{WG,y}$$

$$\text{or } 0 = -(215 \text{ km/h}) \sin \theta + (65.0 \text{ km/h})(\cos 20.0^\circ).$$

Solving for θ gives us

$$\theta = \sin^{-1} \frac{(65.0 \text{ km/h})(\cos 20.0^\circ)}{215 \text{ km/h}} = 16.5^\circ. \quad (\text{Answer})$$

Similarly, for the x components we find

$$v_{PG,x} = v_{PW,x} + v_{WG,x}.$$

Here, because \vec{v}_{PG} is parallel to the x axis, the component $v_{PG,x}$ is equal to the magnitude v_{PG} . Substituting this notation and the value $\theta = 16.5^\circ$, we find

$$\begin{aligned} v_{PG} &= (215 \text{ km/h})(\cos 16.5^\circ) + (65.0 \text{ km/h})(\sin 20.0^\circ) \\ &= 228 \text{ km/h}. \end{aligned} \quad (\text{Answer})$$

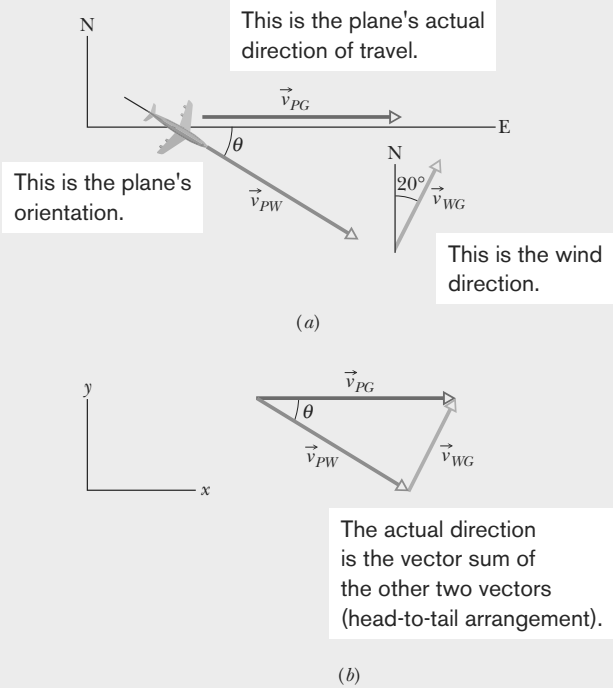


Figure 4-20 A plane flying in a wind.



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Review & Summary

Position Vector The location of a particle relative to the origin of a coordinate system is given by a *position vector* \vec{r} , which in unit-vector notation is

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}. \quad (4-1)$$

Here $x\hat{i}$, $y\hat{j}$, and $z\hat{k}$ are the vector components of position vector \vec{r} , and x , y , and z are its scalar components (as well as the coordinates of the particle). A position vector is described either by a magnitude and one or two angles for orientation, or by its vector or scalar components.

Displacement If a particle moves so that its position vector changes from \vec{r}_1 to \vec{r}_2 , the particle's *displacement* $\Delta\vec{r}$ is

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1. \quad (4-2)$$

The displacement can also be written as

$$\Delta\vec{r} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k} \quad (4-3)$$

$$= \Delta x\hat{i} + \Delta y\hat{j} + \Delta z\hat{k}. \quad (4-4)$$

Average Velocity and Instantaneous Velocity If a particle undergoes a displacement $\Delta\vec{r}$ in time interval Δt , its *average velocity* \vec{v}_{avg} for that time interval is

$$\vec{v}_{\text{avg}} = \frac{\Delta\vec{r}}{\Delta t}. \quad (4-8)$$

As Δt in Eq. 4-8 is shrunk to 0, \vec{v}_{avg} reaches a limit called either the *velocity* or the *instantaneous velocity* \vec{v} :

$$\vec{v} = \frac{d\vec{r}}{dt}, \quad (4-10)$$

which can be rewritten in unit-vector notation as

$$\vec{v} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}, \quad (4-11)$$

where $v_x = dx/dt$, $v_y = dy/dt$, and $v_z = dz/dt$. The instantaneous velocity \vec{v} of a particle is always directed along the tangent to the particle's path at the particle's position.

Average Acceleration and Instantaneous Acceleration

If a particle's velocity changes from \vec{v}_1 to \vec{v}_2 in time interval Δt , its *average acceleration* during Δt is

$$\vec{a}_{\text{avg}} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t} = \frac{\Delta\vec{v}}{\Delta t}. \quad (4-15)$$

As Δt in Eq. 4-15 is shrunk to 0, \vec{a}_{avg} reaches a limiting value called either the *acceleration* or the *instantaneous acceleration* \vec{a} :

$$\vec{a} = \frac{d\vec{v}}{dt}. \quad (4-16)$$

In unit-vector notation,

$$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}, \quad (4-17)$$

where $a_x = dv_x/dt$, $a_y = dv_y/dt$, and $a_z = dv_z/dt$.

Projectile Motion *Projectile motion* is the motion of a particle that is launched with an initial velocity \vec{v}_0 . During its flight, the particle's horizontal acceleration is zero and its vertical acceleration is the free-fall acceleration $-g$. (Upward is taken to be a positive direction.) If \vec{v}_0 is expressed as a magnitude (the speed v_0) and an angle θ_0 (measured from the horizontal), the particle's equations of motion along the horizontal x axis and vertical y axis are

$$x - x_0 = (v_0 \cos \theta_0)t, \quad (4-21)$$

$$y - y_0 = (v_0 \sin \theta_0)t - \frac{1}{2}gt^2, \quad (4-22)$$

$$v_y = v_0 \sin \theta_0 - gt, \quad (4-23)$$

$$v_y^2 = (v_0 \sin \theta_0)^2 - 2g(y - y_0). \quad (4-24)$$

The **trajectory** (path) of a particle in projectile motion is parabolic and is given by

$$y = (\tan \theta_0)x - \frac{gx^2}{2(v_0 \cos \theta_0)^2}, \quad (4-25)$$

if x_0 and y_0 of Eqs. 4-21 to 4-24 are zero. The particle's **horizontal range** R , which is the horizontal distance from the launch point to the point at which the particle returns to the launch height, is

$$R = \frac{v_0^2}{g} \sin 2\theta_0. \quad (4-26)$$

Uniform Circular Motion If a particle travels along a circle or circular arc of radius r at constant speed v , it is said to be in *uniform circular motion* and has an acceleration \vec{a} of constant magnitude

$$a = \frac{v^2}{r}. \quad (4-34)$$

The direction of \vec{a} is toward the center of the circle or circular arc, and \vec{a} is said to be *centripetal*. The time for the particle to complete a circle is

$$T = \frac{2\pi r}{v}. \quad (4-35)$$

T is called the *period of revolution*, or simply the *period*, of the motion.

Relative Motion When two frames of reference A and B are moving relative to each other at constant velocity, the velocity of a particle P as measured by an observer in frame A usually differs from that measured from frame B . The two measured velocities are related by

$$\vec{v}_{PA} = \vec{v}_{PB} + \vec{v}_{BA}, \quad (4-44)$$

where \vec{v}_{BA} is the velocity of B with respect to A . Both observers measure the same acceleration for the particle:

$$\vec{a}_{PA} = \vec{a}_{PB}. \quad (4-45)$$

Questions

1 Figure 4-21 shows the path taken by a skunk foraging for trash food, from initial point i . The skunk took the same time T to go from each labeled point to the next along its path. Rank points a , b , and c according to the magnitude of the average velocity of the skunk to reach them from initial point i , greatest first.

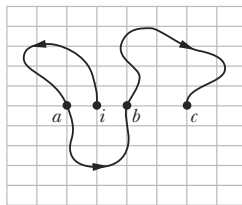


Figure 4-21
Question 1.

2 Figure 4-22 shows the initial position i and the final position f of a particle. What are the (a) initial position vector \vec{r}_i and (b) final position vector \vec{r}_f , both in unit-vector notation? (c) What is the x component of displacement $\Delta \vec{r}$?

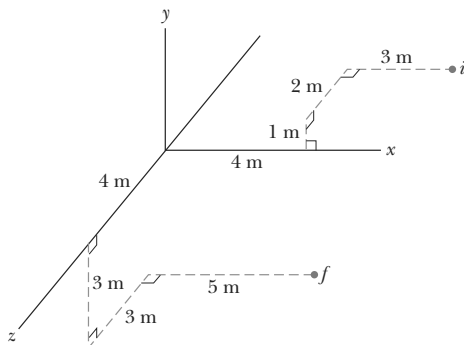


Figure 4-22 Question 2.

3 When Paris was shelled from 100 km away with the WWI long-range artillery piece "Big Bertha," the shells were fired at an angle greater than 45° to give them a greater range, possibly even

twice as long as at 45° . Does that result mean that the air density at high altitudes increases with altitude or decreases?

4 You are to launch a rocket, from just above the ground, with one of the following initial velocity vectors: (1) $\vec{v}_0 = 20\hat{i} + 70\hat{j}$, (2) $\vec{v}_0 = -20\hat{i} + 70\hat{j}$, (3) $\vec{v}_0 = 20\hat{i} - 70\hat{j}$, (4) $\vec{v}_0 = -20\hat{i} - 70\hat{j}$. In your coordinate system, x runs along level ground and y increases upward. (a) Rank the vectors according to the launch speed of the projectile, greatest first. (b) Rank the vectors according to the time of flight of the projectile, greatest first.

5 Figure 4-23 shows three situations in which identical projectiles are launched (at the same level) at identical initial speeds and angles. The projectiles do not land on the same terrain, however. Rank the situations according to the final speeds of the projectiles just before they land, greatest first.

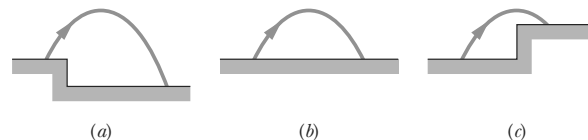


Figure 4-23 Question 5.

6 The only good use of a fruitcake is in catapult practice. Curve 1 in Fig. 4-24 gives the height y of a catapulted fruitcake versus the angle θ between its velocity vector and its acceleration vector during flight. (a) Which of the lettered points on that curve corresponds to the landing of the fruitcake on the ground? (b) Curve 2 is a similar plot for the same

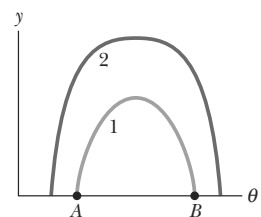


Figure 4-24 Question 6.

launch speed but for a different launch angle. Does the fruitcake now land farther away or closer to the launch point?

7 An airplane flying horizontally at a constant speed of 350 km/h over level ground releases a bundle of food supplies. Ignore the effect of the air on the bundle. What are the bundle's initial (a) vertical and (b) horizontal components of velocity? (c) What is its horizontal component of velocity just before hitting the ground? (d) If the airplane's speed were, instead, 450 km/h, would the time of fall be longer, shorter, or the same?

8 In Fig. 4-25, a cream tangerine is thrown up past windows 1, 2, and 3, which are identical in size and regularly spaced vertically. Rank those three windows according to (a) the time the cream tangerine takes to pass them and (b) the average speed of the cream tangerine during the passage, greatest first.

The cream tangerine then moves down past windows 4, 5, and 6, which are identical in size and irregularly spaced horizontally. Rank those three windows according to (c) the time the cream tangerine takes to pass them and (d) the average speed of the cream tangerine during the passage, greatest first.

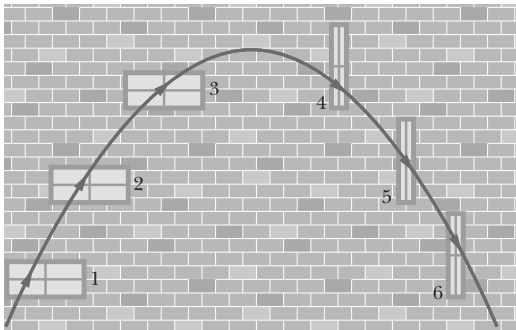


Figure 4-25 Question 8.

9 Figure 4-26 shows three paths for a football kicked from ground level. Ignoring the effects of air, rank the paths according to (a) time of flight, (b) initial vertical velocity component, (c) initial horizontal velocity component, and (d) initial speed, greatest first.

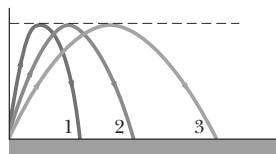


Figure 4-26 Question 9.

10 A ball is shot from ground level over level ground at a certain initial speed. Figure 4-27 gives the range R of the ball versus its launch angle θ_0 . Rank the three lettered points on the plot according to (a) the total flight time of the ball and (b) the ball's speed at maximum height, greatest first.

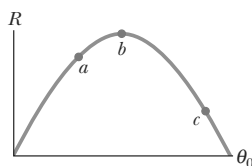


Figure 4-27 Question 10.

11 Figure 4-28 shows four tracks (either half- or quarter-circles) that can be taken by a train, which moves at a constant speed. Rank the tracks according to the magnitude of a train's acceleration on the curved portion, greatest first.

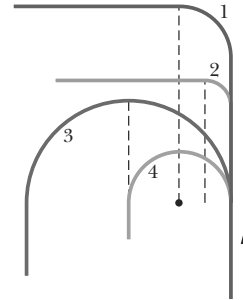


Figure 4-28 Question 11.

12 In Fig. 4-29, particle P is in uniform circular motion, centered on the origin of an xy coordinate system. (a) At what values of θ is the vertical component r_y of the position vector greatest in magnitude? (b) At what values of θ is the vertical component v_y of the particle's velocity greatest in magnitude? (c) At what values of θ is the vertical component a_y of the particle's acceleration greatest in magnitude?

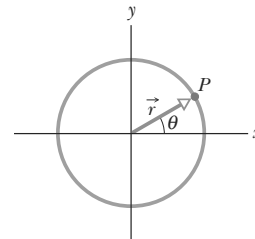


Figure 4-29 Question 12.

13 (a) Is it possible to be accelerating while traveling at constant speed? Is it possible to round a curve with (b) zero acceleration and (c) a constant magnitude of acceleration?

14 While riding in a moving car, you toss an egg directly upward. Does the egg tend to land behind you, in front of you, or back in your hands if the car is (a) traveling at a constant speed, (b) increasing in speed, and (c) decreasing in speed?

15 A snowball is thrown from ground level (by someone in a hole) with initial speed v_0 at an angle of 45° relative to the (level) ground, on which the snowball later lands. If the launch angle is increased, do (a) the range and (b) the flight time increase, decrease, or stay the same?

16 You are driving directly behind a pickup truck, going at the same speed as the truck. A crate falls from the bed of the truck to the road. (a) Will your car hit the crate before the crate hits the road if you neither brake nor swerve? (b) During the fall, is the horizontal speed of the crate more than, less than, or the same as that of the truck?

17 At what point in the path of a projectile is the speed a minimum?

18 In shot put, the shot is put (thrown) from above the athlete's shoulder level. Is the launch angle that produces the greatest range 45° , less than 45° , or greater than 45° ?

Problems



Tutoring problem available (at instructor's discretion) in WileyPLUS and WebAssign



Worked-out solution available in Student Solutions Manual

WWW Worked-out solution is at



Number of dots indicates level of problem difficulty



Interactive solution is at

<http://www.wiley.com/college/halliday>



Additional information available in *The Flying Circus of Physics* and at flyingcircusofphysics.com

Module 4-1 Position and Displacement

•1 The position vector for an electron is $\vec{r} = (5.0 \text{ m})\hat{i} - (3.0 \text{ m})\hat{j} + (2.0 \text{ m})\hat{k}$. (a) Find the magnitude of \vec{r} . (b) Sketch the vector on a right-handed coordinate system.

•2 A watermelon seed has the following coordinates: $x = -5.0 \text{ m}$, $y = 8.0 \text{ m}$, and $z = 0 \text{ m}$. Find its position vector (a) in unit-vector notation and as (b) a magnitude and (c) an angle relative to the positive direction of the x axis. (d) Sketch the vector on a right-handed coordinate system. If the seed is moved to the xyz coordinates $(3.00 \text{ m}, 0 \text{ m}, 0 \text{ m})$, what is its displacement (e) in unit-vector notation and as (f) a magnitude and (g) an angle relative to the positive x direction?

•3 A positron undergoes a displacement $\Delta\vec{r} = 2.0\hat{i} - 3.0\hat{j} + 6.0\hat{k}$, ending with the position vector $\vec{r} = 3.0\hat{j} - 4.0\hat{k}$, in meters. What was the positron's initial position vector?

•4 The minute hand of a wall clock measures 10 cm from its tip to the axis about which it rotates. The magnitude and angle of the displacement vector of the tip are to be determined for three time intervals. What are the (a) magnitude and (b) angle from a quarter after the hour to half past, the (c) magnitude and (d) angle for the next half hour, and the (e) magnitude and (f) angle for the hour after that?

Module 4-2 Average Velocity and Instantaneous Velocity

•5 SSM A train at a constant 60.0 km/h moves east for 40.0 min, then in a direction 50.0° east of due north for 20.0 min, and then west for 50.0 min. What are the (a) magnitude and (b) angle of its average velocity during this trip?

•6 An electron's position is given by $\vec{r} = 3.00t\hat{i} - 4.00t^2\hat{j} + 2.00\hat{k}$, with t in seconds and \vec{r} in meters. (a) In unit-vector notation, what is the electron's velocity $\vec{v}(t)$? At $t = 2.00 \text{ s}$, what is \vec{v} (b) in unit-vector notation and as (c) a magnitude and (d) an angle relative to the positive direction of the x axis?

•7 An ion's position vector is initially $\vec{r} = 5.0\hat{i} - 6.0\hat{j} + 2.0\hat{k}$, and 10 s later it is $\vec{r} = -2.0\hat{i} + 8.0\hat{j} - 2.0\hat{k}$, all in meters. In unit-vector notation, what is its \vec{v}_{avg} during the 10 s?

•8 A plane flies 483 km east from city A to city B in 45.0 min and then 966 km south from city B to city C in 1.50 h. For the total trip, what are the (a) magnitude and (b) direction of the plane's displacement, the (c) magnitude and (d) direction of its average velocity, and (e) its average speed?

•9 Figure 4-30 gives the path of a squirrel moving about on level ground, from point A (at time $t = 0$), to points B (at $t = 5.00 \text{ min}$), C (at $t = 10.0 \text{ min}$), and finally D (at $t = 15.0 \text{ min}$). Consider the average velocities of the squirrel from point A to each of the other three points. Of them, what are the (a) magnitude

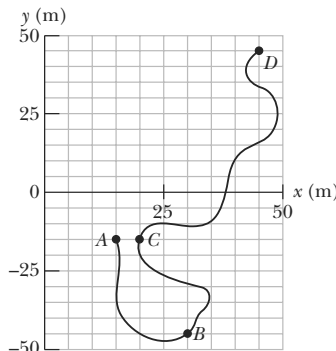


Figure 4-30 Problem 9.

and (b) angle of the one with the least magnitude and the (c) magnitude and (d) angle of the one with the greatest magnitude?

•10 The position vector $\vec{r} = 5.00t\hat{i} + (et + ft^2)\hat{j}$ locates a particle as a function of time t . Vector \vec{r} is in meters, t is in seconds, and factors e and f are constants. Figure 4-31 gives the angle θ of the particle's direction of travel as a function of t (θ is measured from the positive x direction). What are (a) e and (b) f , including units?

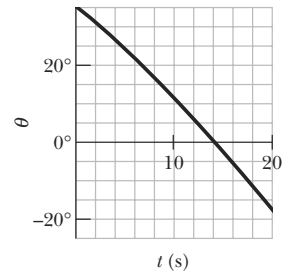


Figure 4-31 Problem 10.

Module 4-3 Average Acceleration and Instantaneous Acceleration

•11 GO The position \vec{r} of a particle moving in an xy plane is given by $\vec{r} = (2.00t^3 - 5.00t)\hat{i} + (6.00 - 7.00t^4)\hat{j}$, with \vec{r} in meters and t in seconds. In unit-vector notation, calculate (a) \vec{r} , (b) \vec{v} , and (c) \vec{a} for $t = 2.00 \text{ s}$. (d) What is the angle between the positive direction of the x axis and a line tangent to the particle's path at $t = 2.00 \text{ s}$?

•12 At one instant a bicyclist is 40.0 m due east of a park's flagpole, going due south with a speed of 10.0 m/s. Then 30.0 s later, the cyclist is 40.0 m due north of the flagpole, going due east with a speed of 10.0 m/s. For the cyclist in this 30.0 s interval, what are the (a) magnitude and (b) direction of the displacement, the (c) magnitude and (d) direction of the average velocity, and the (e) magnitude and (f) direction of the average acceleration?

•13 SSM A particle moves so that its position (in meters) as a function of time (in seconds) is $\vec{r} = \hat{i} + 4t^2\hat{j} + t\hat{k}$. Write expressions for (a) its velocity and (b) its acceleration as functions of time.

•14 A proton initially has $\vec{v} = 4.0\hat{i} - 2.0\hat{j} + 3.0\hat{k}$ and then 4.0 s later has $\vec{v} = -2.0\hat{i} - 2.0\hat{j} + 5.0\hat{k}$ (in meters per second). For that 4.0 s, what are (a) the proton's average acceleration \vec{a}_{avg} in unit-vector notation, (b) the magnitude of \vec{a}_{avg} , and (c) the angle between \vec{a}_{avg} and the positive direction of the x axis?

•15 SSM ILW A particle leaves the origin with an initial velocity $\vec{v} = (3.00\hat{i}) \text{ m/s}$ and a constant acceleration $\vec{a} = (-1.00\hat{i} - 0.500\hat{j}) \text{ m/s}^2$. When it reaches its maximum x coordinate, what are its (a) velocity and (b) position vector?

•16 GO The velocity \vec{v} of a particle moving in the xy plane is given by $\vec{v} = (6.0t - 4.0t^2)\hat{i} + 8.0\hat{j}$, with \vec{v} in meters per second and $t (> 0)$ in seconds. (a) What is the acceleration when $t = 3.0 \text{ s}$? (b) When (if ever) is the acceleration zero? (c) When (if ever) is the velocity zero? (d) When (if ever) does the speed equal 10 m/s?

•17 A cart is propelled over an xy plane with acceleration components $a_x = 4.0 \text{ m/s}^2$ and $a_y = -2.0 \text{ m/s}^2$. Its initial velocity has components $v_{0x} = 8.0 \text{ m/s}$ and $v_{0y} = 12 \text{ m/s}$. In unit-vector notation, what is the velocity of the cart when it reaches its greatest y coordinate?

•18 A moderate wind accelerates a pebble over a horizontal xy plane with a constant acceleration $\vec{a} = (5.00 \text{ m/s}^2)\hat{i} + (7.00 \text{ m/s}^2)\hat{j}$.

At time $t = 0$, the velocity is $(4.00 \text{ m/s})\hat{i}$. What are the (a) magnitude and (b) angle of its velocity when it has been displaced by 12.0 m parallel to the x axis?

••19 The acceleration of a particle moving only on a horizontal xy plane is given by $\vec{a} = 3t\hat{i} + 4t\hat{j}$, where \vec{a} is in meters per second-squared and t is in seconds. At $t = 0$, the position vector $\vec{r} = (20.0 \text{ m})\hat{i} + (40.0 \text{ m})\hat{j}$ locates the particle, which then has the velocity vector $\vec{v} = (5.00 \text{ m/s})\hat{i} + (2.00 \text{ m/s})\hat{j}$. At $t = 4.00 \text{ s}$, what are (a) its position vector in unit-vector notation and (b) the angle between its direction of travel and the positive direction of the x axis?

••20 GO In Fig. 4-32, particle A moves along the line $y = 30 \text{ m}$ with a constant velocity \vec{v} of magnitude 3.0 m/s and parallel to the x axis. At the instant particle A passes the y axis, particle B leaves the origin with a zero initial speed and a constant acceleration \vec{a} of magnitude 0.40 m/s^2 . What angle θ between \vec{a} and the positive direction of the y axis would result in a collision?

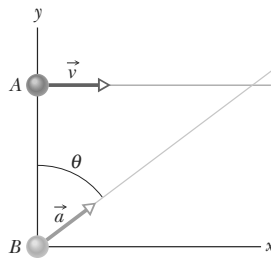


Figure 4-32 Problem 20.

Module 4-4 Projectile Motion

•21 A dart is thrown horizontally with an initial speed of 10 m/s toward point P , the bull's-eye on a dart board. It hits at point Q on the rim, vertically below P , 0.19 s later. (a) What is the distance PQ ? (b) How far away from the dart board is the dart released?

•22 A small ball rolls horizontally off the edge of a tabletop that is 1.20 m high. It strikes the floor at a point 1.52 m horizontally from the table edge. (a) How long is the ball in the air? (b) What is its speed at the instant it leaves the table?

•23 A projectile is fired horizontally from a gun that is 45.0 m above flat ground, emerging from the gun with a speed of 250 m/s. (a) How long does the projectile remain in the air? (b) At what horizontal distance from the firing point does it strike the ground? (c) What is the magnitude of the vertical component of its velocity as it strikes the ground?

•24 In the 1991 World Track and Field Championships in Tokyo, Mike Powell jumped 8.95 m, breaking by a full 5 cm the 23-year long-jump record set by Bob Beamon. Assume that Powell's speed on takeoff was 9.5 m/s (about equal to that of a sprinter) and that $g = 9.80 \text{ m/s}^2$ in Tokyo. How much less was Powell's range than the maximum possible range for a particle launched at the same speed?

•25 The current world-record motorcycle jump is 77.0 m, set by Jason Renie. Assume that he left the take-off ramp at 12.0° to the horizontal and that the take-off and landing heights are the same. Neglecting air drag, determine his take-off speed.

•26 A stone is catapulted at time $t = 0$, with an initial velocity of magnitude 20.0 m/s and at an angle of 40.0° above the horizontal. What are the magnitudes of the (a) horizontal and (b) vertical components of its displacement from the catapult site at $t = 1.10 \text{ s}$? Repeat for the (c) horizontal and (d) vertical components at $t = 1.80 \text{ s}$, and for the (e) horizontal and (f) vertical components at $t = 5.00 \text{ s}$.

••27 ILW A certain airplane has a speed of 290.0 km/h and is diving at an angle of $\theta = 30.0^\circ$ below the horizontal when the pilot releases a radar decoy (Fig. 4-33). The horizontal distance between the release point and the point where the decoy strikes the ground is $d = 700 \text{ m}$. (a) How long is the decoy in the air? (b) How high was the release point?

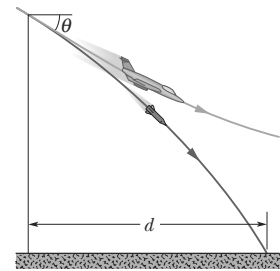


Figure 4-33 Problem 27.

••28 GO In Fig. 4-34, a stone is projected at a cliff of height h with an initial speed of 42.0 m/s directed at angle $\theta_0 = 60.0^\circ$ above the horizontal. The stone strikes at A , 5.50 s after launching. Find (a) the height h of the cliff, (b) the speed of the stone just before impact at A , and (c) the maximum height H reached above the ground.

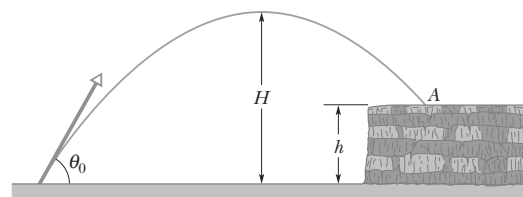


Figure 4-34 Problem 28.

••29 A projectile's launch speed is five times its speed at maximum height. Find launch angle θ_0 .

••30 GO A soccer ball is kicked from the ground with an initial speed of 19.5 m/s at an upward angle of 45° . A player 55 m away in the direction of the kick starts running to meet the ball at that instant. What must be his average speed if he is to meet the ball just before it hits the ground?

••31 In a jump spike, a volleyball player slams the ball from overhead and toward the opposite floor. Controlling the angle of the spike is difficult. Suppose a ball is spiked from a height of 2.30 m with an initial speed of 20.0 m/s at a downward angle of 18.00° . How much farther on the opposite floor would it have landed if the downward angle were, instead, 8.00° ?

••32 GO You throw a ball toward a wall at speed 25.0 m/s and at angle $\theta_0 = 40.0^\circ$ above the horizontal (Fig. 4-35). The wall is distance $d = 22.0 \text{ m}$ from the release point of the ball. (a) How far above the release point does the ball hit the wall? What are the (b) horizontal and (c) vertical components of its velocity as it hits the wall? (d) When it hits, has it passed the highest point on its trajectory?

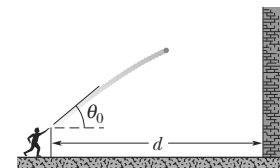


Figure 4-35 Problem 32.

••33 SSM A plane, diving with constant speed at an angle of 53.0° with the vertical, releases a projectile at an altitude of 730 m. The projectile hits the ground 5.00 s after release. (a) What is the speed of the plane? (b) How far does the projectile travel horizontally during its flight? What are the (c) horizontal and (d) vertical components of its velocity just before striking the ground?

••34 A trebuchet was a hurling machine built to attack the walls of a castle under siege. A large stone could be hurled against a wall to break apart the wall. The machine was not placed near the

wall because then arrows could reach it from the castle wall. Instead, it was positioned so that the stone hit the wall during the second half of its flight. Suppose a stone is launched with a speed of $v_0 = 28.0$ m/s and at an angle of $\theta_0 = 40.0^\circ$. What is the speed of the stone if it hits the wall (a) just as it reaches the top of its parabolic path and (b) when it has descended to half that height? (c) As a percentage, how much faster is it moving in part (b) than in part (a)?

••35 SSM A rifle that shoots bullets at 460 m/s is to be aimed at a target 45.7 m away. If the center of the target is level with the rifle, how high above the target must the rifle barrel be pointed so that the bullet hits dead center?

••36 GO During a tennis match, a player serves the ball at 23.6 m/s, with the center of the ball leaving the racquet horizontally 2.37 m above the court surface. The net is 12 m away and 0.90 m high. When the ball reaches the net, (a) does the ball clear it and (b) what is the distance between the center of the ball and the top of the net? Suppose that, instead, the ball is served as before but now it leaves the racquet at 5.00° below the horizontal. When the ball reaches the net, (c) does the ball clear it and (d) what now is the distance between the center of the ball and the top of the net?

••37 SSM WWW A lowly high diver pushes off horizontally with a speed of 2.00 m/s from the platform edge 10.0 m above the surface of the water. (a) At what horizontal distance from the edge is the diver 0.800 s after pushing off? (b) At what vertical distance above the surface of the water is the diver just then? (c) At what horizontal distance from the edge does the diver strike the water?

••38 A golf ball is struck at ground level. The speed of the golf ball as a function of the time is shown in Fig. 4-36, where $t = 0$ at the instant the ball is struck. The scaling on the vertical axis is set by $v_a = 19$ m/s and $v_b = 31$ m/s. (a) How far does the golf ball travel horizontally before returning to ground level? (b) What is the maximum height above ground level attained by the ball?

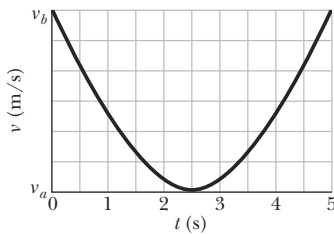


Figure 4-36 Problem 38.

••39 In Fig. 4-37, a ball is thrown leftward from the left edge of the roof, at height h above the ground. The ball hits the ground 1.50 s later, at distance $d = 25.0$ m from the building and at angle $\theta = 60.0^\circ$ with the horizontal. (a) Find h . (Hint: One way is to reverse the motion, as if on video.) What are the (b) magnitude and (c) angle relative to the horizontal of the velocity at which the ball is thrown? (d) Is the angle above or below the horizontal?

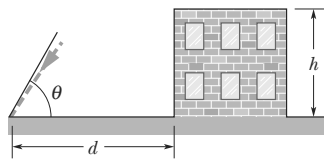


Figure 4-37 Problem 39.

••40 Suppose that a shot putter can put a shot at the world-class speed $v_0 = 15.00$ m/s and at a height of 2.160 m. What horizontal distance would the shot travel if the launch angle θ_0 is (a) 45.00° and (b) 42.00° ? The answers indicate that the angle of 45° , which maximizes the range of projectile motion, does not maximize the horizontal distance when the launch and landing are at different heights.

••41 GO Upon spotting an insect on a twig overhanging water, an archer fish squirts water drops at the insect to knock it into the water (Fig. 4-38). Although the fish sees the insect along a straight-line path at angle ϕ and distance d , a drop must be launched at a different angle θ_0 if its parabolic path is to intersect the insect. If $\phi = 36.0^\circ$ and $d = 0.900$ m, what launch angle θ_0 is required for the drop to be at the top of the parabolic path when it reaches the insect?

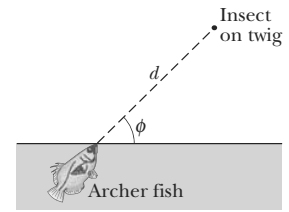


Figure 4-38 Problem 41.

••42 In 1939 or 1940, Emanuel Zacchini took his human-cannonball act to an extreme: After being shot from a cannon, he soared over three Ferris wheels and into a net (Fig. 4-39). Assume that he is launched with a speed of 26.5 m/s and at an angle of 53.0° . (a) Treating him as a particle, calculate his clearance over the first wheel. (b) If he reached maximum height over the middle wheel, by how much did he clear it? (c) How far from the cannon should the net's center have been positioned (neglect air drag)?

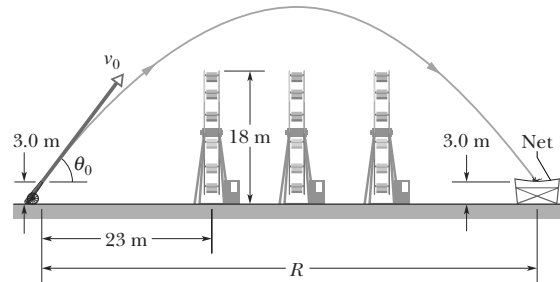


Figure 4-39 Problem 42.

••43 ILW A ball is shot from the ground into the air. At a height of 9.1 m, its velocity is $\vec{v} = (7.6\hat{i} + 6.1\hat{j})$ m/s, with \hat{i} horizontal and \hat{j} upward. (a) To what maximum height does the ball rise? (b) What total horizontal distance does the ball travel? What are the (c) magnitude and (d) angle (below the horizontal) of the ball's velocity just before it hits the ground?

••44 A baseball leaves a pitcher's hand horizontally at a speed of 161 km/h. The distance to the batter is 18.3 m. (a) How long does the ball take to travel the first half of that distance? (b) The second half? (c) How far does the ball fall freely during the first half? (d) During the second half? (e) Why aren't the quantities in (c) and (d) equal?

••45 In Fig. 4-40, a ball is launched with a velocity of magnitude 10.0 m/s, at an angle of 50.0° to the horizontal. The launch point is at the base of a ramp of horizontal length $d_1 = 6.00$ m and height $d_2 = 3.60$ m. A plateau is located at the top of the ramp. (a) Does the ball land on the ramp or the plateau? When it lands, what are the (b) magnitude and (c) angle of its displacement from the launch point?

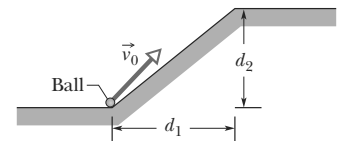


Figure 4-40 Problem 45.

••46 GO In basketball, *hang* is an illusion in which a player seems to weaken the gravitational acceleration while in midair. The illusion depends much on a skilled player's ability to rapidly shift

the ball between hands during the flight, but it might also be supported by the longer horizontal distance the player travels in the upper part of the jump than in the lower part. If a player jumps with an initial speed of $v_0 = 7.00$ m/s at an angle of $\theta_0 = 35.0^\circ$, what percent of the jump's range does the player spend in the upper half of the jump (between maximum height and half maximum height)?

••47 SSM WWW A batter hits a pitched ball when the center of the ball is 1.22 m above the ground. The ball leaves the bat at an angle of 45° with the ground. With that launch, the ball should have a horizontal range (returning to the launch level) of 107 m. (a) Does the ball clear a 7.32-m-high fence that is 97.5 m horizontally from the launch point? (b) At the fence, what is the distance between the fence top and the ball center?

••48 GO In Fig. 4-41, a ball is thrown up onto a roof, landing 4.00 s later at height $h = 20.0$ m above the release level. The ball's path just before landing is angled at $\theta = 60.0^\circ$ with the roof. (a) Find the horizontal distance d it travels. (See the hint to Problem 39.) What are the (b) magnitude and (c) angle (relative to the horizontal) of the ball's initial velocity?

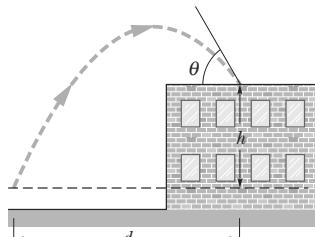


Figure 4-41 Problem 48.

••49 SSM A football kicker can give the ball an initial speed of 25 m/s. What are the (a) least and (b) greatest elevation angles at which he can kick the ball to score a field goal from a point 50 m in front of goalposts whose horizontal bar is 3.44 m above the ground?

••50 GO Two seconds after being projected from ground level, a projectile is displaced 40 m horizontally and 53 m vertically above its launch point. What are the (a) horizontal and (b) vertical components of the initial velocity of the projectile? (c) At the instant the projectile achieves its maximum height above ground level, how far is it displaced horizontally from the launch point?

••51 A skilled skier knows to jump upward before reaching a downward slope. Consider a jump in which the launch speed is $v_0 = 10$ m/s, the launch angle is $\theta_0 = 11.3^\circ$, the initial course is approximately flat, and the steeper track has a slope of 9.0° . Figure 4-42a shows a *prejump* that allows the skier to land on the top portion of the steeper track. Figure 4-42b shows a jump at the edge of the steeper track. In Fig. 4-42a, the skier lands at approximately the launch level. (a) In the landing, what is the angle ϕ between the skier's path and the slope? In Fig. 4-42b, (b) how far below the launch level does the skier land and (c) what is ϕ ? (The greater fall and greater ϕ can result in loss of control in the landing.)



Figure 4-42 Problem 51.

••52 A ball is to be shot from level ground toward a wall at distance x (Fig. 4-43a). Figure 4-43b shows the y component v_y of the ball's velocity just as it would reach the wall, as a function of that

distance x . The scaling is set by $v_{ys} = 5.0$ m/s and $x_s = 20$ m. What is the launch angle?

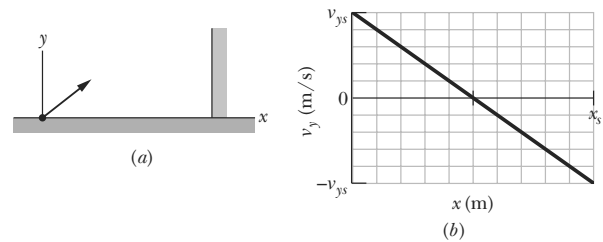


Figure 4-43 Problem 52.

••53 GO In Fig. 4-44, a baseball is hit at a height $h = 1.00$ m and then caught at the same height. It travels alongside a wall, moving up past the top of the wall 1.00 s after it is hit and then down past the top of the wall 4.00 s later, at distance $D = 50.0$ m farther along the wall. (a) What horizontal distance is traveled by the ball from hit to catch? What are the (b) magnitude and (c) angle (relative to the horizontal) of the ball's velocity just after being hit? (d) How high is the wall?

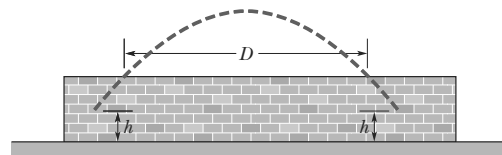


Figure 4-44 Problem 53.

••54 GO A ball is to be shot from level ground with a certain speed. Figure 4-45 shows the range R it will have versus the launch angle θ_0 . The value of θ_0 determines the flight time; let t_{\max} represent the maximum flight time. What is the least speed the ball will have during its flight if θ_0 is chosen such that the flight time is $0.500t_{\max}$?

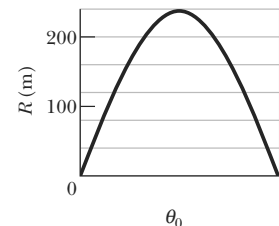


Figure 4-45 Problem 54.

••55 SSM A ball rolls horizontally off the top of a stairway with a speed of 1.52 m/s. The steps are 20.3 cm high and 20.3 cm wide. Which step does the ball hit first?

Module 4-5 Uniform Circular Motion

•56 An Earth satellite moves in a circular orbit 640 km (uniform circular motion) above Earth's surface with a period of 98.0 min. What are (a) the speed and (b) the magnitude of the centripetal acceleration of the satellite?

•57 A carnival merry-go-round rotates about a vertical axis at a constant rate. A man standing on the edge has a constant speed of 3.66 m/s and a centripetal acceleration \vec{a} of magnitude 1.83 m/s². Position vector \vec{r} locates him relative to the rotation axis. (a) What is the magnitude of \vec{r} ? What is the direction of \vec{r} when \vec{a} is directed (b) due east and (c) due south?

•58 A rotating fan completes 1200 revolutions every minute. Consider the tip of a blade, at a radius of 0.15 m. (a) Through what distance does the tip move in one revolution? What are (b) the

tip's speed and (c) the magnitude of its acceleration? (d) What is the period of the motion?

•59 **ILW** A woman rides a carnival Ferris wheel at radius 15 m, completing five turns about its horizontal axis every minute. What are (a) the period of the motion, the (b) magnitude and (c) direction of her centripetal acceleration at the highest point, and the (d) magnitude and (e) direction of her centripetal acceleration at the lowest point?

•60 A centripetal-acceleration addict rides in uniform circular motion with radius $r = 3.00$ m. At one instant his acceleration is $\vec{a} = (6.00 \text{ m/s}^2)\hat{i} + (-4.00 \text{ m/s}^2)\hat{j}$. At that instant, what are the values of (a) $\vec{v} \cdot \vec{a}$ and (b) $\vec{r} \times \vec{a}$?

•61 When a large star becomes a *supernova*, its core may be compressed so tightly that it becomes a *neutron star*, with a radius of about 20 km (about the size of the San Francisco area). If a neutron star rotates once every second, (a) what is the speed of a particle on the star's equator and (b) what is the magnitude of the particle's centripetal acceleration? (c) If the neutron star rotates faster, do the answers to (a) and (b) increase, decrease, or remain the same?

•62 What is the magnitude of the acceleration of a sprinter running at 10 m/s when rounding a turn of radius 25 m?

•63 **GO** At $t_1 = 2.00$ s, the acceleration of a particle in counter-clockwise circular motion is $(6.00 \text{ m/s}^2)\hat{i} + (4.00 \text{ m/s}^2)\hat{j}$. It moves at constant speed. At time $t_2 = 5.00$ s, the particle's acceleration is $(4.00 \text{ m/s}^2)\hat{i} + (-6.00 \text{ m/s}^2)\hat{j}$. What is the radius of the path taken by the particle if $t_2 - t_1$ is less than one period?

•64 **GO** A particle moves horizontally in uniform circular motion, over a horizontal xy plane. At one instant, it moves through the point at coordinates (4.00 m, 4.00 m) with a velocity of $-5.00\hat{i}$ m/s and an acceleration of $+12.5\hat{j}$ m/s². What are the (a) x and (b) y coordinates of the center of the circular path?

•65 A purse at radius 2.00 m and a wallet at radius 3.00 m travel in uniform circular motion on the floor of a merry-go-round as the ride turns. They are on the same radial line. At one instant, the acceleration of the purse is $(2.00 \text{ m/s}^2)\hat{i} + (4.00 \text{ m/s}^2)\hat{j}$. At that instant and in unit-vector notation, what is the acceleration of the wallet?

•66 A particle moves along a circular path over a horizontal xy coordinate system, at constant speed. At time $t_1 = 4.00$ s, it is at point (5.00 m, 6.00 m) with velocity $(3.00 \text{ m/s})\hat{j}$ and acceleration in the positive x direction. At time $t_2 = 10.0$ s, it has velocity $(-3.00 \text{ m/s})\hat{i}$ and acceleration in the positive y direction. What are the (a) x and (b) y coordinates of the center of the circular path if $t_2 - t_1$ is less than one period?

•67 **SSM WWW** A boy whirls a stone in a horizontal circle of radius 1.5 m and at height 2.0 m above level ground. The string breaks, and the stone flies off horizontally and strikes the ground after traveling a horizontal distance of 10 m. What is the magnitude of the centripetal acceleration of the stone during the circular motion?

•68 **GO** A cat rides a merry-go-round turning with uniform circular motion. At time $t_1 = 2.00$ s, the cat's velocity is $\vec{v}_1 = (3.00 \text{ m/s})\hat{i} + (4.00 \text{ m/s})\hat{j}$, measured on a horizontal xy coordinate system. At $t_2 = 5.00$ s, the cat's velocity is $\vec{v}_2 = (-3.00 \text{ m/s})\hat{i} + (-4.00 \text{ m/s})\hat{j}$. What are (a) the magnitude of the cat's centripetal acceleration and (b) the cat's average acceleration during the time interval $t_2 - t_1$, which is less than one period?

Module 4-6 Relative Motion in One Dimension

•69 A cameraman on a pickup truck is traveling westward at 20 km/h while he records a cheetah that is moving westward 30 km/h faster than the truck. Suddenly, the cheetah stops, turns, and then runs at 45 km/h eastward, as measured by a suddenly nervous crew member who stands alongside the cheetah's path. The change in the animal's velocity takes 2.0 s. What are the (a) magnitude and (b) direction of the animal's acceleration according to the cameraman and the (c) magnitude and (d) direction according to the nervous crew member?

•70 A boat is traveling upstream in the positive direction of an x axis at 14 km/h with respect to the water of a river. The water is flowing at 9.0 km/h with respect to the ground. What are the (a) magnitude and (b) direction of the boat's velocity with respect to the ground? A child on the boat walks from front to rear at 6.0 km/h with respect to the boat. What are the (c) magnitude and (d) direction of the child's velocity with respect to the ground?

•71 A suspicious-looking man runs as fast as he can along a moving sidewalk from one end to the other, taking 2.50 s. Then security agents appear, and the man runs as fast as he can back along the sidewalk to his starting point, taking 10.0 s. What is the ratio of the man's running speed to the sidewalk's speed?

Module 4-7 Relative Motion in Two Dimensions

•72 A rugby player runs with the ball directly toward his opponent's goal, along the positive direction of an x axis. He can legally pass the ball to a teammate as long as the ball's velocity relative to the field does not have a positive x component. Suppose the player runs at speed 4.0 m/s relative to the field while he passes the ball with velocity \vec{v}_{BP} relative to himself. If \vec{v}_{BP} has magnitude 6.0 m/s, what is the smallest angle it can have for the pass to be legal?

•73 Two highways intersect as shown in Fig. 4-46. At the instant shown, a police car P is distance $d_P = 800$ m from the intersection and moving at speed $v_P = 80$ km/h. Motorist M is distance $d_M = 600$ m from the intersection and moving at speed $v_M = 60$ km/h.

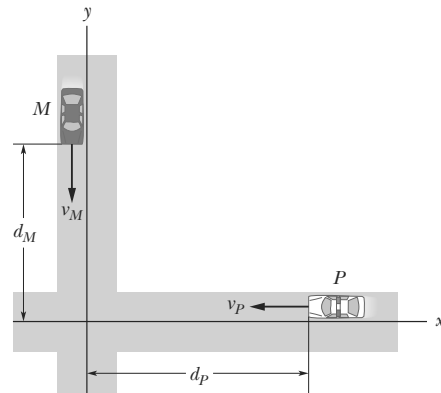


Figure 4-46 Problem 73.

(a) In unit-vector notation, what is the velocity of the motorist with respect to the police car? (b) For the instant shown in Fig. 4-46, what is the angle between the velocity found in (a) and the line of sight between the two cars? (c) If the cars maintain their velocities, do the answers to (a) and (b) change as the cars move nearer the intersection?

••74 After flying for 15 min in a wind blowing 42 km/h at an angle of 20° south of east, an airplane pilot is over a town that is 55 km due north of the starting point. What is the speed of the airplane relative to the air?

••75 **SSM** A train travels due south at 30 m/s (relative to the ground) in a rain that is blown toward the south by the wind. The path of each raindrop makes an angle of 70° with the vertical, as measured by an observer stationary on the ground. An observer on the train, however, sees the drops fall perfectly vertically. Determine the speed of the raindrops relative to the ground.

••76 A light plane attains an airspeed of 500 km/h. The pilot sets out for a destination 800 km due north but discovers that the plane must be headed 20.0° east of due north to fly there directly. The plane arrives in 2.00 h. What were the (a) magnitude and (b) direction of the wind velocity?

••77 **SSM** Snow is falling vertically at a constant speed of 8.0 m/s. At what angle from the vertical do the snowflakes appear to be falling as viewed by the driver of a car traveling on a straight, level road with a speed of 50 km/h?

••78 In the overhead view of Fig. 4-47, Jeeps P and B race along straight lines, across flat terrain, and past stationary border guard A . Relative to the guard, B travels at a constant speed of 20.0 m/s, at the angle $\theta_2 = 30.0^\circ$. Relative to the guard, P has accelerated from rest at a constant rate of 0.400 m/s^2 at the angle $\theta_1 = 60.0^\circ$. At a certain time during the acceleration, P has a speed of 40.0 m/s. At that time, what are the (a) magnitude and (b) direction of the velocity of P relative to B and the (c) magnitude and (d) direction of the acceleration of P relative to B ?

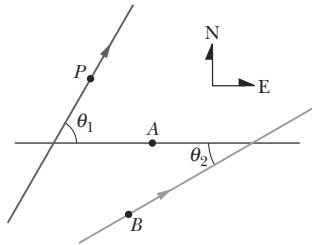


Figure 4-47 Problem 78.

••79 **SSM ILW** Two ships, A and B , leave port at the same time. Ship A travels northwest at 24 knots, and ship B travels at 28 knots in a direction 40° west of south. (1 knot = 1 nautical mile per hour; see Appendix D.) What are the (a) magnitude and (b) direction of the velocity of ship A relative to B ? (c) After what time will the ships be 160 nautical miles apart? (d) What will be the bearing of B (the direction of B 's position) relative to A at that time?

••80 **GO** A 200-m-wide river flows due east at a uniform speed of 2.0 m/s. A boat with a speed of 8.0 m/s relative to the water leaves the south bank pointed in a direction 30° west of north. What are the (a) magnitude and (b) direction of the boat's velocity relative to the ground? (c) How long does the boat take to cross the river?

••81 **GO** Ship A is located 4.0 km north and 2.5 km east of ship B . Ship A has a velocity of 22 km/h toward the south, and ship B has a velocity of 40 km/h in a direction 37° north of east. (a) What is the velocity of A relative to B in unit-vector notation with \hat{i} toward the east? (b) Write an expression (in terms of \hat{i} and \hat{j}) for the position of A relative to B as a function of t , where $t = 0$ when the ships are in the positions described above. (c) At what time is the separation between the ships least? (d) What is that least separation?

••82 **GO** A 200-m-wide river has a uniform flow speed of 1.1 m/s through a jungle and toward the east. An explorer wishes to

leave a small clearing on the south bank and cross the river in a powerboat that moves at a constant speed of 4.0 m/s with respect to the water. There is a clearing on the north bank 82 m upstream from a point directly opposite the clearing on the south bank. (a) In what direction must the boat be pointed in order to travel in a straight line and land in the clearing on the north bank? (b) How long will the boat take to cross the river and land in the clearing?

Additional Problems

83 A woman who can row a boat at 6.4 km/h in still water faces a long, straight river with a width of 6.4 km and a current of 3.2 km/h. Let \hat{i} point directly across the river and \hat{j} point directly downstream. If she rows in a straight line to a point directly opposite her starting position, (a) at what angle to \hat{i} must she point the boat and (b) how long will she take? (c) How long will she take if, instead, she rows 3.2 km down the river and then back to her starting point? (d) How long if she rows 3.2 km up the river and then back to her starting point? (e) At what angle to \hat{i} should she point the boat if she wants to cross the river in the shortest possible time? (f) How long is that shortest time?

84 In Fig. 4-48a, a sled moves in the negative x direction at constant speed v_s while a ball of ice is shot from the sled with a velocity $\vec{v}_0 = v_{0x}\hat{i} + v_{0y}\hat{j}$ relative to the sled. When the ball lands, its horizontal displacement Δx_{bg} relative to the ground (from its launch position to its landing position) is measured. Figure 4-48b gives Δx_{bg} as a function of v_s . Assume the ball lands at approximately its launch height. What are the values of (a) v_{0x} and (b) v_{0y} ? The ball's displacement Δx_{bs} relative to the sled can also be measured. Assume that the sled's velocity is not changed when the ball is shot. What is Δx_{bs} when v_s is (c) 5.0 m/s and (d) 15 m/s?

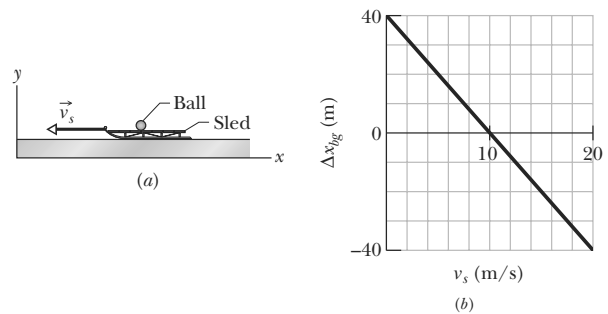


Figure 4-48 Problem 84.

85 You are kidnapped by political-science majors (who are upset because you told them political science is not a real science). Although blindfolded, you can tell the speed of their car (by the whine of the engine), the time of travel (by mentally counting off seconds), and the direction of travel (by turns along the rectangular street system). From these clues, you know that you are taken along the following course: 50 km/h for 2.0 min, turn 90° to the right, 20 km/h for 4.0 min, turn 90° to the right, 20 km/h for 60 s, turn 90° to the left, 50 km/h for 60 s, turn 90° to the right, 20 km/h for 2.0 min, turn 90° to the left, 50 km/h for 30 s. At that point, (a) how far are you from your starting point, and (b) in what direction relative to your initial direction of travel are you?

86 A radar station detects an airplane approaching directly from the east. At first observation, the airplane is at distance $d_1 = 360$ m from the station and at angle $\theta_1 = 40^\circ$ above the horizon (Fig. 4-49). The airplane is tracked through an angular change $\Delta\theta = 123^\circ$ in the vertical east–west plane; its distance is then $d_2 = 790$ m. Find the (a) magnitude and (b) direction of the airplane's displacement during this period.

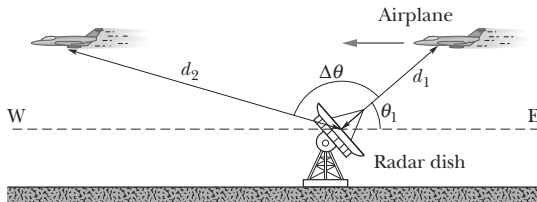


Figure 4-49 Problem 86.

87 SSM A baseball is hit at ground level. The ball reaches its maximum height above ground level 3.0 s after being hit. Then 2.5 s after reaching its maximum height, the ball barely clears a fence that is 97.5 m from where it was hit. Assume the ground is level. (a) What maximum height above ground level is reached by the ball? (b) How high is the fence? (c) How far beyond the fence does the ball strike the ground?

88 Long flights at midlatitudes in the Northern Hemisphere encounter the jet stream, an eastward airflow that can affect a plane's speed relative to Earth's surface. If a pilot maintains a certain speed relative to the air (the plane's *airspeed*), the speed relative to the surface (the plane's *ground speed*) is more when the flight is in the direction of the jet stream and less when the flight is opposite the jet stream. Suppose a round-trip flight is scheduled between two cities separated by 4000 km, with the outgoing flight in the direction of the jet stream and the return flight opposite it. The airline computer advises an airspeed of 1000 km/h, for which the difference in flight times for the outgoing and return flights is 70.0 min. What jet-stream speed is the computer using?

89 SSM A particle starts from the origin at $t = 0$ with a velocity of $8.0\hat{j}$ m/s and moves in the xy plane with constant acceleration $(4.0\hat{i} + 2.0\hat{j})$ m/s². When the particle's x coordinate is 29 m, what are its (a) y coordinate and (b) speed?

90 At what initial speed must the basketball player in Fig. 4-50 throw the ball, at angle $\theta_0 = 55^\circ$ above the horizontal, to make the foul shot? The horizontal distances are $d_1 = 1.0$ ft and $d_2 = 14$ ft, and the heights are $h_1 = 7.0$ ft and $h_2 = 10$ ft.

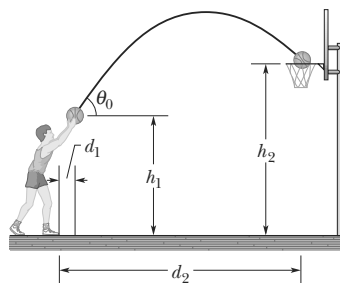


Figure 4-50 Problem 90.

91 During volcanic eruptions, chunks of solid rock can be blasted out of the volcano; these projectiles are called *volcanic bombs*. Figure 4-51 shows a cross section of Mt. Fuji, in Japan. (a) At what initial speed would a bomb have to be ejected, at angle $\theta_0 = 35^\circ$ to the horizontal, from the vent at A in order to fall at the foot of the volcano at B, at vertical distance $h = 3.30$ km and horizontal distance $d = 9.40$ km? Ignore, for the

moment, the effects of air on the bomb's travel. (b) What would be the time of flight? (c) Would the effect of the air increase or decrease your answer in (a)?

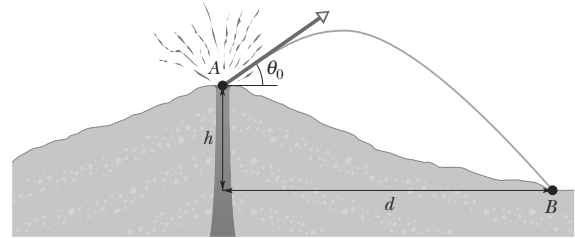


Figure 4-51 Problem 91.

92 An astronaut is rotated in a horizontal centrifuge at a radius of 5.0 m. (a) What is the astronaut's speed if the centripetal acceleration has a magnitude of $7.0g$? (b) How many revolutions per minute are required to produce this acceleration? (c) What is the period of the motion?

93 SSM Oasis A is 90 km due west of oasis B. A desert camel leaves A and takes 50 h to walk 75 km at 37° north of due east. Next it takes 35 h to walk 65 km due south. Then it rests for 5.0 h. What are the (a) magnitude and (b) direction of the camel's displacement relative to A at the resting point? From the time the camel leaves A until the end of the rest period, what are the (c) magnitude and (d) direction of its average velocity and (e) its average speed? The camel's last drink was at A; it must be at B no more than 120 h later for its next drink. If it is to reach B just in time, what must be the (f) magnitude and (g) direction of its average velocity after the rest period?

94 *Curtain of death.* A large metallic asteroid strikes Earth and quickly digs a crater into the rocky material below ground level by launching rocks upward and outward. The following table gives five pairs of launch speeds and angles (from the horizontal) for such rocks, based on a model of crater formation. (Other rocks, with intermediate speeds and angles, are also launched.) Suppose that you are at $x = 20$ km when the asteroid strikes the ground at time $t = 0$ and position $x = 0$ (Fig. 4-52). (a) At $t = 20$ s, what are the x and y coordinates of the rocks headed in your direction from launches A through E? (b) Plot these coordinates and then sketch a curve through the points to include rocks with intermediate launch speeds and angles. The curve should indicate what you would see as you look up into the approaching rocks.

Launch	Speed (m/s)	Angle (degrees)
A	520	14.0
B	630	16.0
C	750	18.0
D	870	20.0
E	1000	22.0

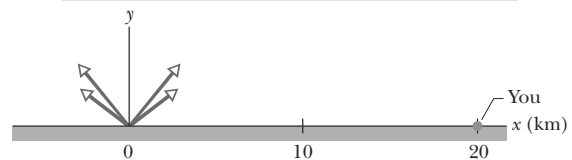


Figure 4-52 Problem 94.

95 Figure 4-53 shows the straight path of a particle across an xy coordinate system as the particle is accelerated from rest during time interval Δt_1 . The acceleration is constant. The xy coordinates for point A are (4.00 m, 6.00 m); those for point B are (12.0 m, 18.0 m). (a) What is the ratio a_y/a_x of the acceleration components? (b) What are the coordinates of the particle if the motion is continued for another interval equal to Δt_1 ?

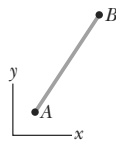


Figure 4-53
Problem 95.

96 For women's volleyball the top of the net is 2.24 m above the floor and the court measures 9.0 m by 9.0 m on each side of the net. Using a jump serve, a player strikes the ball at a point that is 3.0 m above the floor and a horizontal distance of 8.0 m from the net. If the initial velocity of the ball is horizontal, (a) what minimum magnitude must it have if the ball is to clear the net and (b) what maximum magnitude can it have if the ball is to strike the floor inside the back line on the other side of the net?

97 SSM A rifle is aimed horizontally at a target 30 m away. The bullet hits the target 1.9 cm below the aiming point. What are (a) the bullet's time of flight and (b) its speed as it emerges from the rifle?

98 A particle is in uniform circular motion about the origin of an xy coordinate system, moving clockwise with a period of 7.00 s. At one instant, its position vector (measured from the origin) is $\vec{r} = (2.00 \text{ m})\hat{i} - (3.00 \text{ m})\hat{j}$. At that instant, what is its velocity in unit-vector notation?

99 In Fig. 4-54, a lump of wet putty moves in uniform circular motion as it rides at a radius of 20.0 cm on the rim of a wheel rotating counterclockwise with a period of 5.00 ms. The lump then happens to fly off the rim at the 5 o'clock position (as if on a clock face). It leaves the rim at a height of $h = 1.20$ m from the floor and at a distance $d = 2.50$ m from a wall. At what height on the wall does the lump hit?

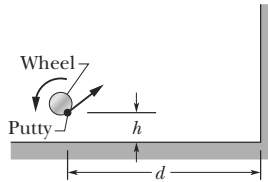


Figure 4-54 Problem 99.

100 An iceboat sails across the surface of a frozen lake with constant acceleration produced by the wind. At a certain instant the boat's velocity is $(6.30\hat{i} - 8.42\hat{j})$ m/s. Three seconds later, because of a wind shift, the boat is instantaneously at rest. What is its average acceleration for this 3.00 s interval?

101 In Fig. 4-55, a ball is shot directly upward from the ground with an initial speed of $v_0 = 7.00$ m/s. Simultaneously, a construction elevator cab begins to move upward from the ground with a constant speed of $v_c = 3.00$ m/s. What maximum height does the ball reach relative to (a) the ground and (b) the cab floor? At what rate does the speed of the ball change relative to (c) the ground and (d) the cab floor?

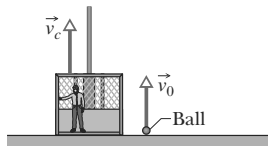


Figure 4-55 Problem 101.

102 A magnetic field forces an electron to move in a circle with radial acceleration 3.0×10^{14} m/s². (a) What is the speed of the electron if the radius of its circular path is 15 cm? (b) What is the period of the motion?

103 In 3.50 h, a balloon drifts 21.5 km north, 9.70 km east, and 2.88 km upward from its release point on the ground. Find (a) the magnitude of its average velocity and (b) the angle its average velocity makes with the horizontal.

104 A ball is thrown horizontally from a height of 20 m and hits the ground with a speed that is three times its initial speed. What is the initial speed?

105 A projectile is launched with an initial speed of 30 m/s at an angle of 60° above the horizontal. What are the (a) magnitude and (b) angle of its velocity 2.0 s after launch, and (c) is the angle above or below the horizontal? What are the (d) magnitude and (e) angle of its velocity 5.0 s after launch, and (f) is the angle above or below the horizontal?

106 The position vector for a proton is initially $\vec{r} = 5.0\hat{i} - 6.0\hat{j} + 2.0\hat{k}$ and then later is $\vec{r} = -2.0\hat{i} + 6.0\hat{j} + 2.0\hat{k}$, all in meters. (a) What is the proton's displacement vector, and (b) to what plane is that vector parallel?

107 A particle P travels with constant speed on a circle of radius $r = 3.00$ m (Fig. 4-56) and completes one revolution in 20.0 s. The particle passes through O at time $t = 0$. State the following vectors in magnitude-angle notation (angle relative to the positive direction of x). With respect to O , find the particle's position vector at the times t of (a) 5.00 s, (b) 7.50 s, and (c) 10.0 s. (d) For the 5.00 s interval from the end of the fifth second to the end of the tenth second, find the particle's displacement. For that interval, find (e) its average velocity and its velocity at the (f) beginning and (g) end. Next, find the acceleration at the (h) beginning and (i) end of that interval.

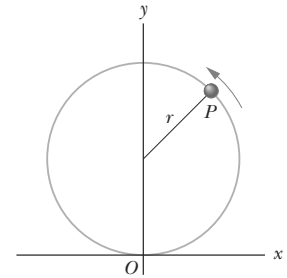


Figure 4-56 Problem 107.

108 The fast French train known as the TGV (Train à Grande Vitesse) has a scheduled average speed of 216 km/h. (a) If the train goes around a curve at that speed and the magnitude of the acceleration experienced by the passengers is to be limited to $0.050g$, what is the smallest radius of curvature for the track that can be tolerated? (b) At what speed must the train go around a curve with a 1.00 km radius to be at the acceleration limit?

109 (a) If an electron is projected horizontally with a speed of 3.0×10^6 m/s, how far will it fall in traversing 1.0 m of horizontal distance? (b) Does the answer increase or decrease if the initial speed is increased?

110 A person walks up a stalled 15-m-long escalator in 90 s. When standing on the same escalator, now moving, the person is carried up in 60 s. How much time would it take that person to walk up the moving escalator? Does the answer depend on the length of the escalator?

111 (a) What is the magnitude of the centripetal acceleration of an object on Earth's equator due to the rotation of Earth? (b) What would Earth's rotation period have to be for objects on the equator to have a centripetal acceleration of magnitude 9.8 m/s^2 ?

112 The range of a projectile depends not only on v_0 and θ_0 but also on the value g of the free-fall acceleration, which varies from place to place. In 1936, Jesse Owens established a world's running broad jump record of 8.09 m at the Olympic Games at Berlin (where $g = 9.8128 \text{ m/s}^2$). Assuming the same values of v_0 and θ_0 , by how much would his record have differed if he had competed instead in 1956 at Melbourne (where $g = 9.7999 \text{ m/s}^2$)?

113 Figure 4-57 shows the path taken by a drunk skunk over level ground, from initial point i to final point f . The angles are $\theta_1 = 30.0^\circ$, $\theta_2 = 50.0^\circ$, and $\theta_3 = 80.0^\circ$, and the distances are $d_1 = 5.00$ m, $d_2 = 8.00$ m, and $d_3 = 12.0$ m. What are the (a) magnitude and (b) angle of the skunk's displacement from i to f ?

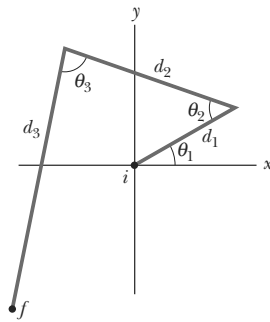


Figure 4-57 Problem 113.

114 The position vector \vec{r} of a particle moving in the xy plane is $\vec{r} = 2t\hat{i} + 2\sin[(\pi/4 \text{ rad/s})t]\hat{j}$, with \vec{r} in meters and t in seconds. (a) Calculate the x and y components of the particle's position at $t = 0, 1.0, 2.0, 3.0$, and 4.0 s and sketch the particle's path in the xy plane for the interval $0 \leq t \leq 4.0$ s. (b) Calculate the components of the particle's velocity at $t = 1.0, 2.0$, and 3.0 s. Show that the velocity is tangent to the path of the particle and in the direction the particle is moving at each time by drawing the velocity vectors on the plot of the particle's path in part (a). (c) Calculate the components of the particle's acceleration at $t = 1.0, 2.0$, and 3.0 s.

115 An electron having an initial horizontal velocity of magnitude 1.00×10^9 cm/s travels into the region between two horizontal metal plates that are electrically charged. In that region, the electron travels a horizontal distance of 2.00 cm and has a constant downward acceleration of magnitude 1.00×10^{17} cm/s² due to the charged plates. Find (a) the time the electron takes to travel the 2.00 cm, (b) the vertical distance it travels during that time, and the magnitudes of its (c) horizontal and (d) vertical velocity components as it emerges from the region.

116 An elevator without a ceiling is ascending with a constant speed of 10 m/s. A boy on the elevator shoots a ball directly upward, from a height of 2.0 m above the elevator floor, just as the elevator floor is 28 m above the ground. The initial speed of the ball with respect to the elevator is 20 m/s. (a) What maximum height above the ground does the ball reach? (b) How long does the ball take to return to the elevator floor?

117 A football player punts the football so that it will have a "hang time" (time of flight) of 4.5 s and land 46 m away. If the ball leaves the player's foot 150 cm above the ground, what must be the (a) magnitude and (b) angle (relative to the horizontal) of the ball's initial velocity?

118 An airport terminal has a moving sidewalk to speed passengers through a long corridor. Larry does not use the moving sidewalk; he takes 150 s to walk through the corridor. Curly, who simply stands on the moving sidewalk, covers the same distance in 70 s. Moe boards the sidewalk and walks along it. How long does Moe take to move through the corridor? Assume that Larry and Moe walk at the same speed.

119 A wooden boxcar is moving along a straight railroad track at speed v_1 . A sniper fires a bullet (initial speed v_2) at it from a high-powered rifle. The bullet passes through both lengthwise walls of the car, its entrance and exit holes being exactly opposite each other as viewed from within the car. From what direction, relative to the track, is the bullet fired? Assume that the bullet is not deflected upon entering the car, but that its speed decreases by 20% . Take $v_1 = 85$ km/h and $v_2 = 650$ m/s. (Why don't you need to know the width of the boxcar?)

120 A sprinter running on a circular track has a velocity of constant magnitude 9.20 m/s and a centripetal acceleration of magnitude 3.80 m/s². What are (a) the track radius and (b) the period of the circular motion?

121 Suppose that a space probe can withstand the stresses of a $20g$ acceleration. (a) What is the minimum turning radius of such a craft moving at a speed of one-tenth the speed of light? (b) How long would it take to complete a 90° turn at this speed?

122 You are to throw a ball with a speed of 12.0 m/s at a target that is height $h = 5.00$ m above the level at which you release the ball (Fig. 4-58). You want the ball's velocity to be horizontal at the instant it reaches the target. (a) At what angle θ above the horizontal must you throw the ball? (b) What is the horizontal distance from the release point to the target? (c) What is the speed of the ball just as it reaches the target?

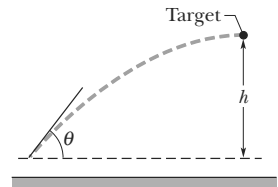


Figure 4-58 Problem 122.

123 A projectile is fired with an initial speed $v_0 = 30.0$ m/s from level ground at a target that is on the ground, at distance $R = 20.0$ m, as shown in Fig. 4-59. What are the (a) least and (b) greatest launch angles that will allow the projectile to hit the target?

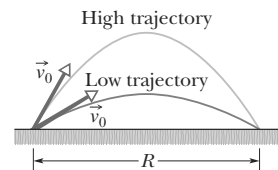


Figure 4-59 Problem 123.

124 *A graphing surprise.* At time $t = 0$, a burrito is launched from level ground, with an initial speed of 16.0 m/s and launch angle θ_0 . Imagine a position vector \vec{r} continuously directed from the launching point to the burrito during the flight. Graph the magnitude r of the position vector for (a) $\theta_0 = 40.0^\circ$ and (b) $\theta_0 = 80.0^\circ$. For $\theta_0 = 40.0^\circ$, (c) when does r reach its maximum value, (d) what is that value, and how far (e) horizontally and (f) vertically is the burrito from the launch point? For $\theta_0 = 80.0^\circ$, (g) when does r reach its maximum value, (h) what is that value, and how far (i) horizontally and (j) vertically is the burrito from the launch point?

125 A cannon located at sea level fires a ball with initial speed 82 m/s and initial angle 45° . The ball lands in the water after traveling a horizontal distance 686 m. How much greater would the horizontal distance have been had the cannon been 30 m higher?

126 The magnitude of the velocity of a projectile when it is at its maximum height above ground level is 10.0 m/s. (a) What is the magnitude of the velocity of the projectile 1.00 s before it achieves its maximum height? (b) What is the magnitude of the velocity of the projectile 1.00 s after it achieves its maximum height? If we take $x = 0$ and $y = 0$ to be at the point of maximum height and positive x to be in the direction of the velocity there, what are the (c) x coordinate and (d) y coordinate of the projectile 1.00 s before it reaches its maximum height and the (e) x coordinate and (f) y coordinate 1.0 s after it reaches its maximum height?

127 A frightened rabbit moving at 6.00 m/s due east runs onto a large area of level ice of negligible friction. As the rabbit slides across the ice, the force of the wind causes it to have a constant acceleration of 1.40 m/s², due north. Choose a coordinate system with the origin at the rabbit's initial position on the ice and the positive x axis directed toward the east. In unit-vector notation, what are the rabbit's (a) velocity and (b) position when it has slid for 3.00 s?

128 The pilot of an aircraft flies due east relative to the ground in a wind blowing 20.0 km/h toward the south. If the speed of the aircraft in the absence of wind is 70.0 km/h, what is the speed of the aircraft relative to the ground?

129 The pitcher in a slow-pitch softball game releases the ball at a point 3.0 ft above ground level. A stroboscopic plot of the position of the ball is shown in Fig. 4-60, where the readings are 0.25 s apart and the ball is released at $t = 0$. (a) What is the initial speed of the ball? (b) What is the speed of the ball at the instant it reaches its maximum height above ground level? (c) What is that maximum height?

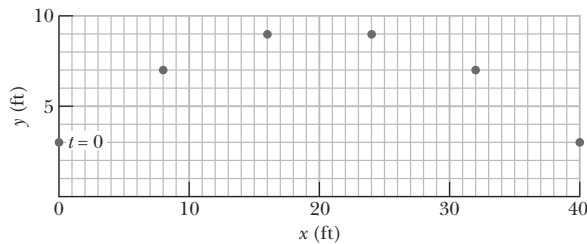


Figure 4-60 Problem 129.

130 Some state trooper departments use aircraft to enforce highway speed limits. Suppose that one of the airplanes has a speed of 135 mi/h in still air. It is flying straight north so that it is at all times directly above a north-south highway. A ground observer tells the pilot by radio that a 70.0 mi/h wind is blowing but neglects to give the wind direction. The pilot observes that in spite of the wind the plane can travel 135 mi along the highway in 1.00 h. In other words, the ground speed is the same as if there were no wind. (a) From what direction is the wind blowing? (b) What is the heading of the plane; that is, in what direction does it point?

131 A golfer tees off from the top of a rise, giving the golf ball an initial velocity of 43.0 m/s at an angle of 30.0° above the horizontal. The ball strikes the fairway a horizontal distance of 180 m from the tee. Assume the fairway is level. (a) How high is the rise above the fairway? (b) What is the speed of the ball as it strikes the fairway?

132 A track meet is held on a planet in a distant solar system. A shot-putter releases a shot at a point 2.0 m above ground level. A stroboscopic plot of the position of the shot is shown in Fig. 4-61,

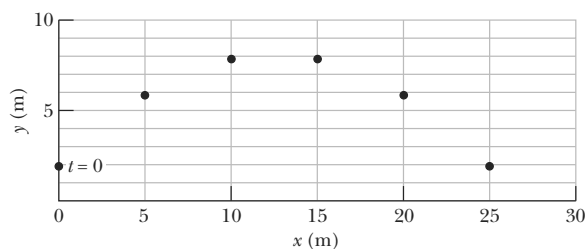


Figure 4-61 Problem 132.

where the readings are 0.50 s apart and the shot is released at time $t = 0$. (a) What is the initial velocity of the shot in unit-vector notation? (b) What is the magnitude of the free-fall acceleration on the planet? (c) How long after it is released does the shot reach the ground? (d) If an identical throw of the shot is made on the surface of Earth, how long after it is released does it reach the ground?

133 A helicopter is flying in a straight line over a level field at a constant speed of 6.20 m/s and at a constant altitude of 9.50 m. A package is ejected horizontally from the helicopter with an initial velocity of 12.0 m/s relative to the helicopter and in a direction opposite the helicopter's motion. (a) Find the initial speed of the package relative to the ground. (b) What is the horizontal distance between the helicopter and the package at the instant the package strikes the ground? (c) What angle does the velocity vector of the package make with the ground at the instant before impact, as seen from the ground?

134 A car travels around a flat circle on the ground, at a constant speed of 12.0 m/s. At a certain instant the car has an acceleration of 3.00 m/s^2 toward the east. What are its distance and direction from the center of the circle at that instant if it is traveling (a) clockwise around the circle and (b) counterclockwise around the circle?

135 You throw a ball from a cliff with an initial velocity of 15.0 m/s at an angle of 20.0° below the horizontal. Find (a) its horizontal displacement and (b) its vertical displacement 2.30 s later.

136 A baseball is hit at Fenway Park in Boston at a point 0.762 m above home plate with an initial velocity of 33.53 m/s directed 55.0° above the horizontal. The ball is observed to clear the 11.28-m-high wall in left field (known as the "green monster") 5.00 s after it is hit, at a point just inside the left-field foul-line pole. Find (a) the horizontal distance down the left-field foul line from home plate to the wall; (b) the vertical distance by which the ball clears the wall; (c) the horizontal and vertical displacements of the ball with respect to home plate 0.500 s before it clears the wall.

137 A transcontinental flight of 4350 km is scheduled to take 50 min longer westward than eastward. The airspeed of the airplane is 966 km/h, and the jet stream it will fly through is presumed to move due east. What is the assumed speed of the jet stream?

138 A woman can row a boat at 6.40 km/h in still water. (a) If she is crossing a river where the current is 3.20 km/h, in what direction must her boat be headed if she wants to reach a point directly opposite her starting point? (b) If the river is 6.40 km wide, how long will she take to cross the river? (c) Suppose that instead of crossing the river she rows 3.20 km down the river and then back to her starting point. How long will she take? (d) How long will she take to row 3.20 km up the river and then back to her starting point? (e) In what direction should she head the boat if she wants to cross in the shortest possible time, and what is that time?

C H A P T E R 5

Force and Motion–I

5-1 NEWTON'S FIRST AND SECOND LAWS

Learning Objectives

After reading this module, you should be able to . . .

- 5.01 Identify that a force is a vector quantity and thus has both magnitude and direction and also components.
- 5.02 Given two or more forces acting on the same particle, add the forces as *vectors* to get the net force.
- 5.03 Identify Newton's first and second laws of motion.
- 5.04 Identify inertial reference frames.
- 5.05 Sketch a free-body diagram for an object, showing the object as a particle and drawing the forces acting on it as vectors with their tails anchored on the particle.
- 5.06 Apply the relationship (Newton's second law) between the net force on an object, the mass of the object, and the acceleration produced by the net force.
- 5.07 Identify that only *external* forces on an object can cause the object to accelerate.

Key Ideas

- The velocity of an object can change (the object can accelerate) when the object is acted on by one or more forces (pushes or pulls) from other objects. Newtonian mechanics relates accelerations and forces.
- Forces are vector quantities. Their magnitudes are defined in terms of the acceleration they would give the standard kilogram. A force that accelerates that standard body by exactly 1 m/s^2 is defined to have a magnitude of 1 N. The direction of a force is the direction of the acceleration it causes. Forces are combined according to the rules of vector algebra. The net force on a body is the vector sum of all the forces acting on the body.
- If there is no net force on a body, the body remains at rest if it is initially at rest or moves in a straight line at constant speed if it is in motion.
- Reference frames in which Newtonian mechanics holds are called inertial reference frames or inertial frames. Reference frames in which Newtonian mechanics does not hold are called noninertial reference frames or noninertial frames.
- The mass of a body is the characteristic of that body that relates the body's acceleration to the net force causing the acceleration. Masses are scalar quantities.
- The net force \vec{F}_{net} on a body with mass m is related to the body's acceleration \vec{a} by

$$\vec{F}_{\text{net}} = m\vec{a},$$
 which may be written in the component versions

$$F_{\text{net},x} = ma_x \quad F_{\text{net},y} = ma_y \quad \text{and} \quad F_{\text{net},z} = ma_z.$$
 The second law indicates that in SI units

$$1 \text{ N} = 1 \text{ kg} \cdot \text{m/s}^2.$$
- A free-body diagram is a stripped-down diagram in which only *one* body is considered. That body is represented by either a sketch or a dot. The external forces on the body are drawn, and a coordinate system is superimposed, oriented so as to simplify the solution.

What Is Physics?

We have seen that part of physics is a study of motion, including accelerations, which are changes in velocities. Physics is also a study of what can *cause* an object to accelerate. That cause is a **force**, which is, loosely speaking, a push or pull on the object. The force is said to *act* on the object to change its velocity. For example, when a dragster accelerates, a force from the track acts on the rear tires to cause the dragster's acceleration. When a defensive guard knocks down a quarterback, a force from the guard acts on the quarterback to cause the quarterback's backward acceleration. When a car slams into a telephone pole, a force on the car from the

pole causes the car to stop. Science, engineering, legal, and medical journals are filled with articles about forces on objects, including people.

A Heads Up. Many students find this chapter to be more challenging than the preceding ones. One reason is that we need to use vectors in setting up equations—we cannot just sum some scalars. So, we need the vector rules from Chapter 3. Another reason is that we shall see a lot of different arrangements: objects will move along floors, ceilings, walls, and ramps. They will move upward on ropes looped around pulleys or by sitting in ascending or descending elevators. Sometimes, objects will even be tied together.

However, in spite of the variety of arrangements, we need only a single key idea (Newton's second law) to solve most of the homework problems. The purpose of this chapter is for us to explore how we can apply that single key idea to any given arrangement. The application will take experience—we need to solve lots of problems, not just read words. So, let's go through some of the words and then get to the sample problems.

Newtonian Mechanics

The relation between a force and the acceleration it causes was first understood by Isaac Newton (1642–1727) and is the subject of this chapter. The study of that relation, as Newton presented it, is called *Newtonian mechanics*. We shall focus on its three primary laws of motion.

Newtonian mechanics does not apply to all situations. If the speeds of the interacting bodies are very large—an appreciable fraction of the speed of light—we must replace Newtonian mechanics with Einstein's special theory of relativity, which holds at any speed, including those near the speed of light. If the interacting bodies are on the scale of atomic structure (for example, they might be electrons in an atom), we must replace Newtonian mechanics with quantum mechanics. Physicists now view Newtonian mechanics as a special case of these two more comprehensive theories. Still, it is a very important special case because it applies to the motion of objects ranging in size from the very small (almost on the scale of atomic structure) to astronomical (galaxies and clusters of galaxies).

Newton's First Law

Before Newton formulated his mechanics, it was thought that some influence, a “force,” was needed to keep a body moving at constant velocity. Similarly, a body was thought to be in its “natural state” when it was at rest. For a body to move with constant velocity, it seemingly had to be propelled in some way, by a push or a pull. Otherwise, it would “naturally” stop moving.

These ideas were reasonable. If you send a puck sliding across a wooden floor, it does indeed slow and then stop. If you want to make it move across the floor with constant velocity, you have to continuously pull or push it.

Send a puck sliding over the ice of a skating rink, however, and it goes a lot farther. You can imagine longer and more slippery surfaces, over which the puck would slide farther and farther. In the limit you can think of a long, extremely slippery surface (said to be a **frictionless surface**), over which the puck would hardly slow. (We can in fact come close to this situation by sending a puck sliding over a horizontal air table, across which it moves on a film of air.)

From these observations, we can conclude that a body will keep moving with constant velocity if no force acts on it. That leads us to the first of Newton's three laws of motion:



Newton's First Law: If no force acts on a body, the body's velocity cannot change; that is, the body cannot accelerate.

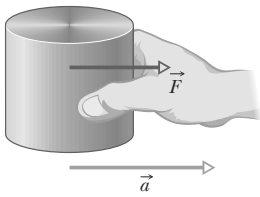


Figure 5-1 A force \vec{F} on the standard kilogram gives that body an acceleration \vec{a} .

In other words, if the body is at rest, it stays at rest. If it is moving, it continues to move with the same velocity (same magnitude *and* same direction).

Force

Before we begin working problems with forces, we need to discuss several features of forces, such as the force unit, the vector nature of forces, the combining of forces, and the circumstances in which we can measure forces (without being fooled by a fictitious force).

Unit. We can define the unit of force in terms of the acceleration a force would give to the standard kilogram (Fig. 1-3), which has a mass defined to be exactly 1 kg. Suppose we put that body on a horizontal, frictionless surface and pull horizontally (Fig. 5-1) such that the body has an acceleration of 1 m/s^2 . Then we can define our applied force as having a magnitude of 1 newton (abbreviated N). If we then pulled with a force magnitude of 2 N, we would find that the acceleration is 2 m/s^2 . Thus, the acceleration is proportional to the force. If the standard body of 1 kg has an acceleration of magnitude a (in meters per second per second), then the force (in newtons) producing the acceleration has a magnitude equal to a . We now have a workable definition of the force unit.

Vectors. Force is a vector quantity and thus has not only magnitude but also direction. So, if two or more forces act on a body, we find the **net force** (or **resultant force**) by adding them as vectors, following the rules of Chapter 3. A single force that has the same magnitude and direction as the calculated net force would then have the same effect as all the individual forces. This fact, called the **principle of superposition for forces**, makes everyday forces reasonable and predictable. The world would indeed be strange and unpredictable if, say, you and a friend each pulled on the standard body with a force of 1 N and somehow the net pull was 14 N and the resulting acceleration was 14 m/s^2 .

In this book, forces are most often represented with a vector symbol such as \vec{F} , and a net force is represented with the vector symbol \vec{F}_{net} . As with other vectors, a force or a net force can have components along coordinate axes. When forces act only along a single axis, they are single-component forces. Then we can drop the overhead arrows on the force symbols and just use signs to indicate the directions of the forces along that axis.

The First Law. Instead of our previous wording, the more proper statement of Newton's First Law is in terms of a *net* force:



Newton's First Law: If no *net* force acts on a body ($\vec{F}_{\text{net}} = 0$), the body's velocity cannot change; that is, the body cannot accelerate.

There may be multiple forces acting on a body, but if their net force is zero, the body cannot accelerate. So, if we happen to know that a body's velocity is constant, we can immediately say that the net force on it is zero.

Inertial Reference Frames

Newton's first law is not true in all reference frames, but we can always find reference frames in which it (as well as the rest of Newtonian mechanics) is true. Such special frames are referred to as **inertial reference frames**, or simply **inertial frames**.



An inertial reference frame is one in which Newton's laws hold.

For example, we can assume that the ground is an inertial frame provided we can neglect Earth's astronomical motions (such as its rotation).

That assumption works well if, say, a puck is sent sliding along a *short* strip of frictionless ice—we would find that the puck's motion obeys Newton's laws. However, suppose the puck is sent sliding along a *long* ice strip extending from the north pole (Fig. 5-2a). If we view the puck from a stationary frame in space, the puck moves south along a simple straight line because Earth's rotation around the north pole merely slides the ice beneath the puck. However, if we view the puck from a point on the ground so that we rotate with Earth, the puck's path is not a simple straight line. Because the eastward speed of the ground beneath the puck is greater the farther south the puck slides, from our ground-based view the puck appears to be deflected westward (Fig. 5-2b). However, this apparent deflection is caused not by a force as required by Newton's laws but by the fact that we see the puck from a rotating frame. In this situation, the ground is a **noninertial frame**, and trying to explain the deflection in terms of a force would lead us to a fictitious force. A more common example of inventing such a nonexistent force can occur in a car that is rapidly increasing in speed. You might claim that a force to the rear shoves you hard into the seat back.

In this book we usually assume that the ground is an inertial frame and that measured forces and accelerations are from this frame. If measurements are made in, say, a vehicle that is accelerating relative to the ground, then the measurements are being made in a noninertial frame and the results can be surprising.

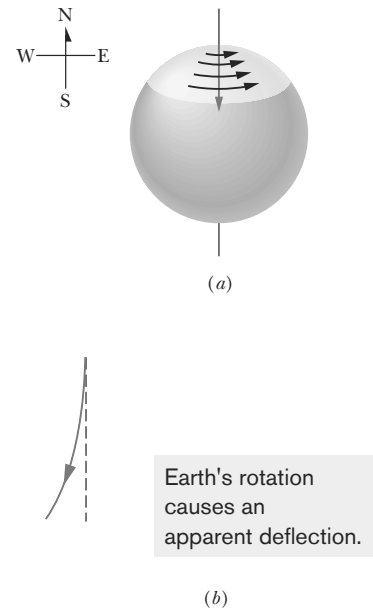
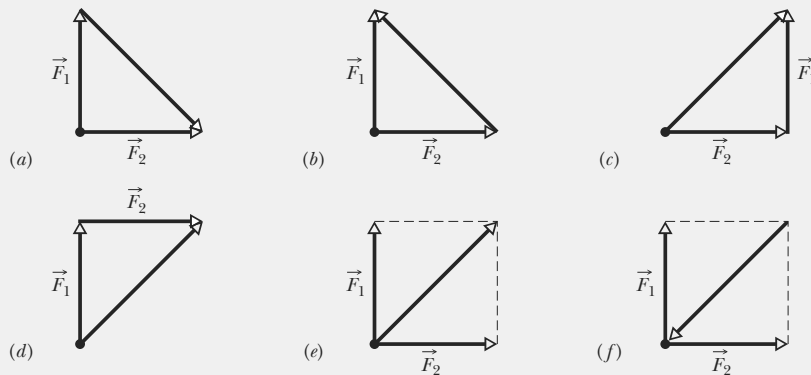


Figure 5-2 (a) The path of a puck sliding from the north pole as seen from a stationary point in space. Earth rotates to the east. (b) The path of the puck as seen from the ground.



Checkpoint 1

Which of the figure's six arrangements correctly show the vector addition of forces \vec{F}_1 and \vec{F}_2 to yield the third vector, which is meant to represent their net force \vec{F}_{net} ?



Mass

From everyday experience you already know that applying a given force to bodies (say, a baseball and a bowling ball) results in different accelerations. The common explanation is correct: The object with the larger mass is accelerated less. But we can be more precise. The acceleration is actually inversely related to the mass (rather than, say, the square of the mass).

Let's justify that inverse relationship. Suppose, as previously, we push on the standard body (defined to have a mass of exactly 1 kg) with a force of magnitude 1 N. The body accelerates with a magnitude of 1 m/s². Next we push on body *X* with the same force and find that it accelerates at 0.25 m/s². Let's make the (correct) assumption that with the same force,

$$\frac{m_X}{m_0} = \frac{a_0}{a_X},$$

and thus

$$m_X = m_0 \frac{a_0}{a_X} = (1.0 \text{ kg}) \frac{1.0 \text{ m/s}^2}{0.25 \text{ m/s}^2} = 4.0 \text{ kg}.$$

Defining the mass of X in this way is useful only if the procedure is consistent. Suppose we apply an 8.0 N force first to the standard body (getting an acceleration of 8.0 m/s²) and then to body X (getting an acceleration of 2.0 m/s²). We would then calculate the mass of X as

$$m_X = m_0 \frac{a_0}{a_X} = (1.0 \text{ kg}) \frac{8.0 \text{ m/s}^2}{2.0 \text{ m/s}^2} = 4.0 \text{ kg},$$

which means that our procedure is consistent and thus usable.

The results also suggest that mass is an intrinsic characteristic of a body—it automatically comes with the existence of the body. Also, it is a scalar quantity. However, the nagging question remains: What, exactly, is mass?

Since the word *mass* is used in everyday English, we should have some intuitive understanding of it, maybe something that we can physically sense. Is it a body's size, weight, or density? The answer is no, although those characteristics are sometimes confused with mass. We can say only that *the mass of a body is the characteristic that relates a force on the body to the resulting acceleration*. Mass has no more familiar definition; you can have a physical sensation of mass only when you try to accelerate a body, as in the kicking of a baseball or a bowling ball.

Newton's Second Law

All the definitions, experiments, and observations we have discussed so far can be summarized in one neat statement:



Newton's Second Law: The net force on a body is equal to the product of the body's mass and its acceleration.

In equation form,

$$\vec{F}_{\text{net}} = m\vec{a} \quad (\text{Newton's second law}). \quad (5-1)$$

Identify the Body. This simple equation is the key idea for nearly all the homework problems in this chapter, but we must use it cautiously. First, we must be certain about which body we are applying it to. Then \vec{F}_{net} must be the vector sum of *all* the forces that act on *that* body. Only forces that act on *that* body are to be included in the vector sum, not forces acting on other bodies that might be involved in the given situation. For example, if you are in a rugby scrum, the net force on *you* is the vector sum of all the pushes and pulls on *your* body. It does not include any push or pull on another player from you or from anyone else. Every time you work a force problem, your first step is to clearly state the body to which you are applying Newton's law.

Separate Axes. Like other vector equations, Eq. 5-1 is equivalent to three component equations, one for each axis of an xyz coordinate system:

$$F_{\text{net},x} = ma_x, \quad F_{\text{net},y} = ma_y, \quad \text{and} \quad F_{\text{net},z} = ma_z. \quad (5-2)$$

Each of these equations relates the net force component along an axis to the acceleration along that same axis. For example, the first equation tells us that the sum of all the force components along the x axis causes the x component a_x of the body's acceleration, but causes no acceleration in the y and z directions. Turned around, the acceleration component a_x is caused only by the sum of the

force components along the x axis and is *completely* unrelated to force components along another axis. In general,



The acceleration component along a given axis is caused *only* by the sum of the force components along that *same* axis, and not by force components along any other axis.

Forces in Equilibrium. Equation 5-1 tells us that if the net force on a body is zero, the body's acceleration $\vec{a} = 0$. If the body is at rest, it stays at rest; if it is moving, it continues to move at constant velocity. In such cases, any forces on the body *balance* one another, and both the forces and the body are said to be in *equilibrium*. Commonly, the forces are also said to *cancel* one another, but the term “cancel” is tricky. It does *not* mean that the forces cease to exist (canceling forces is not like canceling dinner reservations). The forces still act on the body but cannot change the velocity.

Units. For SI units, Eq. 5-1 tells us that

$$1 \text{ N} = (1 \text{ kg})(1 \text{ m/s}^2) = 1 \text{ kg} \cdot \text{m/s}^2. \quad (5-3)$$

Some force units in other systems of units are given in Table 5-1 and Appendix D.

Diagrams. To solve problems with Newton's second law, we often draw a **free-body diagram** in which the only body shown is the one for which we are summing forces. A sketch of the body itself is preferred by some teachers but, to save space in these chapters, we shall usually represent the body with a dot. Also, each force on the body is drawn as a vector arrow with its tail anchored on the body. A coordinate system is usually included, and the acceleration of the body is sometimes shown with a vector arrow (labeled as an acceleration). This whole procedure is designed to focus our attention on the body of interest.

Table 5-1 Units in Newton's Second Law (Eqs. 5-1 and 5-2)

System	Force	Mass	Acceleration
SI	newton (N)	kilogram (kg)	m/s^2
CGS ^a	dyne	gram (g)	cm/s^2
British ^b	pound (lb)	slug	ft/s^2

^a1 dyne = $1 \text{ g} \cdot \text{cm/s}^2$.

^b1 lb = $1 \text{ slug} \cdot \text{ft/s}^2$.

External Forces Only. A **system** consists of one or more bodies, and any force on the bodies inside the system from bodies outside the system is called an **external force**. If the bodies making up a system are rigidly connected to one another, we can treat the system as one composite body, and the net force \vec{F}_{net} on it is the vector sum of all external forces. (We do not include **internal forces**—that is, forces between two bodies inside the system. Internal forces cannot accelerate the system.) For example, a connected railroad engine and car form a system. If, say, a tow line pulls on the front of the engine, the force due to the tow line acts on the whole engine–car system. Just as for a single body, we can relate the net external force on a system to its acceleration with Newton's second law, $\vec{F}_{\text{net}} = m\vec{a}$, where m is the total mass of the system.



Checkpoint 2

The figure here shows two horizontal forces acting on a block on a frictionless floor. If a third horizontal force \vec{F}_3 also acts on the block, what are the magnitude and direction of \vec{F}_3 when the block is (a) stationary and (b) moving to the left with a constant speed of 5 m/s?



Sample Problem 5.01 One- and two-dimensional forces, puck

Here are examples of how to use Newton's second law for a puck when one or two forces act on it. Parts A, B, and C of Fig. 5-3 show three situations in which one or two forces act on a puck that moves over frictionless ice along an x axis, in one-dimensional motion. The puck's mass is $m = 0.20$ kg. Forces \vec{F}_1 and \vec{F}_2 are directed along the axis and have magnitudes $F_1 = 4.0$ N and $F_2 = 2.0$ N. Force \vec{F}_3 is directed at angle $\theta = 30^\circ$ and has magnitude $F_3 = 1.0$ N. In each situation, what is the acceleration of the puck?

KEY IDEA

In each situation we can relate the acceleration \vec{a} to the net force \vec{F}_{net} acting on the puck with Newton's second law, $\vec{F}_{\text{net}} = m\vec{a}$. However, because the motion is along only the x axis, we can simplify each situation by writing the second law for x components only:

$$F_{\text{net},x} = ma_x \quad (5-4)$$

The free-body diagrams for the three situations are also given in Fig. 5-3, with the puck represented by a dot.

Situation A: For Fig. 5-3b, where only one horizontal force acts, Eq. 5-4 gives us

$$F_1 = ma_x,$$

which, with given data, yields

$$a_x = \frac{F_1}{m} = \frac{4.0 \text{ N}}{0.20 \text{ kg}} = 20 \text{ m/s}^2. \quad (\text{Answer})$$

The positive answer indicates that the acceleration is in the positive direction of the x axis.

Situation B: In Fig. 5-3d, two horizontal forces act on the puck, \vec{F}_1 in the positive direction of x and \vec{F}_2 in the negative direction. Now Eq. 5-4 gives us

$$F_1 - F_2 = ma_x,$$

which, with given data, yields

$$a_x = \frac{F_1 - F_2}{m} = \frac{4.0 \text{ N} - 2.0 \text{ N}}{0.20 \text{ kg}} = 10 \text{ m/s}^2. \quad (\text{Answer})$$

Thus, the net force accelerates the puck in the positive direction of the x axis.

Situation C: In Fig. 5-3f, force \vec{F}_3 is not directed along the direction of the puck's acceleration; only x component $F_{3,x}$ is. (Force \vec{F}_3 is two-dimensional but the motion is only one-

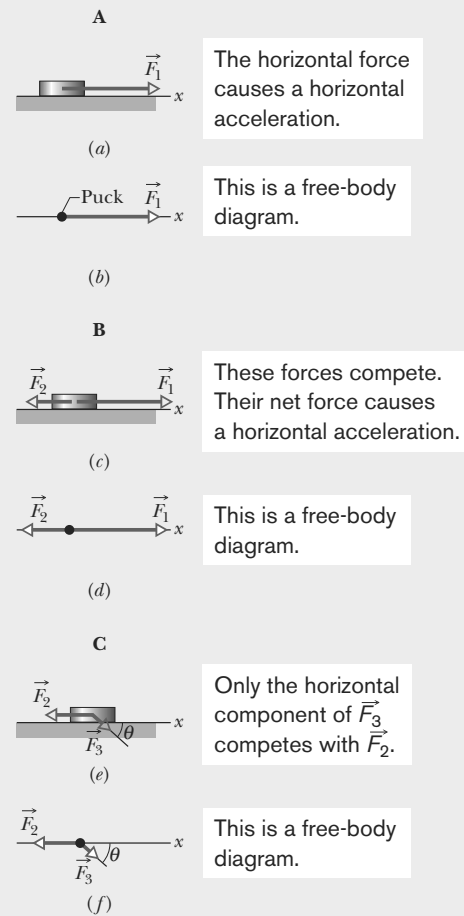


Figure 5-3 In three situations, forces act on a puck that moves along an x axis. Free-body diagrams are also shown.

dimensional.) Thus, we write Eq. 5-4 as

$$F_{3,x} - F_2 = ma_x. \quad (5-5)$$

From the figure, we see that $F_{3,x} = F_3 \cos \theta$. Solving for the acceleration and substituting for $F_{3,x}$ yield

$$\begin{aligned} a_x &= \frac{F_{3,x} - F_2}{m} = \frac{F_3 \cos \theta - F_2}{m} \\ &= \frac{(1.0 \text{ N})(\cos 30^\circ) - 2.0 \text{ N}}{0.20 \text{ kg}} = -5.7 \text{ m/s}^2. \end{aligned} \quad (\text{Answer})$$

Thus, the net force accelerates the puck in the negative direction of the x axis.



Sample Problem 5.02 Two-dimensional forces, cookie tin

Here we find a missing force by using the acceleration. In the overhead view of Fig. 5-4a, a 2.0 kg cookie tin is accelerated at 3.0 m/s^2 in the direction shown by \vec{a} , over a frictionless horizontal surface. The acceleration is caused by three horizontal forces, only two of which are shown: \vec{F}_1 of magnitude 10 N and \vec{F}_2 of magnitude 20 N. What is the third force \vec{F}_3 in unit-vector notation and in magnitude-angle notation?

KEY IDEA

The net force \vec{F}_{net} on the tin is the sum of the three forces and is related to the acceleration \vec{a} via Newton's second law ($\vec{F}_{\text{net}} = m\vec{a}$). Thus,

$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = m\vec{a}, \quad (5-6)$$

which gives us

$$\vec{F}_3 = m\vec{a} - \vec{F}_1 - \vec{F}_2. \quad (5-7)$$

Calculations: Because this is a two-dimensional problem, we *cannot* find \vec{F}_3 merely by substituting the magnitudes for the vector quantities on the right side of Eq. 5-7. Instead, we must vectorially add $m\vec{a}$, $-\vec{F}_1$ (the reverse of \vec{F}_1), and $-\vec{F}_2$ (the reverse of \vec{F}_2), as shown in Fig. 5-4b. This addition can be done directly on a vector-capable calculator because we know both magnitude and angle for all three vectors. However, here we shall evaluate the right side of Eq. 5-7 in terms of components, first along the x axis and then along the y axis. **Caution:** Use only one axis at a time.

x components: Along the x axis we have

$$\begin{aligned} F_{3,x} &= ma_x - F_{1,x} - F_{2,x} \\ &= m(a \cos 50^\circ) - F_1 \cos(-150^\circ) - F_2 \cos 90^\circ. \end{aligned}$$

Then, substituting known data, we find

$$\begin{aligned} F_{3,x} &= (2.0 \text{ kg})(3.0 \text{ m/s}^2) \cos 50^\circ - (10 \text{ N}) \cos(-150^\circ) \\ &\quad - (20 \text{ N}) \cos 90^\circ \\ &= 12.5 \text{ N}. \end{aligned}$$

y components: Similarly, along the y axis we find

$$\begin{aligned} F_{3,y} &= ma_y - F_{1,y} - F_{2,y} \\ &= m(a \sin 50^\circ) - F_1 \sin(-150^\circ) - F_2 \sin 90^\circ \\ &= (2.0 \text{ kg})(3.0 \text{ m/s}^2) \sin 50^\circ - (10 \text{ N}) \sin(-150^\circ) \\ &\quad - (20 \text{ N}) \sin 90^\circ \\ &= -10.4 \text{ N}. \end{aligned}$$

Vector: In unit-vector notation, we can write

$$\begin{aligned} \vec{F}_3 &= F_{3,x}\hat{i} + F_{3,y}\hat{j} = (12.5 \text{ N})\hat{i} - (10.4 \text{ N})\hat{j} \\ &\approx (13 \text{ N})\hat{i} - (10 \text{ N})\hat{j}. \end{aligned} \quad (\text{Answer})$$

We can now use a vector-capable calculator to get the magnitude and the angle of \vec{F}_3 . We can also use Eq. 3-6 to obtain the magnitude and the angle (from the positive direction of the x axis) as

$$F_3 = \sqrt{F_{3,x}^2 + F_{3,y}^2} = 16 \text{ N}$$

$$\text{and} \quad \theta = \tan^{-1} \frac{F_{3,y}}{F_{3,x}} = -40^\circ. \quad (\text{Answer})$$

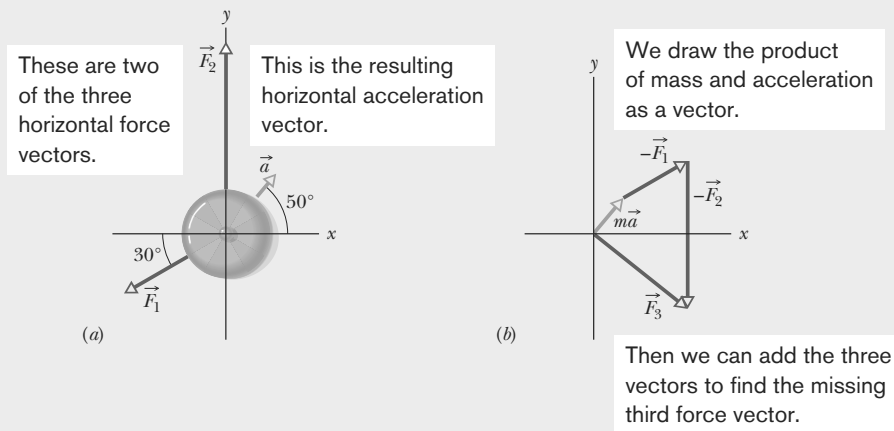


Figure 5-4 (a) An overhead view of two of three horizontal forces that act on a cookie tin, resulting in acceleration \vec{a} . \vec{F}_3 is not shown. (b) An arrangement of vectors $m\vec{a}$, $-\vec{F}_1$, and $-\vec{F}_2$ to find force \vec{F}_3 .



Additional examples, video, and practice available at WileyPLUS



5-2 SOME PARTICULAR FORCES

Learning Objectives

After reading this module, you should be able to . . .

- 5.08** Determine the magnitude and direction of the gravitational force acting on a body with a given mass, at a location with a given free-fall acceleration.
- 5.09** Identify that the weight of a body is the magnitude of the net force required to prevent the body from falling freely, as measured from the reference frame of the ground.
- 5.10** Identify that a scale gives an object's weight when the measurement is done in an inertial frame but not in an accelerating frame, where it gives an apparent weight.
- 5.11** Determine the magnitude and direction of the normal force on an object when the object is pressed or pulled onto a surface.
- 5.12** Identify that the force parallel to the surface is a frictional force that appears when the object slides or attempts to slide along the surface.
- 5.13** Identify that a tension force is said to pull at both ends of a cord (or a cord-like object) when the cord is taut.

Key Ideas

● A gravitational force \vec{F}_g on a body is a pull by another body. In most situations in this book, the other body is Earth or some other astronomical body. For Earth, the force is directed down toward the ground, which is assumed to be an inertial frame. With that assumption, the magnitude of \vec{F}_g is

$$F_g = mg,$$

where m is the body's mass and g is the magnitude of the free-fall acceleration.

● The weight W of a body is the magnitude of the upward force needed to balance the gravitational force on the body. A body's weight is related to the body's mass by

$$W = mg.$$

● A normal force \vec{F}_N is the force on a body from a surface against which the body presses. The normal force is always perpendicular to the surface.

● A frictional force \vec{f} is the force on a body when the body slides or attempts to slide along a surface. The force is always parallel to the surface and directed so as to oppose the sliding. On a frictionless surface, the frictional force is negligible.

● When a cord is under tension, each end of the cord pulls on a body. The pull is directed along the cord, away from the point of attachment to the body. For a massless cord (a cord with negligible mass), the pulls at both ends of the cord have the same magnitude T , even if the cord runs around a massless, frictionless pulley (a pulley with negligible mass and negligible friction on its axle to oppose its rotation).

Some Particular Forces

The Gravitational Force

A **gravitational force** \vec{F}_g on a body is a certain type of pull that is directed toward a second body. In these early chapters, we do not discuss the nature of this force and usually consider situations in which the second body is Earth. Thus, when we speak of *the* gravitational force \vec{F}_g on a body, we usually mean a force that pulls on it directly toward the center of Earth—that is, directly down toward the ground. We shall assume that the ground is an inertial frame.

Free Fall. Suppose a body of mass m is in free fall with the free-fall acceleration of magnitude g . Then, if we neglect the effects of the air, the only force acting on the body is the gravitational force \vec{F}_g . We can relate this downward force and downward acceleration with Newton's second law ($\vec{F} = m\vec{a}$). We place a vertical y axis along the body's path, with the positive direction upward. For this axis, Newton's second law can be written in the form $F_{\text{net},y} = ma_y$, which, in our situation, becomes

$$-F_g = m(-g)$$

or

$$F_g = mg. \quad (5-8)$$

In words, the magnitude of the gravitational force is equal to the product mg .

At Rest. This same gravitational force, with the same magnitude, still acts on the body even when the body is not in free fall but is, say, at rest on a pool table or moving across the table. (For the gravitational force to disappear, Earth would have to disappear.)

We can write Newton's second law for the gravitational force in these vector forms:

$$\vec{F}_g = -F_g \hat{j} = -mg \hat{j} = m\vec{g}, \quad (5-9)$$

where \hat{j} is the unit vector that points upward along a y axis, directly away from the ground, and \vec{g} is the free-fall acceleration (written as a vector), directed downward.

Weight

The **weight** W of a body is the magnitude of the net force required to prevent the body from falling freely, as measured by someone on the ground. For example, to keep a ball at rest in your hand while you stand on the ground, you must provide an upward force to balance the gravitational force on the ball from Earth. Suppose the magnitude of the gravitational force is 2.0 N. Then the magnitude of your upward force must be 2.0 N, and thus the weight W of the ball is 2.0 N. We also say that the ball *weighs* 2.0 N and speak about the ball *weighing* 2.0 N.

A ball with a weight of 3.0 N would require a greater force from you—namely, a 3.0 N force—to keep it at rest. The reason is that the gravitational force you must balance has a greater magnitude—namely, 3.0 N. We say that this second ball is *heavier* than the first ball.

Now let us generalize the situation. Consider a body that has an acceleration \vec{a} of zero relative to the ground, which we again assume to be an inertial frame. Two forces act on the body: a downward gravitational force \vec{F}_g and a balancing upward force of magnitude W . We can write Newton's second law for a vertical y axis, with the positive direction upward, as

$$F_{\text{net},y} = ma_y.$$

In our situation, this becomes

$$W - F_g = m(0) \quad (5-10)$$

$$\text{or} \quad W = F_g \quad (\text{weight, with ground as inertial frame}). \quad (5-11)$$

This equation tells us (assuming the ground is an inertial frame) that



The weight W of a body is equal to the magnitude F_g of the gravitational force on the body.

Substituting mg for F_g from Eq. 5-8, we find

$$W = mg \quad (\text{weight}), \quad (5-12)$$

which relates a body's weight to its mass.

Weighing. To *weigh* a body means to measure its weight. One way to do this is to place the body on one of the pans of an equal-arm balance (Fig. 5-5) and then place reference bodies (whose masses are known) on the other pan until we strike a balance (so that the gravitational forces on the two sides match). The masses on the pans then match, and we know the mass of the body. If we know the value of g for the location of the balance, we can also find the weight of the body with Eq. 5-12.

We can also weigh a body with a spring scale (Fig. 5-6). The body stretches a spring, moving a pointer along a scale that has been calibrated and marked in

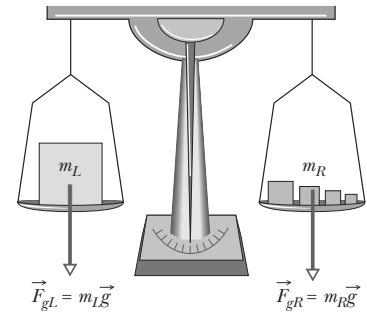


Figure 5-5 An equal-arm balance. When the device is in balance, the gravitational force \vec{F}_{gL} on the body being weighed (on the left pan) and the total gravitational force \vec{F}_{gR} on the reference bodies (on the right pan) are equal. Thus, the mass m_L of the body being weighed is equal to the total mass m_R of the reference bodies.

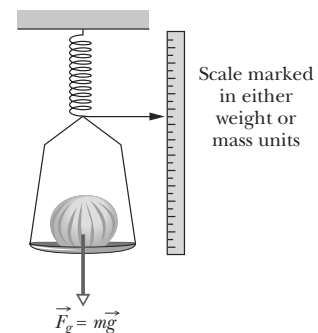


Figure 5-6 A spring scale. The reading is proportional to the *weight* of the object on the pan, and the scale gives that weight if marked in weight units. If, instead, it is marked in mass units, the reading is the object's weight only if the value of g at the location where the scale is being used is the same as the value of g at the location where the scale was calibrated.

either mass or weight units. (Most bathroom scales in the United States work this way and are marked in the force unit pounds.) If the scale is marked in mass units, it is accurate only where the value of g is the same as where the scale was calibrated.

The weight of a body must be measured when the body is not accelerating vertically relative to the ground. For example, you can measure your weight on a scale in your bathroom or on a fast train. However, if you repeat the measurement with the scale in an accelerating elevator, the reading differs from your weight because of the acceleration. Such a measurement is called an *apparent weight*.

Caution: A body's weight is not its mass. Weight is the magnitude of a force and is related to mass by Eq. 5-12. If you move a body to a point where the value of g is different, the body's mass (an intrinsic property) is not different but the weight is. For example, the weight of a bowling ball having a mass of 7.2 kg is 71 N on Earth but only 12 N on the Moon. The mass is the same on Earth and Moon, but the free-fall acceleration on the Moon is only 1.6 m/s^2 .

The Normal Force

If you stand on a mattress, Earth pulls you downward, but you remain stationary. The reason is that the mattress, because it deforms downward due to you, pushes up on you. Similarly, if you stand on a floor, it deforms (it is compressed, bent, or buckled ever so slightly) and pushes up on you. Even a seemingly rigid concrete floor does this (if it is not sitting directly on the ground, enough people on the floor could break it).

The push on you from the mattress or floor is a **normal force** \vec{F}_N . The name comes from the mathematical term *normal*, meaning perpendicular: The force on you from, say, the floor is perpendicular to the floor.



When a body presses against a surface, the surface (even a seemingly rigid one) deforms and pushes on the body with a normal force \vec{F}_N that is perpendicular to the surface.

Figure 5-7a shows an example. A block of mass m presses down on a table, deforming it somewhat because of the gravitational force \vec{F}_g on the block. The table pushes up on the block with normal force \vec{F}_N . The free-body diagram for the block is given in Fig. 5-7b. Forces \vec{F}_g and \vec{F}_N are the only two forces on the block and they are both vertical. Thus, for the block we can write Newton's second law for a positive-upward y axis ($F_{\text{net}, y} = ma_y$) as

$$F_N - F_g = ma_y.$$

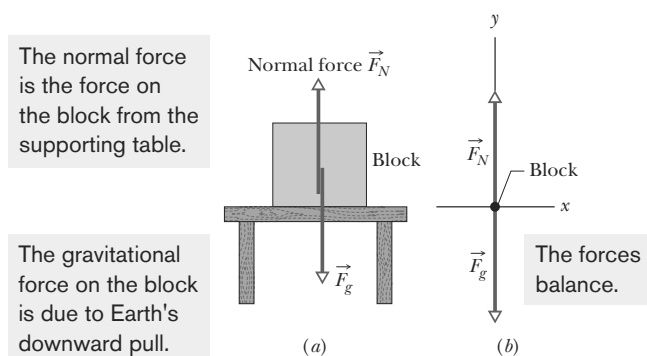


Figure 5-7 (a) A block resting on a table experiences a normal force \vec{F}_N perpendicular to the tabletop. (b) The free-body diagram for the block.

From Eq. 5-8, we substitute mg for F_g , finding

$$F_N - mg = ma_y.$$

Then the magnitude of the normal force is

$$F_N = mg + ma_y = m(g + a_y) \quad (5-13)$$

for any vertical acceleration a_y of the table and block (they might be in an accelerating elevator). (*Caution:* We have already included the sign for g but a_y can be positive or negative here.) If the table and block are not accelerating relative to the ground, then $a_y = 0$ and Eq. 5-13 yields

$$F_N = mg. \quad (5-14)$$



Checkpoint 3

In Fig. 5-7, is the magnitude of the normal force \vec{F}_N greater than, less than, or equal to mg if the block and table are in an elevator moving upward (a) at constant speed and (b) at increasing speed?

Friction

If we either slide or attempt to slide a body over a surface, the motion is resisted by a bonding between the body and the surface. (We discuss this bonding more in the next chapter.) The resistance is considered to be a single force \vec{f} , called either the **frictional force** or simply **friction**. This force is directed along the surface, opposite the direction of the intended motion (Fig. 5-8). Sometimes, to simplify a situation, friction is assumed to be negligible (the surface, or even the body, is said to be *frictionless*).

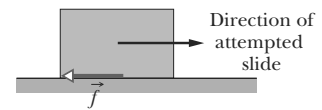


Figure 5-8 A frictional force \vec{f} opposes the attempted slide of a body over a surface.

Tension

When a cord (or a rope, cable, or other such object) is attached to a body and pulled taut, the cord pulls on the body with a force \vec{T} directed away from the body and along the cord (Fig. 5-9a). The force is often called a *tension force* because the cord is said to be in a state of *tension* (or to be *under tension*), which means that it is being pulled taut. The *tension in the cord* is the magnitude T of the force on the body. For example, if the force on the body from the cord has magnitude $T = 50$ N, the tension in the cord is 50 N.

A cord is often said to be *massless* (meaning its mass is negligible compared to the body's mass) and *unstretchable*. The cord then exists only as a connection between two bodies. It pulls on both bodies with the same force magnitude T ,

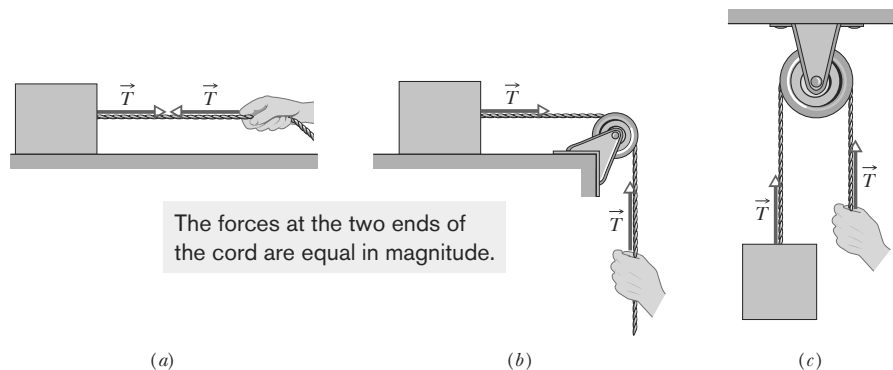


Figure 5-9 (a) The cord, pulled taut, is under tension. If its mass is negligible, the cord pulls on the body and the hand with force \vec{T} , even if the cord runs around a massless, frictionless pulley as in (b) and (c).

even if the bodies and the cord are accelerating and even if the cord runs around a *massless, frictionless pulley* (Figs. 5-9b and c). Such a pulley has negligible mass compared to the bodies and negligible friction on its axle opposing its rotation. If the cord wraps halfway around a pulley, as in Fig. 5-9c, the net force on the pulley from the cord has the magnitude $2T$.



Checkpoint 4

The suspended body in Fig. 5-9c weighs 75 N. Is T equal to, greater than, or less than 75 N when the body is moving upward (a) at constant speed, (b) at increasing speed, and (c) at decreasing speed?

5-3 APPLYING NEWTON'S LAWS

Learning Objectives

After reading this module, you should be able to . . .

5.14 Identify Newton's third law of motion and third-law force pairs.

5.15 For an object that moves vertically or on a horizontal or inclined plane, apply Newton's second law to a free-body diagram of the object.

5.16 For an arrangement where a system of several objects moves rigidly together, draw a free-body diagram and apply Newton's second law for the individual objects and also for the system taken as a composite object.

Key Ideas

● The net force \vec{F}_{net} on a body with mass m is related to the body's acceleration \vec{a} by

$$\vec{F}_{\text{net}} = m\vec{a},$$

which may be written in the component versions

$$F_{\text{net},x} = ma_x \quad F_{\text{net},y} = ma_y \quad \text{and} \quad F_{\text{net},z} = ma_z.$$

● If a force \vec{F}_{BC} acts on body B due to body C , then there is a force \vec{F}_{CB} on body C due to body B :

$$\vec{F}_{BC} = -\vec{F}_{CB}.$$

The forces are equal in magnitude but opposite in directions.

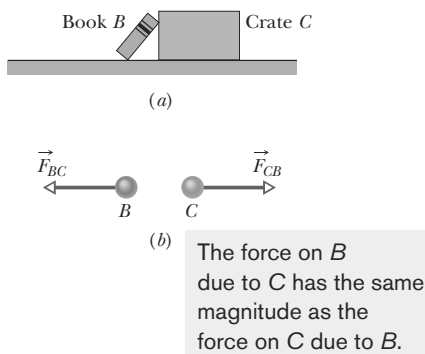


Figure 5-10 (a) Book B leans against crate C . (b) Forces \vec{F}_{BC} (the force on the book from the crate) and \vec{F}_{CB} (the force on the crate from the book) have the same magnitude and are opposite in direction.

Newton's Third Law

Two bodies are said to *interact* when they push or pull on each other—that is, when a force acts on each body due to the other body. For example, suppose you position a book B so it leans against a crate C (Fig. 5-10a). Then the book and crate interact: There is a horizontal force \vec{F}_{BC} on the book from the crate (or due to the crate) and a horizontal force \vec{F}_{CB} on the crate from the book (or due to the book). This pair of forces is shown in Fig. 5-10b. Newton's third law states that



Newton's Third Law: When two bodies interact, the forces on the bodies from each other are always equal in magnitude and opposite in direction.

For the book and crate, we can write this law as the scalar relation

$$F_{BC} = F_{CB} \quad (\text{equal magnitudes})$$

or as the vector relation

$$\vec{F}_{BC} = -\vec{F}_{CB} \quad (\text{equal magnitudes and opposite directions}), \quad (5-15)$$

where the minus sign means that these two forces are in opposite directions. We can call the forces between two interacting bodies a **third-law force pair**. When

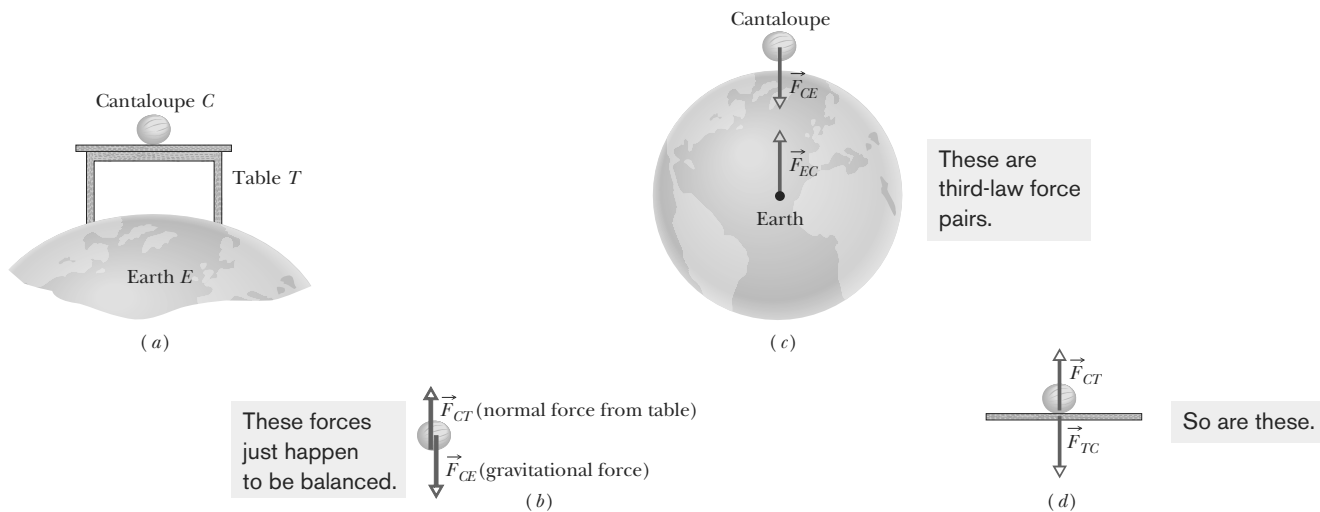


Figure 5-11 (a) A cantaloupe lies on a table that stands on Earth. (b) The forces on the cantaloupe are \vec{F}_{CT} and \vec{F}_{CE} . (c) The third-law force pair for the cantaloupe–Earth interaction. (d) The third-law force pair for the cantaloupe–table interaction.

any two bodies interact in any situation, a third-law force pair is present. The book and crate in Fig. 5-10a are stationary, but the third law would still hold if they were moving and even if they were accelerating.

As another example, let us find the third-law force pairs involving the cantaloupe in Fig. 5-11a, which lies on a table that stands on Earth. The cantaloupe interacts with the table and with Earth (this time, there are three bodies whose interactions we must sort out).

Let's first focus on the forces acting on the cantaloupe (Fig. 5-11b). Force \vec{F}_{CT} is the normal force on the cantaloupe from the table, and force \vec{F}_{CE} is the gravitational force on the cantaloupe due to Earth. Are they a third-law force pair? No, because they are forces on a single body, the cantaloupe, and not on two interacting bodies.

To find a third-law pair, we must focus not on the cantaloupe but on the interaction between the cantaloupe and one other body. In the cantaloupe–Earth interaction (Fig. 5-11c), Earth pulls on the cantaloupe with a gravitational force \vec{F}_{CE} and the cantaloupe pulls on Earth with a gravitational force \vec{F}_{EC} . Are these forces a third-law force pair? Yes, because they are forces on two interacting bodies, the force on each due to the other. Thus, by Newton's third law,

$$\vec{F}_{CE} = -\vec{F}_{EC} \quad (\text{cantaloupe–Earth interaction}).$$

Next, in the cantaloupe–table interaction, the force on the cantaloupe from the table is \vec{F}_{CT} and, conversely, the force on the table from the cantaloupe is \vec{F}_{TC} (Fig. 5-11d). These forces are also a third-law force pair, and so

$$\vec{F}_{CT} = -\vec{F}_{TC} \quad (\text{cantaloupe–table interaction}).$$



Checkpoint 5

Suppose that the cantaloupe and table of Fig. 5-11 are in an elevator cab that begins to accelerate upward. (a) Do the magnitudes of \vec{F}_{TC} and \vec{F}_{CT} increase, decrease, or stay the same? (b) Are those two forces still equal in magnitude and opposite in direction? (c) Do the magnitudes of \vec{F}_{CE} and \vec{F}_{EC} increase, decrease, or stay the same? (d) Are those two forces still equal in magnitude and opposite in direction?

Applying Newton's Laws

The rest of this chapter consists of sample problems. You should pore over them, learning their procedures for attacking a problem. Especially important is knowing how to translate a sketch of a situation into a free-body diagram with appropriate axes, so that Newton's laws can be applied.

Sample Problem 5.03 Block on table, block hanging

Figure 5-12 shows a block S (the *sliding block*) with mass $M = 3.3$ kg. The block is free to move along a horizontal frictionless surface and connected, by a cord that wraps over a frictionless pulley, to a second block H (the *hanging block*), with mass $m = 2.1$ kg. The cord and pulley have negligible masses compared to the blocks (they are “massless”). The hanging block H falls as the sliding block S accelerates to the right. Find (a) the acceleration of block S , (b) the acceleration of block H , and (c) the tension in the cord.

Q What is this problem all about?

You are given two bodies—sliding block and hanging block—but must also consider *Earth*, which pulls on both bodies. (Without Earth, nothing would happen here.) A total of five forces act on the blocks, as shown in Fig. 5-13:

1. The cord pulls to the right on sliding block S with a force of magnitude T .
2. The cord pulls upward on hanging block H with a force of the same magnitude T . This upward force keeps block H from falling freely.
3. Earth pulls down on block S with the gravitational force \vec{F}_{gS} , which has a magnitude equal to Mg .
4. Earth pulls down on block H with the gravitational force \vec{F}_{gH} , which has a magnitude equal to mg .
5. The table pushes up on block S with a normal force \vec{F}_N .

There is another thing you should note. We assume that the cord does not stretch, so that if block H falls 1 mm in a

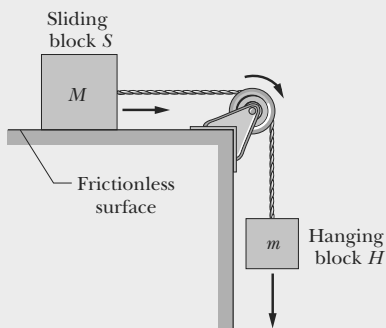


Figure 5-12 A block S of mass M is connected to a block H of mass m by a cord that wraps over a pulley.

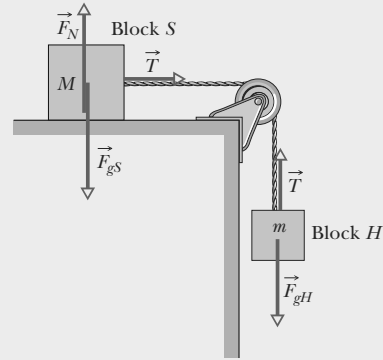


Figure 5-13 The forces acting on the two blocks of Fig. 5-12.

certain time, block S moves 1 mm to the right in that same time. This means that the blocks move together and their accelerations have the same magnitude a .

Q How do I classify this problem? Should it suggest a particular law of physics to me?

Yes. Forces, masses, and accelerations are involved, and they should suggest Newton's second law of motion, $\vec{F}_{\text{net}} = m\vec{a}$. That is our starting key idea.

Q If I apply Newton's second law to this problem, to which body should I apply it?

We focus on two bodies, the sliding block and the hanging block. Although they are *extended objects* (they are not points), we can still treat each block as a particle because every part of it moves in exactly the same way. A second key idea is to apply Newton's second law separately to each block.

Q What about the pulley?

We cannot represent the pulley as a particle because different parts of it move in different ways. When we discuss rotation, we shall deal with pulleys in detail. Meanwhile, we eliminate the pulley from consideration by assuming its mass to be negligible compared with the masses of the two blocks. Its only function is to change the cord's orientation.

Q OK. Now how do I apply $\vec{F}_{\text{net}} = m\vec{a}$ to the sliding block?

Represent block S as a particle of mass M and draw *all* the forces that act *on* it, as in Fig. 5-14a. This is the block's free-body diagram. Next, draw a set of axes. It makes sense

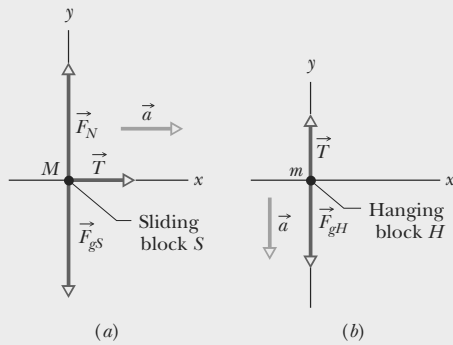


Figure 5-14 (a) A free-body diagram for block S of Fig. 5-12. (b) A free-body diagram for block H of Fig. 5-12.

to draw the x axis parallel to the table, in the direction in which the block moves.

Q Thanks, but you still haven't told me how to apply $\vec{F}_{\text{net}} = m\vec{a}$ to the sliding block. All you've done is explain how to draw a free-body diagram.

You are right, and here's the third key idea: The expression $\vec{F}_{\text{net}} = M\vec{a}$ is a vector equation, so we can write it as three component equations:

$$F_{\text{net},x} = Ma_x \quad F_{\text{net},y} = Ma_y \quad F_{\text{net},z} = Ma_z \quad (5-16)$$

in which $F_{\text{net},x}$, $F_{\text{net},y}$, and $F_{\text{net},z}$ are the components of the net force along the three axes. Now we apply each component equation to its corresponding direction. Because block S does not accelerate vertically, $F_{\text{net},y} = Ma_y$ becomes

$$F_N - F_{gS} = 0 \quad \text{or} \quad F_N = F_{gS}. \quad (5-17)$$

Thus in the y direction, the magnitude of the normal force is equal to the magnitude of the gravitational force.

No force acts in the z direction, which is perpendicular to the page.

In the x direction, there is only one force component, which is T . Thus, $F_{\text{net},x} = Ma_x$ becomes

$$T = Ma. \quad (5-18)$$

This equation contains two unknowns, T and a ; so we cannot yet solve it. Recall, however, that we have not said anything about the hanging block.

Q I agree. How do I apply $\vec{F}_{\text{net}} = m\vec{a}$ to the hanging block?

We apply it just as we did for block S : Draw a free-body diagram for block H , as in Fig. 5-14b. Then apply $\vec{F}_{\text{net}} = m\vec{a}$ in component form. This time, because the acceleration is along the y axis, we use the y part of Eq. 5-16 ($F_{\text{net},y} = ma_y$) to write

$$T - F_{gH} = ma_y. \quad (5-19)$$

We can now substitute mg for F_{gH} and $-a$ for a_y (negative

because block H accelerates in the negative direction of the y axis). We find

$$T - mg = -ma. \quad (5-20)$$

Now note that Eqs. 5-18 and 5-20 are simultaneous equations with the same two unknowns, T and a . Subtracting these equations eliminates T . Then solving for a yields

$$a = \frac{m}{M + m} g. \quad (5-21)$$

Substituting this result into Eq. 5-18 yields

$$T = \frac{Mm}{M + m} g. \quad (5-22)$$

Putting in the numbers gives, for these two quantities,

$$a = \frac{m}{M + m} g = \frac{2.1 \text{ kg}}{3.3 \text{ kg} + 2.1 \text{ kg}} (9.8 \text{ m/s}^2) = 3.8 \text{ m/s}^2 \quad (\text{Answer})$$

$$\text{and } T = \frac{Mm}{M + m} g = \frac{(3.3 \text{ kg})(2.1 \text{ kg})}{3.3 \text{ kg} + 2.1 \text{ kg}} (9.8 \text{ m/s}^2) = 13 \text{ N}. \quad (\text{Answer})$$

Q The problem is now solved, right?

That's a fair question, but the problem is not really finished until we have examined the results to see whether they make sense. (If you made these calculations on the job, wouldn't you want to see whether they made sense before you turned them in?)

Look first at Eq. 5-21. Note that it is dimensionally correct and that the acceleration a will always be less than g (because of the cord, the hanging block is not in free fall).

Look now at Eq. 5-22, which we can rewrite in the form

$$T = \frac{M}{M + m} mg. \quad (5-23)$$

In this form, it is easier to see that this equation is also dimensionally correct, because both T and mg have dimensions of forces. Equation 5-23 also lets us see that the tension in the cord is always less than mg , and thus is always less than the gravitational force on the hanging block. That is a comforting thought because, if T were greater than mg , the hanging block would accelerate upward.

We can also check the results by studying special cases, in which we can guess what the answers must be. A simple example is to put $g = 0$, as if the experiment were carried out in interstellar space. We know that in that case, the blocks would not move from rest, there would be no forces on the ends of the cord, and so there would be no tension in the cord. Do the formulas predict this? Yes, they do. If you put $g = 0$ in Eqs. 5-21 and 5-22, you find $a = 0$ and $T = 0$. Two more special cases you might try are $M = 0$ and $m \rightarrow \infty$.



Sample Problem 5.04 Cord accelerates box up a ramp

Many students consider problems involving ramps (inclined planes) to be especially hard. The difficulty is probably visual because we work with (a) a tilted coordinate system and (b) the components of the gravitational force, not the full force. Here is a typical example with all the tilting and angles explained. (In *WileyPLUS*, the figure is available as an animation with voiceover.) In spite of the tilt, the key idea is to apply Newton's second law to the axis along which the motion occurs.

In Fig. 5-15a, a cord pulls a box of sea biscuits up along a frictionless plane inclined at angle $\theta = 30.0^\circ$. The box has mass $m = 5.00$ kg, and the force from the cord has magnitude $T = 25.0$ N. What is the box's acceleration a along the inclined plane?

KEY IDEA

The acceleration along the plane is set by the force components along the plane (not by force components perpendicular to the plane), as expressed by Newton's second law (Eq. 5-1).

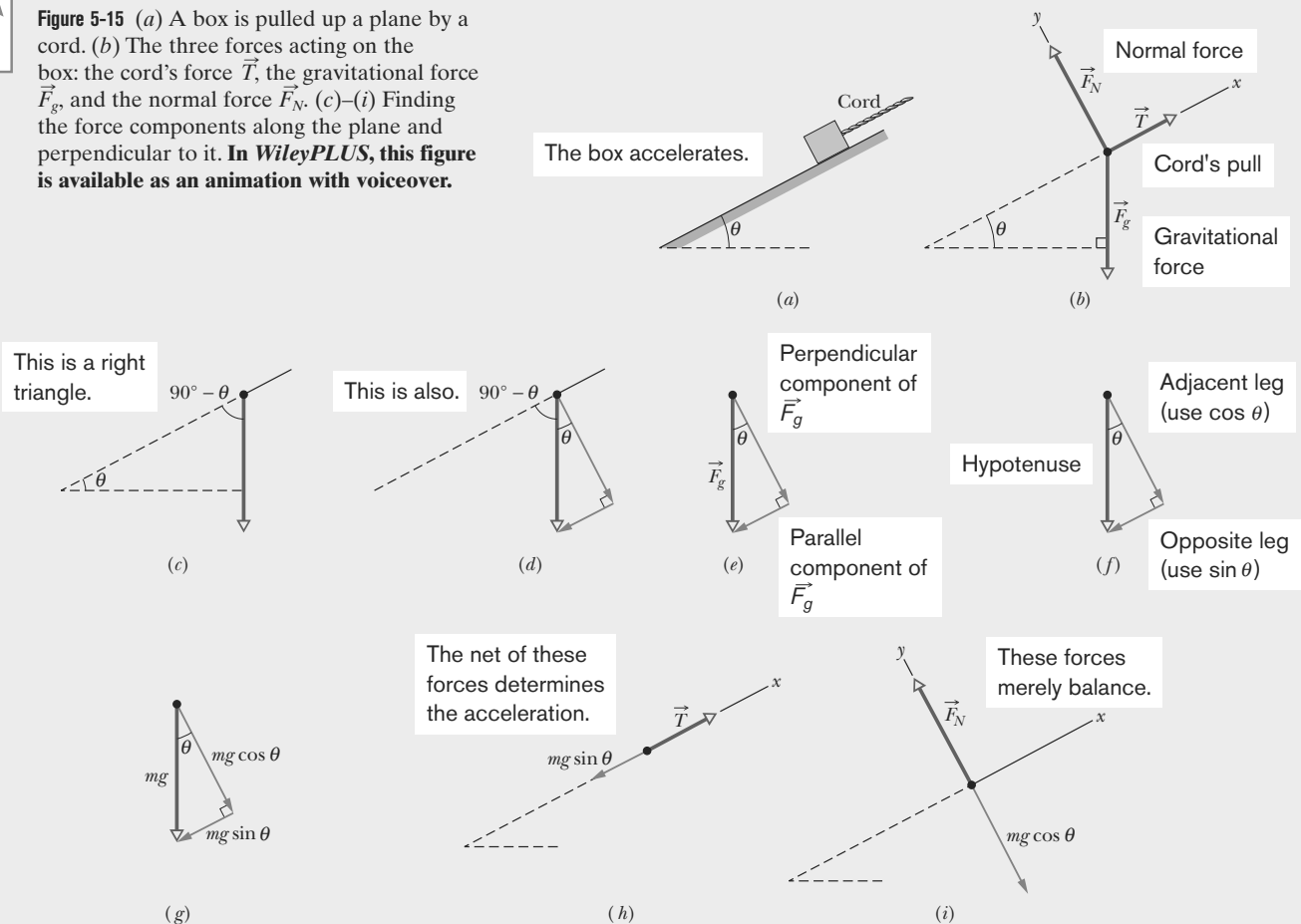
(Eq. 5-1).

Calculations: We need to write Newton's second law for motion along an axis. Because the box moves along the inclined plane, placing an x axis along the plane seems reasonable (Fig. 5-15b). (There is nothing wrong with using our usual coordinate system, but the expressions for components would be a lot messier because of the misalignment of the x axis with the motion.)

After choosing a coordinate system, we draw a free-body diagram with a dot representing the box (Fig. 5-15b). Then we draw all the vectors for the forces acting on the box, with the tails of the vectors anchored on the dot. (Drawing the vectors willy-nilly on the diagram can easily lead to errors, especially on exams, so always anchor the tails.)

Force \vec{T} from the cord is up the plane and has magnitude $T = 25.0$ N. The gravitational force \vec{F}_g is downward (of

Figure 5-15 (a) A box is pulled up a plane by a cord. (b) The three forces acting on the box: the cord's force \vec{T} , the gravitational force \vec{F}_g , and the normal force \vec{F}_N . (c)–(i) Finding the force components along the plane and perpendicular to it. **In WileyPLUS, this figure is available as an animation with voiceover.**



course) and has magnitude $mg = (5.00 \text{ kg})(9.80 \text{ m/s}^2) = 49.0 \text{ N}$. That direction means that only a component of the force is along the plane, and only that component (not the full force) affects the box's acceleration along the plane. Thus, before we can write Newton's second law for motion along the x axis, we need to find an expression for that important component.

Figures 5-15c to h indicate the steps that lead to the expression. We start with the given angle of the plane and work our way to a triangle of the force components (they are the legs of the triangle and the full force is the hypotenuse). Figure 5-15c shows that the angle between the ramp and \vec{F}_g is $90^\circ - \theta$. (Do you see a right triangle there?) Next, Figs. 5-15d to f show \vec{F}_g and its components: One component is parallel to the plane (that is the one we want) and the other is perpendicular to the plane.

Because the perpendicular component is perpendicular, the angle between it and \vec{F}_g must be θ (Fig. 5-15d). The component we want is the far leg of the component right triangle. The magnitude of the hypotenuse is mg (the magnitude of the gravitational force). Thus, the component we want has magnitude $mg \sin \theta$ (Fig. 5-15g).

We have one more force to consider, the normal force \vec{F}_N shown in Fig. 5-15b. However, it is perpendicular to the

plane and thus cannot affect the motion along the plane. (It has no component along the plane to accelerate the box.)

We are now ready to write Newton's second law for motion along the tilted x axis:

$$F_{\text{net},x} = ma_x.$$

The component a_x is the only component of the acceleration (the box is not leaping up from the plane, which would be strange, or descending into the plane, which would be even stranger). So, let's simply write a for the acceleration along the plane. Because \vec{T} is in the positive x direction and the component $mg \sin \theta$ is in the negative x direction, we next write

$$T - mg \sin \theta = ma. \quad (5-24)$$

Substituting data and solving for a , we find

$$a = 0.100 \text{ m/s}^2. \quad (\text{Answer})$$

The result is positive, indicating that the box accelerates up the inclined plane, in the positive direction of the tilted x axis. If we decreased the magnitude of \vec{T} enough to make $a = 0$, the box would move up the plane at constant speed. And if we decrease the magnitude of \vec{T} even more, the acceleration would be negative in spite of the cord's pull.

Sample Problem 5.05 Reading a force graph

Here is an example of where you must dig information out of a graph, not just read off a number. In Fig. 5-16a, two forces are applied to a 4.00 kg block on a frictionless floor, but only force \vec{F}_1 is indicated. That force has a fixed magnitude but can be applied at an adjustable angle θ to the positive direction of the x axis. Force \vec{F}_2 is horizontal and fixed in both magnitude and angle. Figure 5-16b gives the horizontal acceleration a_x of the block for any given value of θ from 0° to 90° . What is the value of a_x for $\theta = 180^\circ$?

KEY IDEAS

(1) The horizontal acceleration a_x depends on the net horizontal force $F_{\text{net},x}$, as given by Newton's second law. (2) The net horizontal force is the sum of the horizontal components of forces \vec{F}_1 and \vec{F}_2 .

Calculations: The x component of \vec{F}_2 is F_2 because the vector is horizontal. The x component of \vec{F}_1 is $F_1 \cos \theta$. Using these expressions and a mass m of 4.00 kg, we can write Newton's second law ($\vec{F}_{\text{net}} = m\vec{a}$) for motion along the x axis as

$$F_1 \cos \theta + F_2 = 4.00a_x. \quad (5-25)$$

From this equation we see that when angle $\theta = 90^\circ$, $F_1 \cos \theta$ is zero and $F_2 = 4.00a_x$. From the graph we see that the

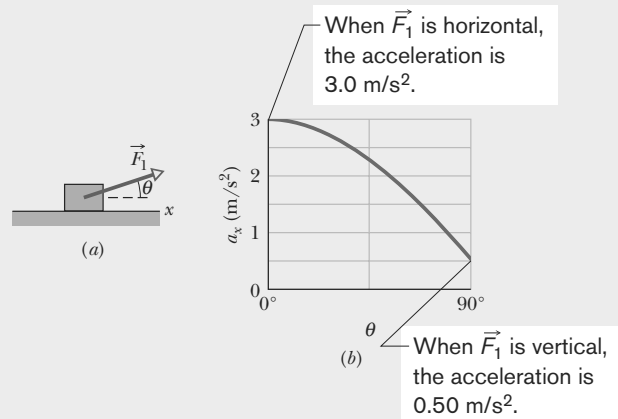


Figure 5-16 (a) One of the two forces applied to a block is shown. Its angle θ can be varied. (b) The block's acceleration component a_x versus θ .

corresponding acceleration is 0.50 m/s^2 . Thus, $F_2 = 2.00 \text{ N}$ and \vec{F}_2 must be in the positive direction of the x axis.

From Eq. 5-25, we find that when $\theta = 0^\circ$,

$$F_1 \cos 0^\circ + 2.00 = 4.00a_x. \quad (5-26)$$

From the graph we see that the corresponding acceleration is 3.0 m/s^2 . From Eq. 5-26, we then find that $F_1 = 10 \text{ N}$.

Substituting $F_1 = 10 \text{ N}$, $F_2 = 2.00 \text{ N}$, and $\theta = 180^\circ$ into Eq. 5-25 leads to

$$a_x = -2.00 \text{ m/s}^2. \quad (\text{Answer})$$



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Sample Problem 5.06 Forces within an elevator cab

Although people would surely avoid getting into the elevator with you, suppose that you weigh yourself while on an elevator that is moving. Would you weigh more than, less than, or the same as when the scale is on a stationary floor?

In Fig. 5-17a, a passenger of mass $m = 72.2 \text{ kg}$ stands on a platform scale in an elevator cab. We are concerned with the scale readings when the cab is stationary and when it is moving up or down.

(a) Find a general solution for the scale reading, whatever the vertical motion of the cab.

KEY IDEAS

(1) The reading is equal to the magnitude of the normal force \vec{F}_N on the passenger from the scale. The only other force acting on the passenger is the gravitational force \vec{F}_g , as shown in the free-body diagram of Fig. 5-17b. (2) We can relate the forces on the passenger to his acceleration \vec{a} by using Newton's second law ($\vec{F}_{\text{net}} = m\vec{a}$). However, recall that we can use this law only in an inertial frame. If the cab accelerates, then it is *not* an inertial frame. So we choose the ground to be our inertial frame and make any measure of the passenger's acceleration relative to it.

Calculations: Because the two forces on the passenger and his acceleration are all directed vertically, along the y axis in Fig. 5-17b, we can use Newton's second law written for y components ($F_{\text{net},y} = ma_y$) to get

$$\begin{aligned} F_N - F_g &= ma \\ \text{or} \quad F_N &= F_g + ma. \end{aligned} \quad (5-27)$$

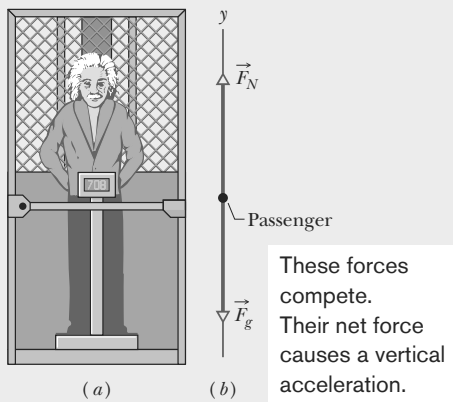


Figure 5-17 (a) A passenger stands on a platform scale that indicates either his weight or his apparent weight. (b) The free-body diagram for the passenger, showing the normal force \vec{F}_N on him from the scale and the gravitational force \vec{F}_g .

This tells us that the scale reading, which is equal to normal force magnitude F_N , depends on the vertical acceleration. Substituting mg for F_g gives us

$$F_N = m(g + a) \quad (\text{Answer}) \quad (5-28)$$

for any choice of acceleration a . If the acceleration is upward, a is positive; if it is downward, a is negative.

(b) What does the scale read if the cab is stationary or moving upward at a constant 0.50 m/s ?

KEY IDEA

For any constant velocity (zero or otherwise), the acceleration a of the passenger is zero.

Calculation: Substituting this and other known values into Eq. 5-28, we find

$$F_N = (72.2 \text{ kg})(9.8 \text{ m/s}^2 + 0) = 708 \text{ N}. \quad (\text{Answer})$$

This is the weight of the passenger and is equal to the magnitude F_g of the gravitational force on him.

(c) What does the scale read if the cab accelerates upward at 3.20 m/s^2 and downward at 3.20 m/s^2 ?

Calculations: For $a = 3.20 \text{ m/s}^2$, Eq. 5-28 gives

$$\begin{aligned} F_N &= (72.2 \text{ kg})(9.8 \text{ m/s}^2 + 3.20 \text{ m/s}^2) \\ &= 939 \text{ N}, \end{aligned} \quad (\text{Answer})$$

and for $a = -3.20 \text{ m/s}^2$, it gives

$$\begin{aligned} F_N &= (72.2 \text{ kg})(9.8 \text{ m/s}^2 - 3.20 \text{ m/s}^2) \\ &= 477 \text{ N}. \end{aligned} \quad (\text{Answer})$$

For an upward acceleration (either the cab's upward speed is increasing or its downward speed is decreasing), the scale reading is greater than the passenger's weight. That reading is a measurement of an apparent weight, because it is made in a noninertial frame. For a downward acceleration (either decreasing upward speed or increasing downward speed), the scale reading is less than the passenger's weight.

(d) During the upward acceleration in part (c), what is the magnitude F_{net} of the net force on the passenger, and what is the magnitude $a_{\text{p,cab}}$ of his acceleration as measured in the frame of the cab? Does $\vec{F}_{\text{net}} = m\vec{a}_{\text{p,cab}}$?

Calculation: The magnitude F_g of the gravitational force on the passenger does not depend on the motion of the passenger or the cab; so, from part (b), F_g is 708 N . From part (c), the magnitude F_N of the normal force on the passenger during

the upward acceleration is the 939 N reading on the scale. Thus, the net force on the passenger is

$$F_{\text{net}} = F_N - F_g = 939 \text{ N} - 708 \text{ N} = 231 \text{ N}, \quad (\text{Answer})$$

Sample Problem 5.07 Acceleration of block pushing on block

Some homework problems involve objects that move together, because they are either shoved together or tied together. Here is an example in which you apply Newton's second law to the composite of two blocks and then to the individual blocks.

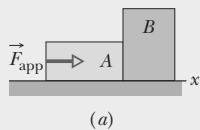
In Fig. 5-18a, a constant horizontal force \vec{F}_{app} of magnitude 20 N is applied to block A of mass $m_A = 4.0 \text{ kg}$, which pushes against block B of mass $m_B = 6.0 \text{ kg}$. The blocks slide over a frictionless surface, along an x axis.

(a) What is the acceleration of the blocks?

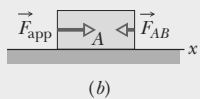
Serious Error: Because force \vec{F}_{app} is applied directly to block A, we use Newton's second law to relate that force to the acceleration \vec{a} of block A. Because the motion is along the x axis, we use that law for x components ($F_{\text{net},x} = ma_x$), writing it as

$$F_{\text{app}} = m_A a.$$

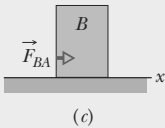
However, this is seriously wrong because \vec{F}_{app} is not the only horizontal force acting on block A. There is also the force \vec{F}_{AB} from block B (Fig. 5-18b).



This force causes the acceleration of the full two-block system.



These are the two forces acting on just block A. Their net force causes its acceleration.



This is the only force causing the acceleration of block B.

Figure 5-18 (a) A constant horizontal force \vec{F}_{app} is applied to block A, which pushes against block B. (b) Two horizontal forces act on block A. (c) Only one horizontal force acts on block B.

during the upward acceleration. However, his acceleration $a_{\text{p,cab}}$ relative to the frame of the cab is zero. Thus, in the non-inertial frame of the accelerating cab, F_{net} is not equal to $ma_{\text{p,cab}}$, and Newton's second law does not hold.

Dead-End Solution: Let us now include force \vec{F}_{AB} by writing, again for the x axis,

$$F_{\text{app}} - F_{AB} = m_A a.$$

(We use the minus sign to include the direction of \vec{F}_{AB} .) Because F_{AB} is a second unknown, we cannot solve this equation for a .

Successful Solution: Because of the direction in which force \vec{F}_{app} is applied, the two blocks form a rigidly connected system. We can relate the net force *on the system* to the acceleration *of the system* with Newton's second law. Here, once again for the x axis, we can write that law as

$$F_{\text{app}} = (m_A + m_B)a,$$

where now we properly apply \vec{F}_{app} to the system with total mass $m_A + m_B$. Solving for a and substituting known values, we find

$$a = \frac{F_{\text{app}}}{m_A + m_B} = \frac{20 \text{ N}}{4.0 \text{ kg} + 6.0 \text{ kg}} = 2.0 \text{ m/s}^2. \quad (\text{Answer})$$

Thus, the acceleration of the system and of each block is in the positive direction of the x axis and has the magnitude 2.0 m/s^2 .

(b) What is the (horizontal) force \vec{F}_{BA} on block B from block A (Fig. 5-18c)?

KEY IDEA

We can relate the net force on block B to the block's acceleration with Newton's second law.

Calculation: Here we can write that law, still for components along the x axis, as

$$F_{BA} = m_B a,$$

which, with known values, gives

$$F_{BA} = (6.0 \text{ kg})(2.0 \text{ m/s}^2) = 12 \text{ N}. \quad (\text{Answer})$$

Thus, force \vec{F}_{BA} is in the positive direction of the x axis and has a magnitude of 12 N.



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Review & Summary

Newtonian Mechanics The velocity of an object can change (the object can accelerate) when the object is acted on by one or more **forces** (pushes or pulls) from other objects. *Newtonian mechanics* relates accelerations and forces.

Force Forces are vector quantities. Their magnitudes are defined in terms of the acceleration they would give the standard kilogram. A force that accelerates that standard body by exactly 1 m/s^2 is defined to have a magnitude of 1 N . The direction of a force is the direction of the acceleration it causes. Forces are combined according to the rules of vector algebra. The **net force** on a body is the vector sum of all the forces acting on the body.

Newton's First Law If there is no net force on a body, the body remains at rest if it is initially at rest or moves in a straight line at constant speed if it is in motion.

Inertial Reference Frames Reference frames in which Newtonian mechanics holds are called *inertial reference frames* or *inertial frames*. Reference frames in which Newtonian mechanics does not hold are called *noninertial reference frames* or *noninertial frames*.

Mass The **mass** of a body is the characteristic of that body that relates the body's acceleration to the net force causing the acceleration. Masses are scalar quantities.

Newton's Second Law The net force \vec{F}_{net} on a body with mass m is related to the body's acceleration \vec{a} by

$$\vec{F}_{\text{net}} = m\vec{a}, \quad (5-1)$$

which may be written in the component versions

$$F_{\text{net},x} = ma_x \quad F_{\text{net},y} = ma_y \quad \text{and} \quad F_{\text{net},z} = ma_z. \quad (5-2)$$

The second law indicates that in SI units

$$1 \text{ N} = 1 \text{ kg} \cdot \text{m/s}^2. \quad (5-3)$$

A **free-body diagram** is a stripped-down diagram in which only *one* body is considered. That body is represented by either a sketch or a dot. The external forces on the body are drawn, and a coordinate system is superimposed, oriented so as to simplify the solution.

Some Particular Forces A **gravitational force** \vec{F}_g on a body is a pull by another body. In most situations in this book, the other body is Earth or some other astronomical body. For Earth, the force is directed down toward the ground, which is assumed to be an inertial frame. With that assumption, the magnitude of \vec{F}_g is

$$F_g = mg, \quad (5-8)$$

where m is the body's mass and g is the magnitude of the free-fall acceleration.

The **weight** W of a body is the magnitude of the upward force needed to balance the gravitational force on the body. A body's weight is related to the body's mass by

$$W = mg. \quad (5-12)$$

A **normal force** \vec{F}_N is the force on a body from a surface against which the body presses. The normal force is always perpendicular to the surface.

A **frictional force** \vec{f} is the force on a body when the body slides or attempts to slide along a surface. The force is always parallel to the surface and directed so as to oppose the sliding. On a *frictionless surface*, the frictional force is negligible.

When a cord is under **tension**, each end of the cord pulls on a body. The pull is directed along the cord, away from the point of attachment to the body. For a *massless cord* (a cord with negligible mass), the pulls at both ends of the cord have the same magnitude T , even if the cord runs around a *massless, frictionless pulley* (a pulley with negligible mass and negligible friction on its axle to oppose its rotation).

Newton's Third Law If a force \vec{F}_{BC} acts on body B due to body C , then there is a force \vec{F}_{CB} on body C due to body B :

$$\vec{F}_{BC} = -\vec{F}_{CB}.$$

Questions

1 Figure 5-19 gives the free-body diagram for four situations in which an object is pulled by several forces across a frictionless floor, as seen from overhead. In which situations does the acceleration \vec{a} of the object have (a) an x component and (b) a y component?

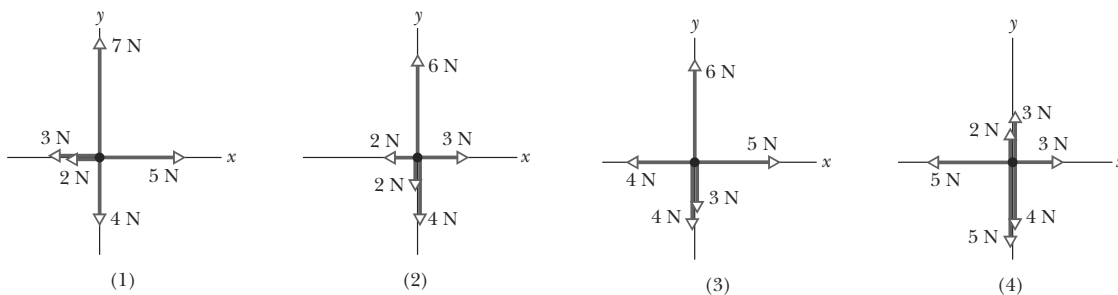


Figure 5-19 Question 1.

(c) In each situation, give the direction of \vec{a} by naming either a quadrant or a direction along an axis. (Don't reach for the calculator because this can be answered with a few mental calculations.)

- 2 Two horizontal forces,

$$\vec{F}_1 = (3 \text{ N})\hat{i} - (4 \text{ N})\hat{j} \quad \text{and} \quad \vec{F}_2 = -(1 \text{ N})\hat{i} - (2 \text{ N})\hat{j}$$

pull a banana split across a frictionless lunch counter. Without using a calculator, determine which of the vectors in the free-body diagram of Fig. 5-20 best represent (a) \vec{F}_1 and (b) \vec{F}_2 . What is the net-force component along (c) the x axis and (d) the y axis? Into which quadrants do (e) the net-force vector and (f) the split's acceleration vector point?

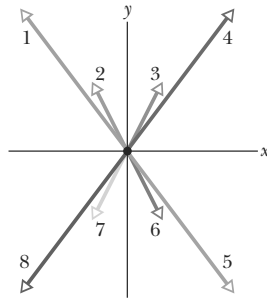


Figure 5-20 Question 2.

- 3 In Fig. 5-21, forces \vec{F}_1 and \vec{F}_2 are applied to a lunchbox as it slides at constant velocity over a frictionless floor. We are to decrease angle θ without changing the magnitude of \vec{F}_1 . For constant velocity, should we increase, decrease, or maintain the magnitude of \vec{F}_2 ?

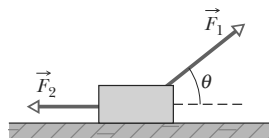


Figure 5-21 Question 3.

- 4 At time $t = 0$, constant \vec{F} begins to act on a rock moving through deep space in the $+x$ direction. (a) For time $t > 0$, which are possible functions $x(t)$ for the rock's position: (1) $x = 4t - 3$, (2) $x = -4t^2 + 6t - 3$, (3) $x = 4t^2 + 6t - 3$? (b) For which function is \vec{F} directed opposite the rock's initial direction of motion?

- 5 Figure 5-22 shows overhead views of four situations in which forces act on a block that lies on a frictionless floor. If the force magnitudes are chosen properly, in which situations is it possible that the block is (a) stationary and (b) moving with a constant velocity?

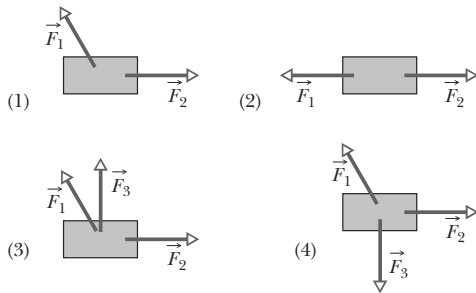


Figure 5-22 Question 5.

- 6 Figure 5-23 shows the same breadbox in four situations where horizontal forces are applied. Rank the situations according to the magnitude of the box's acceleration, greatest first.

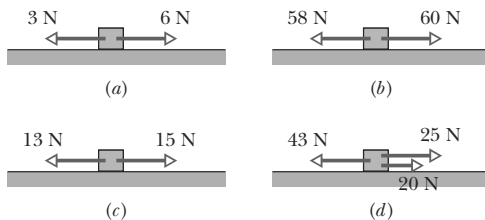


Figure 5-23 Question 6.

- 7 July 17, 1981, Kansas City: The newly opened Hyatt Regency is packed with people listening and dancing to a band playing favorites from the 1940s. Many of the people are crowded onto the walkways that hang like bridges across the wide atrium. Suddenly two of the walkways collapse, falling onto the merry-makers on the main floor.

The walkways were suspended one above another on vertical rods and held in place by nuts threaded onto the rods. In the original design, only two long rods were to be used, each extending through all three walkways (Fig. 5-24a). If each walkway and the merry-makers on it have a combined mass of M , what is the total mass supported by the threads and two nuts on (a) the lowest walkway and (b) the highest walkway?

Apparently someone responsible for the actual construction realized that threading nuts on a rod is impossible except at the ends, so the design was changed: Instead, six rods were used, each connecting two walkways (Fig. 5-24b). What now is the total mass supported by the threads and two nuts on (c) the lowest walkway, (d) the upper side of the highest walkway, and (e) the lower side of the highest walkway? It was this design that failed on that tragic night—a simple engineering error.

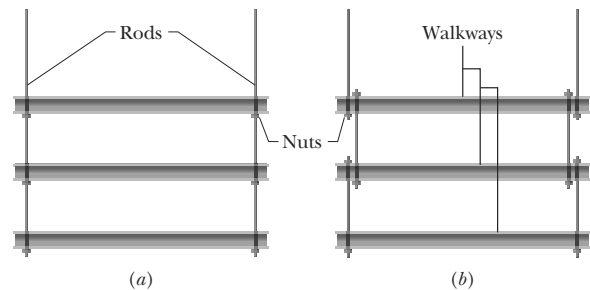


Figure 5-24 Question 7.

- 8 Figure 5-25 gives three graphs of velocity component $v_x(t)$ and three graphs of velocity component $v_y(t)$. The graphs are not to scale. Which $v_x(t)$ graph and which $v_y(t)$ graph best correspond to each of the four situations in Question 1 and Fig. 5-19?

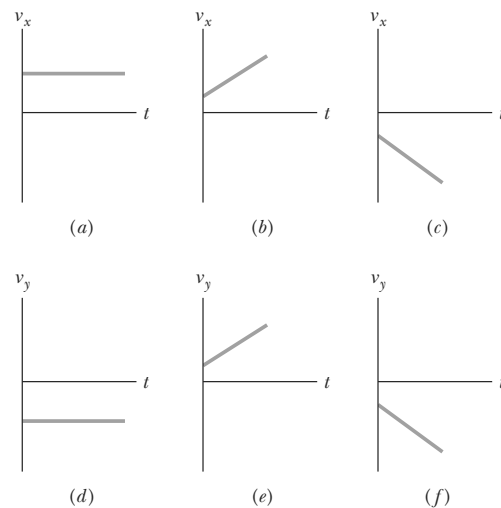


Figure 5-25 Question 8.

9 Figure 5-26 shows a train of four blocks being pulled across a frictionless floor by force \vec{F} . What total mass is accelerated to the right by (a) force \vec{F} , (b) cord 3, and (c) cord 1? (d) Rank the blocks according to their accelerations, greatest first. (e) Rank the cords according to their tension, greatest first.

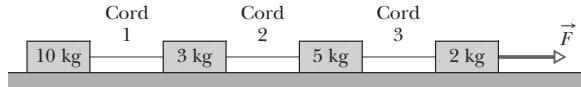


Figure 5-26 Question 9.

10 Figure 5-27 shows three blocks being pushed across a frictionless floor by horizontal force \vec{F} . What total mass is accelerated to the right by (a) force \vec{F} , (b) force \vec{F}_{21} on block 2 from block 1, and (c) force

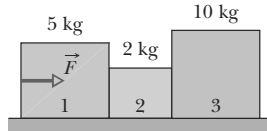


Figure 5-27 Question 10.

\vec{F}_{32} on block 3 from block 2? (d) Rank the blocks according to their acceleration magnitudes, greatest first. (e) Rank forces \vec{F} , \vec{F}_{21} , and \vec{F}_{32} according to magnitude, greatest first.

11 A vertical force \vec{F} is applied to a block of mass m that lies on a floor. What happens to the magnitude of the normal force \vec{F}_N on the block from the floor as magnitude F is increased from zero if force \vec{F} is (a) downward and (b) upward?

12 Figure 5-28 shows four choices for the direction of a force of magnitude F to be applied to a block on an inclined plane. The directions are either horizontal or vertical. (For choice b , the force is not enough to lift the block off the plane.) Rank the choices according to the magnitude of the normal force acting on the block from the plane, greatest first.

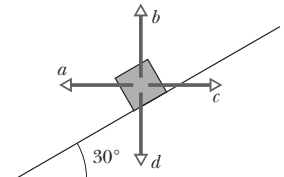


Figure 5-28 Question 12.

Problems



Tutoring problem available (at instructor's discretion) in WileyPLUS and WebAssign



Worked-out solution available in Student Solutions Manual

WWW Worked-out solution is at

<http://www.wiley.com/college/halliday>



Number of dots indicates level of problem difficulty

ILW Interactive solution is at



Additional information available in *The Flying Circus of Physics* and at [flyingcircusofphysics.com](http://www.flyingcircusofphysics.com)

Module 5-1 Newton's First and Second Laws

•1 Only two horizontal forces act on a 3.0 kg body that can move over a frictionless floor. One force is 9.0 N, acting due east, and the other is 8.0 N, acting 62° north of west. What is the magnitude of the body's acceleration?

•2 Two horizontal forces act on a 2.0 kg chopping block that can slide over a frictionless kitchen counter, which lies in an xy plane. One force is $\vec{F}_1 = (3.0 \text{ N})\hat{i} + (4.0 \text{ N})\hat{j}$. Find the acceleration of the chopping block in unit-vector notation when the other force is (a) $\vec{F}_2 = (-3.0 \text{ N})\hat{i} + (-4.0 \text{ N})\hat{j}$, (b) $\vec{F}_2 = (-3.0 \text{ N})\hat{i} + (4.0 \text{ N})\hat{j}$, and (c) $\vec{F}_2 = (3.0 \text{ N})\hat{i} + (-4.0 \text{ N})\hat{j}$.

•3 If the 1 kg standard body has an acceleration of 2.00 m/s^2 at 20.0° to the positive direction of an x axis, what are (a) the x component and (b) the y component of the net force acting on the body, and (c) what is the net force in unit-vector notation?

•4 While two forces act on it, a particle is to move at the constant velocity $\vec{v} = (3 \text{ m/s})\hat{i} - (4 \text{ m/s})\hat{j}$. One of the forces is $\vec{F}_1 = (2 \text{ N})\hat{i} + (-6 \text{ N})\hat{j}$. What is the other force?

•5 GO Three astronauts, propelled by jet backpacks, push and guide a 120 kg asteroid toward a processing dock, exerting the forces shown in Fig. 5-29, with $F_1 = 32 \text{ N}$, $F_2 = 55 \text{ N}$, $F_3 = 41 \text{ N}$, $\theta_1 = 30^\circ$, and $\theta_3 = 60^\circ$. What is the asteroid's acceleration (a) in unit-vector notation and as (b) a magnitude and (c) a direction relative to the positive direction of the x axis?

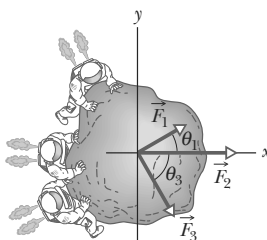


Figure 5-29 Problem 5.

••6 In a two-dimensional tug-of-war, Alex, Betty, and Charles pull horizontally on an automobile tire at the angles shown in the overhead view of Fig. 5-30. The tire remains stationary in spite of the three pulls. Alex pulls with force \vec{F}_A of magnitude 220 N, and Charles pulls with force \vec{F}_C of magnitude 170 N. Note that the direction of \vec{F}_C is not given. What is the magnitude of Betty's force \vec{F}_B ?

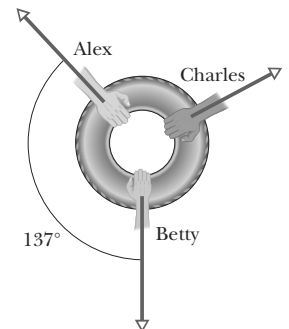


Figure 5-30 Problem 6.

••7 SSM There are two forces on the 2.00 kg box in the overhead view of Fig. 5-31, but only one is shown. For $F_1 = 20.0 \text{ N}$, $a = 12.0 \text{ m/s}^2$, and $\theta = 30.0^\circ$, find the second force (a) in unit-vector notation and as (b) a magnitude and (c) an angle relative to the positive direction of the x axis.

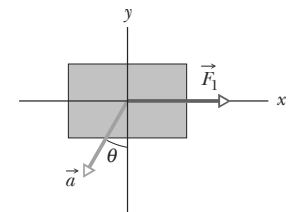



Figure 5-31 Problem 7.


••8 A 2.00 kg object is subjected to three forces that give it an acceleration $\vec{a} = -(8.00 \text{ m/s}^2)\hat{i} + (6.00 \text{ m/s}^2)\hat{j}$. If two of the three forces are $\vec{F}_1 = (30.0 \text{ N})\hat{i} + (16.0 \text{ N})\hat{j}$ and $\vec{F}_2 = -(12.0 \text{ N})\hat{i} + (8.00 \text{ N})\hat{j}$, find the third force.

••9 A 0.340 kg particle moves in an xy plane according to $x(t) = -15.00 + 2.00t - 4.00t^3$ and $y(t) = 25.00 + 7.00t - 9.00t^2$, with x and y in meters and t in seconds. At $t = 0.700 \text{ s}$, what are

(a) the magnitude and (b) the angle (relative to the positive direction of the x axis) of the net force on the particle, and (c) what is the angle of the particle's direction of travel?

••10  A 0.150 kg particle moves along an x axis according to $x(t) = -13.00 + 2.00t + 4.00t^2 - 3.00t^3$, with x in meters and t in seconds. In unit-vector notation, what is the net force acting on the particle at $t = 3.40$ s?

••11 A 2.0 kg particle moves along an x axis, being propelled by a variable force directed along that axis. Its position is given by $x = 3.0 \text{ m} + (4.0 \text{ m/s})t + ct^2 - (2.0 \text{ m/s}^3)t^3$, with x in meters and t in seconds. The factor c is a constant. At $t = 3.0$ s, the force on the particle has a magnitude of 36 N and is in the negative direction of the axis. What is c ?

••12  Two horizontal forces \vec{F}_1 and \vec{F}_2 act on a 4.0 kg disk that slides over frictionless ice, on which an xy coordinate system is laid out. Force \vec{F}_1 is in the positive direction of the x axis and has a magnitude of 7.0 N. Force \vec{F}_2 has a magnitude of 9.0 N. Figure 5-32 gives the x component v_x of the velocity of the disk as a function of time t during the sliding. What is the angle between the constant directions of forces \vec{F}_1 and \vec{F}_2 ?

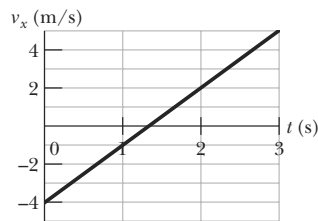


Figure 5-32 Problem 12.

Module 5-2 Some Particular Forces

•13 Figure 5-33 shows an arrangement in which four disks are suspended by cords. The longer, top cord loops over a frictionless pulley and pulls with a force of magnitude 98 N on the wall to which it is attached. The tensions in the three shorter cords are $T_1 = 58.8$ N, $T_2 = 49.0$ N, and $T_3 = 9.8$ N. What are the masses of (a) disk A, (b) disk B, (c) disk C, and (d) disk D?

•14 A block with a weight of 3.0 N is at rest on a horizontal surface. A 1.0 N upward force is applied to the block by means of an attached vertical string. What are the (a) magnitude and (b) direction of the force of the block on the horizontal surface?

•15 **SSM** (a) An 11.0 kg salami is supported by a cord that runs to a spring scale, which is supported by a cord hung from the ceiling (Fig. 5-34a). What is the reading on the scale, which is marked in SI weight units? (This is a way to measure weight by a deli owner.) (b) In Fig. 5-34b the salami is supported by a cord that runs around a pulley and to a scale. The opposite end of the scale is attached by a cord to a wall. What is the reading on the scale? (This is the way by a physics major.) (c) In Fig. 5-34c the wall has been replaced with a second 11.0 kg salami, and the assembly is stationary. What is the

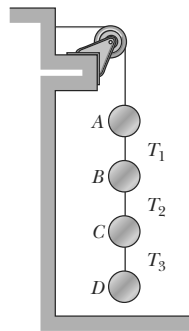


Figure 5-33 Problem 13.

reading on the scale? (This is the way by a deli owner who was once a physics major.)

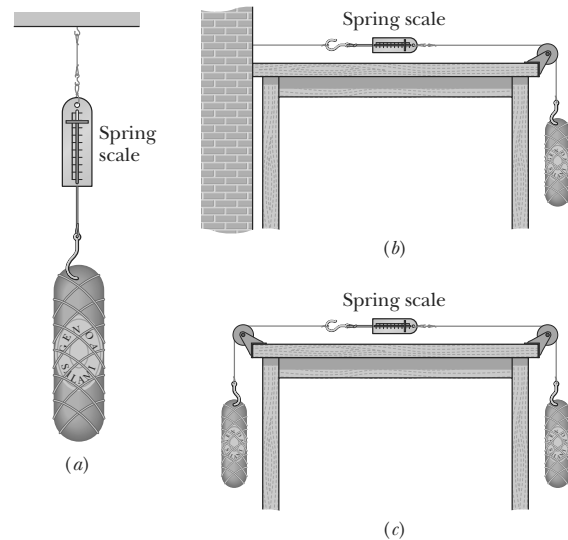


Figure 5-34 Problem 15.

••16 Some insects can walk below a thin rod (such as a twig) by hanging from it. Suppose that such an insect has mass m and hangs from a horizontal rod as shown in Fig. 5-35, with angle $\theta = 40^\circ$. Its six legs are all under the same tension, and the leg sections nearest the body are horizontal. (a) What is the ratio of the tension in each tibia (forepart of a leg) to the insect's weight? (b) If the insect straightens out its legs somewhat, does the tension in each tibia increase, decrease, or stay the same?

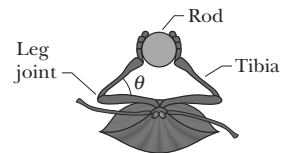


Figure 5-35 Problem 16.

Module 5-3 Applying Newton's Laws

•17 **SSM WWW** In Fig. 5-36, let the mass of the block be 8.5 kg and the angle θ be 30° . Find (a) the tension in the cord and (b) the normal force acting on the block. (c) If the cord is cut, find the magnitude of the resulting acceleration of the block.

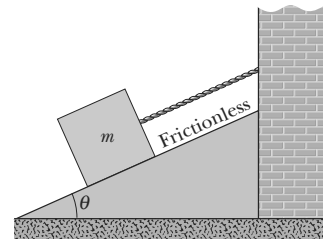



Figure 5-36 Problem 17.

•18  In April 1974, John Massis of Belgium managed to move two passenger railroad cars. He did so by clamping his teeth down on a bit that was attached to the cars with a rope and then leaning backward while pressing his feet against the railway ties. The cars together weighed 700 kN (about 80 tons). Assume that he pulled with a constant force that was 2.5 times his body weight, at an upward angle θ of 30° from the horizontal. His mass was 80 kg, and he moved the cars by 1.0 m. Neglecting any retarding force from the wheel rotation, find the speed of the cars at the end of the pull.

•19 **SSM** A 500 kg rocket sled can be accelerated at a constant rate from rest to 1600 km/h in 1.8 s. What is the magnitude of the required net force?

•20 A car traveling at 53 km/h hits a bridge abutment. A passenger in the car moves forward a distance of 65 cm (with respect to the road) while being brought to rest by an inflated air bag. What magnitude of force (assumed constant) acts on the passenger's upper torso, which has a mass of 41 kg?

•21 A constant horizontal force \vec{F}_a pushes a 2.00 kg FedEx package across a frictionless floor on which an xy coordinate system has been drawn. Figure 5-37 gives the package's x and y velocity components versus time t . What are the (a) magnitude and (b) direction of \vec{F}_a ?

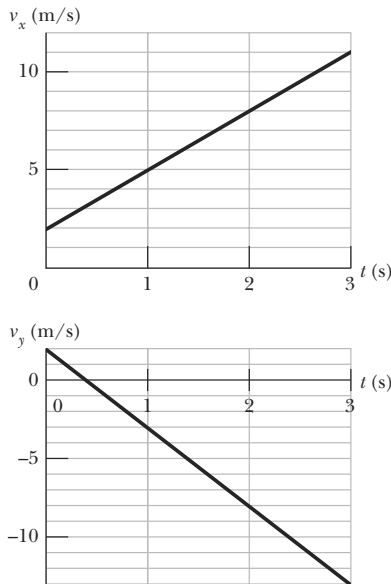


Figure 5-37 Problem 21.

•22 **SSM** A customer sits in an amusement park ride in which the compartment is to be pulled downward in the negative direction of a y axis with an acceleration magnitude of $1.24g$, with $g = 9.80 \text{ m/s}^2$. A 0.567 g coin rests on the customer's knee. Once the motion begins and in unit-vector notation, what is the coin's acceleration relative to (a) the ground and (b) the customer? (c) How long does the coin take to reach the compartment ceiling, 2.20 m above the knee? In unit-vector notation, what are (d) the actual force on the coin and (e) the apparent force according to the customer's measure of the coin's acceleration?

•23 Tarzan, who weighs 820 N, swings from a cliff at the end of a 20.0 m vine that hangs from a high tree limb and initially makes an angle of 22.0° with the vertical. Assume that an x axis extends horizontally away from the cliff edge and a y axis extends upward. Immediately after Tarzan steps off the cliff, the tension in the vine is 760 N. Just then, what are (a) the force on him from the vine in unit-vector notation and the net force on him (b) in unit-vector notation and as (c) a magnitude and (d) an angle relative to the positive direction of the x axis? What are the (e) magnitude and (f) angle of Tarzan's acceleration just then?

•24 There are two horizontal forces on the 2.0 kg box in the overhead view of Fig. 5-38 but only one (of magnitude $F_1 = 20 \text{ N}$) is shown. The box moves along the x axis. For each of the following values for the acceleration a_x of the box, find the second force in unit-vector notation: (a) 10 m/s^2 , (b) 20 m/s^2 , (c) 0, (d) -10 m/s^2 , and (e) -20 m/s^2 .



Figure 5-38 Problem 24.

•25 **Sunjamming.** A "sun yacht" is a spacecraft with a large sail that is pushed by sunlight. Although such a push is tiny in everyday circumstances, it can be large enough to send the spacecraft outward from the Sun on a cost-free but slow trip. Suppose that the spacecraft has a mass of 900 kg and receives a push of 20 N. (a) What is the magnitude of the resulting acceleration? If the craft starts from rest, (b) how far will it travel in 1 day and (c) how fast will it then be moving?

•26 The tension at which a fishing line snaps is commonly called the line's "strength." What minimum strength is needed for a line that is to stop a salmon of weight 85 N in 11 cm if the fish is initially drifting at 2.8 m/s? Assume a constant deceleration.

•27 **SSM** An electron with a speed of $1.2 \times 10^7 \text{ m/s}$ moves horizontally into a region where a constant vertical force of $4.5 \times 10^{-16} \text{ N}$ acts on it. The mass of the electron is $9.11 \times 10^{-31} \text{ kg}$. Determine the vertical distance the electron is deflected during the time it has moved 30 mm horizontally.

•28 A car that weighs $1.30 \times 10^4 \text{ N}$ is initially moving at 40 km/h when the brakes are applied and the car is brought to a stop in 15 m. Assuming the force that stops the car is constant, find (a) the magnitude of that force and (b) the time required for the change in speed. If the initial speed is doubled, and the car experiences the same force during the braking, by what factors are (c) the stopping distance and (d) the stopping time multiplied? (There could be a lesson here about the danger of driving at high speeds.)

•29 A firefighter who weighs 712 N slides down a vertical pole with an acceleration of 3.00 m/s^2 , directed downward. What are the (a) magnitude and (b) direction (up or down) of the vertical force on the firefighter from the pole and the (c) magnitude and (d) direction of the vertical force on the pole from the firefighter?

•30 **SSM** The high-speed winds around a tornado can drive projectiles into trees, building walls, and even metal traffic signs. In a laboratory simulation, a standard wood toothpick was shot by pneumatic gun into an oak branch. The toothpick's mass was 0.13 g, its speed before entering the branch was 220 m/s, and its penetration depth was 15 mm. If its speed was decreased at a uniform rate, what was the magnitude of the force of the branch on the toothpick?

•31 **SSM WWW** A block is projected up a frictionless inclined plane with initial speed $v_0 = 3.50 \text{ m/s}$. The angle of incline is $\theta = 32.0^\circ$. (a) How far up the plane does the block go? (b) How long does it take to get there? (c) What is its speed when it gets back to the bottom?

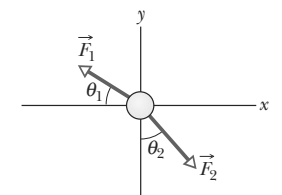


Figure 5-39 Problem 32.

•32 Figure 5-39 shows an overhead view of a 0.0250 kg lemon half and

two of the three horizontal forces that act on it as it is on a frictionless table. Force \vec{F}_1 has a magnitude of 6.00 N and is at $\theta_1 = 30.0^\circ$. Force \vec{F}_2 has a magnitude of 7.00 N and is at $\theta_2 = 30.0^\circ$. In unit-vector notation, what is the third force if the lemon half (a) is stationary, (b) has the constant velocity $\vec{v} = (13.0\hat{i} - 14.0\hat{j})$ m/s, and (c) has the varying velocity $\vec{v} = (13.0t\hat{i} - 14.0t\hat{j})$ m/s², where t is time?

••33 An elevator cab and its load have a combined mass of 1600 kg. Find the tension in the supporting cable when the cab, originally moving downward at 12 m/s, is brought to rest with constant acceleration in a distance of 42 m.

••34 **GO** In Fig. 5-40, a crate of mass $m = 100$ kg is pushed at constant speed up a frictionless ramp ($\theta = 30.0^\circ$) by a horizontal force \vec{F} . What are the magnitudes of (a) \vec{F} and (b) the force on the crate from the ramp?

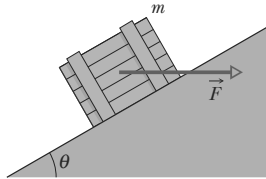


Figure 5-40 Problem 34.

••35 The velocity of a 3.00 kg particle is given by $\vec{v} = (8.00\hat{i} + 3.00t^2\hat{j})$ m/s, with time t in seconds. At the instant the net force on the particle has a magnitude of 35.0 N, what are the direction (relative to the positive direction of the x axis) of (a) the net force and (b) the particle's direction of travel?

••36 Holding on to a towrope moving parallel to a frictionless ski slope, a 50 kg skier is pulled up the slope, which is at an angle of 8.0° with the horizontal. What is the magnitude F_{rope} of the force on the skier from the rope when (a) the magnitude v of the skier's velocity is constant at 2.0 m/s and (b) $v = 2.0$ m/s as v increases at a rate of 0.10 m/s²?

••37 A 40 kg girl and an 8.4 kg sled are on the frictionless ice of a frozen lake, 15 m apart but connected by a rope of negligible mass. The girl exerts a horizontal 5.2 N force on the rope. What are the acceleration magnitudes of (a) the sled and (b) the girl? (c) How far from the girl's initial position do they meet?

••38 A 40 kg skier skis directly down a frictionless slope angled at 10° to the horizontal. Assume the skier moves in the negative direction of an x axis along the slope. A wind force with component F_x acts on the skier. What is F_x if the magnitude of the skier's velocity is (a) constant, (b) increasing at a rate of 1.0 m/s², and (c) increasing at a rate of 2.0 m/s²?

••39 **ILW** A sphere of mass 3.0×10^{-4} kg is suspended from a cord. A steady horizontal breeze pushes the sphere so that the cord makes a constant angle of 37° with the vertical. Find (a) the push magnitude and (b) the tension in the cord.

••40 **GO** A dated box of dates, of mass 5.00 kg, is sent sliding up a frictionless ramp at an angle of θ to the horizontal. Figure 5-41 gives,

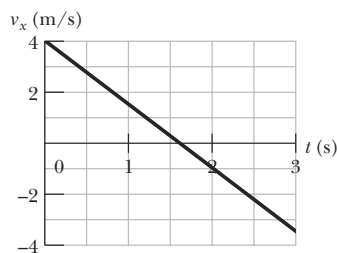


Figure 5-41 Problem 40.

as a function of time t , the component v_x of the box's velocity along an x axis that extends directly up the ramp. What is the magnitude of the normal force on the box from the ramp?

••41 Using a rope that will snap if the tension in it exceeds 387 N, you need to lower a bundle of old roofing material weighing 449 N from a point 6.1 m above the ground. Obviously if you hang the bundle on the rope, it will snap. So, you allow the bundle to accelerate downward. (a) What magnitude of the bundle's acceleration will put the rope on the verge of snapping? (b) At that acceleration, with what speed would the bundle hit the ground?

••42 **GO** In earlier days, horses pulled barges down canals in the manner shown in Fig. 5-42. Suppose the horse pulls on the rope with a force of 7900 N at an angle of $\theta = 18^\circ$ to the direction of motion of the barge, which is headed straight along the positive direction of an x axis. The mass of the barge is 9500 kg, and the magnitude of its acceleration is 0.12 m/s². What are the (a) magnitude and (b) direction (relative to positive x) of the force on the barge from the water?

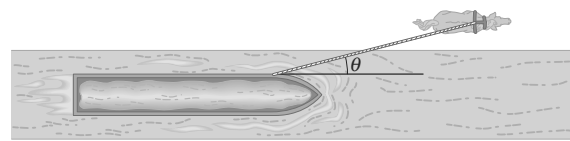


Figure 5-42 Problem 42.

••43 **SSM** In Fig. 5-43, a chain consisting of five links, each of mass 0.100 kg, is lifted vertically with constant acceleration of magnitude $a = 2.50$ m/s². Find the magnitudes of (a) the force on link 1 from link 2, (b) the force on link 2 from link 3, (c) the force on link 3 from link 4, and (d) the force on link 4 from link 5. Then find the magnitudes of (e) the force \vec{F} on the top link from the person lifting the chain and (f) the *net* force accelerating each link.

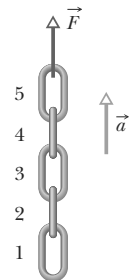


Figure 5-43 Problem 43.

••44 A lamp hangs vertically from a cord in a descending elevator that decelerates at 2.4 m/s². (a) If the tension in the cord is 89 N, what is the lamp's mass? (b) What is the cord's tension when the elevator ascends with an upward acceleration of 2.4 m/s²?

••45 An elevator cab that weighs 27.8 kN moves upward. What is the tension in the cable if the cab's speed is (a) increasing at a rate of 1.22 m/s² and (b) decreasing at a rate of 1.22 m/s²?

••46 An elevator cab is pulled upward by a cable. The cab and its single occupant have a combined mass of 2000 kg. When that occupant drops a coin, its acceleration relative to the cab is 8.00 m/s² downward. What is the tension in the cable?

••47 **GO** The Zacchini family was renowned for their human-cannonball act in which a family member was shot from a cannon using either elastic bands or compressed air. In one version of the act, Emanuel Zacchini was shot over three Ferris wheels to land in a net at the same height as the open end of the cannon and at a range of 69 m. He was propelled inside the barrel for 5.2 m and launched at an angle of 53° . If his mass was 85 kg and he underwent constant acceleration inside the barrel, what was the magnitude of the force propelling him? (*Hint*: Treat the launch as though it were along a ramp at 53° . Neglect air drag.)

••48 GO In Fig. 5-44, elevator cabs *A* and *B* are connected by a short cable and can be pulled upward or lowered by the cable above cab *A*. Cab *A* has mass 1700 kg; cab *B* has mass 1300 kg. A 12.0 kg box of catnip lies on the floor of cab *A*. The tension in the cable connecting the cabs is 1.91×10^4 N. What is the magnitude of the normal force on the box from the floor?

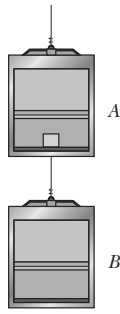


Figure 5-44
Problem 48.

••49 In Fig. 5-45, a block of mass $m = 5.00$ kg is pulled along a horizontal frictionless floor by a cord that exerts a force of magnitude $F = 12.0$ N at an angle $\theta = 25.0^\circ$. (a) What is the magnitude of the block's acceleration? (b) The force magnitude F is slowly increased. What is its value just before the block is lifted (completely) off the floor? (c) What is the magnitude of the block's acceleration just before it is lifted (completely) off the floor?

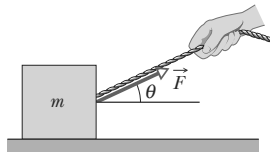


Figure 5-45
Problems 49 and 60.

••50 GO In Fig. 5-46, three ballot boxes are connected by cords, one of which wraps over a pulley having negligible friction on its axle and negligible mass. The three masses are $m_A = 30.0$ kg, $m_B = 40.0$ kg, and $m_C = 10.0$ kg. When the assembly is released from rest, (a) what is the tension in the cord connecting *B* and *C*, and (b) how far does *A* move in the first 0.250 s (assuming it does not reach the pulley)?

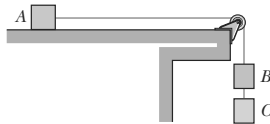


Figure 5-46 Problem 50.

••51 GO Figure 5-47 shows two blocks connected by a cord (of negligible mass) that passes over a frictionless pulley (also of negligible mass). The arrangement is known as *Atwood's machine*. One block has mass $m_1 = 1.30$ kg; the other has mass $m_2 = 2.80$ kg. What are (a) the magnitude of the blocks' acceleration and (b) the tension in the cord?

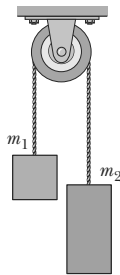


Figure 5-47
Problems 51 and 65.

••52 An 85 kg man lowers himself to the ground from a height of 10.0 m by holding onto a rope that runs over a frictionless pulley to a 65 kg sandbag. With what speed does the man hit the ground if he started from rest?

••53 In Fig. 5-48, three connected blocks are pulled to the right on a horizontal frictionless table by a force of magnitude $T_3 = 65.0$ N. If $m_1 = 12.0$ kg, $m_2 = 24.0$ kg, and $m_3 = 31.0$ kg, calculate (a) the magnitude of the system's acceleration, (b) the tension T_1 , and (c) the tension T_2 .

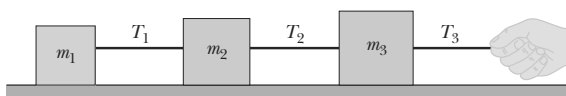


Figure 5-48 Problem 53.

••54 GO Figure 5-49 shows four penguins that are being playfully pulled along very slippery (frictionless) ice by a curator. The masses of three penguins and the tension in two of the cords are $m_1 = 12$ kg, $m_3 = 15$ kg, $m_4 = 20$ kg, $T_2 = 111$ N, and $T_4 = 222$ N. Find the penguin mass m_2 that is not given.

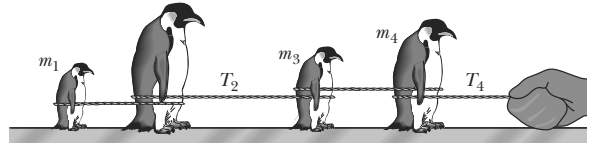


Figure 5-49 Problem 54.

••55 SSM ILW WWW Two blocks are in contact on a frictionless table. A horizontal force is applied to the larger block, as shown in Fig. 5-50. (a) If $m_1 = 2.3$ kg, $m_2 = 1.2$ kg, and $F = 3.2$ N, find the magnitude of the force between the two blocks. (b) Show that if a force of the same magnitude F is applied to the smaller block but in the opposite direction, the magnitude of the force between the blocks is 2.1 N, which is not the same value calculated in (a). (c) Explain the difference.

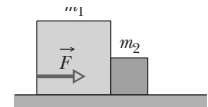


Figure 5-50
Problem 55.

••56 GO In Fig. 5-51a, a constant horizontal force \vec{F}_a is applied to block *A*, which pushes against block *B* with a 20.0 N force directed horizontally to the right. In Fig. 5-51b, the same force \vec{F}_a is applied to block *B*; now block *A* pushes on block *B* with a 10.0 N force directed horizontally to the left. The blocks have a combined mass of 12.0 kg. What are the magnitudes of (a) their acceleration in Fig. 5-51a and (b) force \vec{F}_a ?

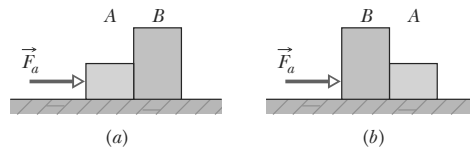


Figure 5-51 Problem 56.

••57 ILW A block of mass $m_1 = 3.70$ kg on a frictionless plane inclined at angle $\theta = 30.0^\circ$ is connected by a cord over a massless, frictionless pulley to a second block of mass $m_2 = 2.30$ kg (Fig. 5-52). What are (a) the magnitude of the acceleration of each block, (b) the direction of the acceleration of the hanging block, and (c) the tension in the cord?

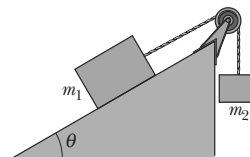


Figure 5-52 Problem 57.

••58 Figure 5-53 shows a man sitting in a bosun's chair that dangles from a massless rope, which runs over a massless, frictionless pulley and back down to the man's hand. The combined mass of man and chair is 95.0 kg. With what force magnitude must the man pull on the rope if he is to rise (a) with a constant velocity and

(b) with an upward acceleration of 1.30 m/s^2 ? (*Hint: A free-body diagram can really help.*) If the rope on the right extends to the ground and is pulled by a co-worker, with what force magnitude must the co-worker pull for the man to rise (c) with a constant velocity and (d) with an upward acceleration of 1.30 m/s^2 ? What is the magnitude of the force on the ceiling from the pulley system in (e) part a, (f) part b, (g) part c, and (h) part d?

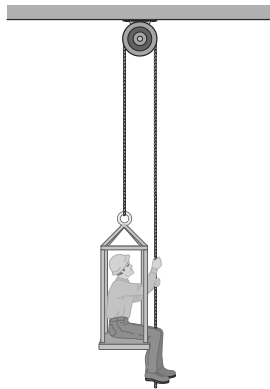


Figure 5-53 Problem 58.

••59 **SSM** A 10 kg monkey climbs up a massless rope that runs over a frictionless tree limb and back down to a 15 kg package on the ground (Fig. 5-54). (a) What is the magnitude of the least acceleration the monkey must have if it is to lift the package off the ground? If, after the package has been lifted, the monkey stops its climb and holds onto the rope, what are the (b) magnitude and (c) direction of the monkey's acceleration and (d) the tension in the rope?

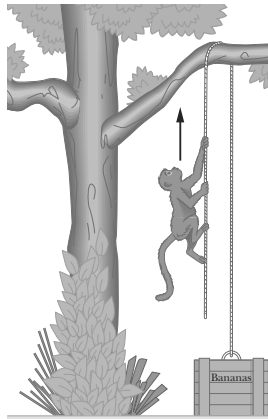


Figure 5-54 Problem 59.

••60 Figure 5-45 shows a 5.00 kg block being pulled along a frictionless floor by a cord that applies a force of constant magnitude 20.0 N but with an angle $\theta(t)$ that varies with time. When angle $\theta = 25.0^\circ$, at what rate is the acceleration of the block changing if (a) $\theta(t) = (2.00 \times 10^{-2} \text{ deg/s})t$ and (b) $\theta(t) = -(2.00 \times 10^{-2} \text{ deg/s})t$? (*Hint: The angle should be in radians.*)

••61 **SSM ILW** A hot-air balloon of mass M is descending vertically with downward acceleration of magnitude a . How much mass (ballast) must be thrown out to give the balloon an upward acceleration of magnitude a ? Assume that the upward force from the air (the lift) does not change because of the decrease in mass.

••62 In shot putting, many athletes elect to launch the shot at an angle that is smaller than the theoretical one (about 42°) at which the distance of a projected ball at the same speed and height is greatest. One reason has to do with the speed the athlete can give the shot during the acceleration phase of the throw. Assume that a 7.260 kg shot is accelerated along a straight path of length 1.650 m by a constant applied force of magnitude 380.0 N , starting with an initial speed of 2.500 m/s (due to the athlete's preliminary motion). What is the shot's speed at the end of the acceleration phase if the angle between the path and the horizontal is (a) 30.00° and (b) 42.00° ? (*Hint: Treat the motion as though it were along a ramp at the given angle.*) (c) By what percent is the launch speed decreased if the athlete increases the angle from 30.00° to 42.00° ?

••63 **GO** Figure 5-55 gives, as a function of time t , the force component F_x that acts on a 3.00 kg ice block that can move only along the x axis. At $t = 0$, the block is moving in the positive direction of

the axis, with a speed of 3.0 m/s . What are its (a) speed and (b) direction of travel at $t = 11 \text{ s}$?

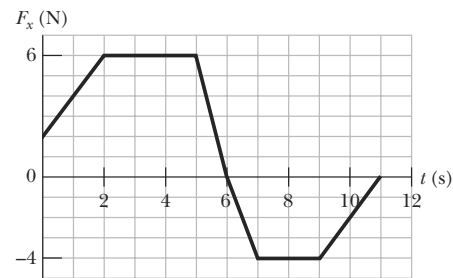


Figure 5-55 Problem 63.

••64 **GO** Figure 5-56 shows a box of mass $m_2 = 1.0 \text{ kg}$ on a frictionless plane inclined at angle $\theta = 30^\circ$. It is connected by a cord of negligible mass to a box of mass $m_1 = 3.0 \text{ kg}$ on a horizontal frictionless surface. The pulley is frictionless and massless. (a) If the magnitude of horizontal force \vec{F} is 2.3 N , what is the tension in the connecting cord? (b) What is the largest value the magnitude of \vec{F} may have without the cord becoming slack?

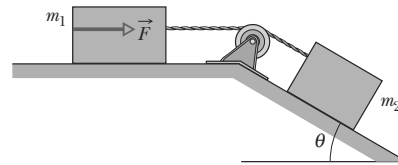


Figure 5-56 Problem 64.

••65 **GO** Figure 5-47 shows *Atwood's machine*, in which two containers are connected by a cord (of negligible mass) passing over a frictionless pulley (also of negligible mass). At time $t = 0$, container 1 has mass 1.30 kg and container 2 has mass 2.80 kg , but container 1 is losing mass (through a leak) at the constant rate of 0.200 kg/s . At what rate is the acceleration magnitude of the containers changing at (a) $t = 0$ and (b) $t = 3.00 \text{ s}$? (c) When does the acceleration reach its maximum value?

••66 **GO** Figure 5-57 shows a section of a cable-car system. The maximum permissible mass of each car with occupants is 2800 kg . The cars, riding on a support cable, are pulled by a second cable attached to the support tower on each car. Assume that the cables

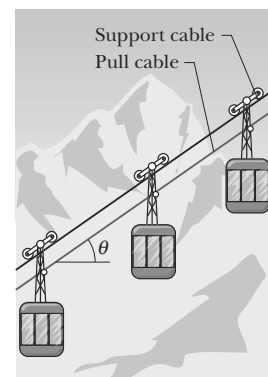


Figure 5-57 Problem 66.

are taut and inclined at angle $\theta = 35^\circ$. What is the difference in tension between adjacent sections of pull cable if the cars are at the maximum permissible mass and are being accelerated up the incline at 0.81 m/s^2 ?

***67 Figure 5-58 shows three blocks attached by cords that loop over frictionless pulleys. Block B lies on a frictionless table; the masses are $m_A = 6.00 \text{ kg}$, $m_B = 8.00 \text{ kg}$, and $m_C = 10.0 \text{ kg}$. When the blocks are released, what is the tension in the cord at the right?

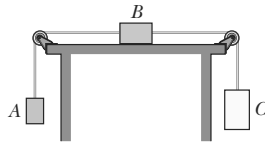


Figure 5-58 Problem 67.

***68 A shot putter launches a 7.260 kg shot by pushing it along a straight line of length 1.650 m and at an angle of 34.10° from the horizontal, accelerating the shot to the launch speed from its initial speed of 2.500 m/s (which is due to the athlete's preliminary motion). The shot leaves the hand at a height of 2.110 m and at an angle of 34.10° , and it lands at a horizontal distance of 15.90 m . What is the magnitude of the athlete's average force on the shot during the acceleration phase? (Hint: Treat the motion during the acceleration phase as though it were along a ramp at the given angle.)

Additional Problems

69 In Fig. 5-59, 4.0 kg block A and 6.0 kg block B are connected by a string of negligible mass. Force $\vec{F}_A = (12 \text{ N})\hat{i}$ acts on block A ; force $\vec{F}_B = (24 \text{ N})\hat{i}$ acts on block B . What is the tension in the string?

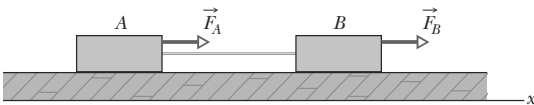


Figure 5-59 Problem 69.

70 An 80 kg man drops to a concrete patio from a window 0.50 m above the patio. He neglects to bend his knees on landing, taking 2.0 cm to stop. (a) What is his average acceleration from when his feet first touch the patio to when he stops? (b) What is the magnitude of the average stopping force exerted on him by the patio?

71 SSM Figure 5-60 shows a box of dirty money (mass $m_1 = 3.0 \text{ kg}$) on a frictionless plane inclined at angle $\theta_1 = 30^\circ$. The box is connected via a cord of negligible mass to a box of laundered money (mass $m_2 = 2.0 \text{ kg}$) on a frictionless plane inclined at angle $\theta_2 = 60^\circ$. The pulley is frictionless and has negligible mass. What is the tension in the cord?

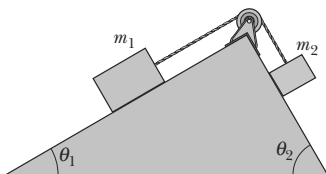


Figure 5-60 Problem 71.

72 Three forces act on a particle that moves with unchanging velocity $\vec{v} = (2 \text{ m/s})\hat{i} - (7 \text{ m/s})\hat{j}$. Two of the forces are $\vec{F}_1 = (2 \text{ N})\hat{i} + (3 \text{ N})\hat{j} + (-2 \text{ N})\hat{k}$ and $\vec{F}_2 = (-5 \text{ N})\hat{i} + (8 \text{ N})\hat{j} + (-2 \text{ N})\hat{k}$. What is the third force?

73 SSM In Fig. 5-61, a tin of antioxidants ($m_1 = 1.0 \text{ kg}$) on a frictionless inclined surface is connected to a tin of corned beef ($m_2 = 2.0 \text{ kg}$). The pulley is massless and frictionless. An upward force of magnitude $F = 6.0 \text{ N}$ acts on the corned beef tin, which has a downward acceleration of 5.5 m/s^2 . What are (a) the tension in the connecting cord and (b) angle β ?

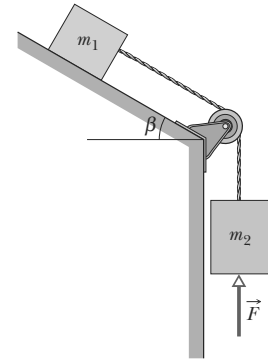


Figure 5-61 Problem 73.

74 The only two forces acting on a body have magnitudes of 20 N and 35 N and directions that differ by 80° . The resulting acceleration has a magnitude of 20 m/s^2 . What is the mass of the body?

75 Figure 5-62 is an overhead view of a 12 kg tire that is to be pulled by three horizontal ropes. One rope's force ($F_1 = 50 \text{ N}$) is indicated. The forces from the other ropes are to be oriented such that the tire's acceleration magnitude a is least. What is that least a if (a) $F_2 = 30 \text{ N}$, $F_3 = 20 \text{ N}$; (b) $F_2 = 30 \text{ N}$, $F_3 = 10 \text{ N}$; and (c) $F_2 = F_3 = 30 \text{ N}$?

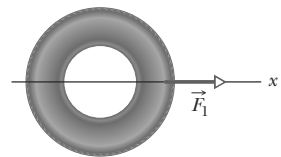


Figure 5-62 Problem 75.

76 A block of mass M is pulled along a horizontal frictionless surface by a rope of mass m , as shown in Fig. 5-63. A horizontal force \vec{F} acts on one end of the rope.

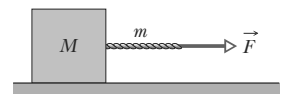


Figure 5-63 Problem 76.

(a) Show that the rope *must* sag, even if only by an imperceptible amount. Then, assuming that the sag is negligible, find (b) the acceleration of rope and block, (c) the force on the block from the rope, and (d) the tension in the rope at its midpoint.

77 SSM A worker drags a crate across a factory floor by pulling on a rope tied to the crate. The worker exerts a force of magnitude $F = 450 \text{ N}$ on the rope, which is inclined at an upward angle $\theta = 38^\circ$ to the horizontal, and the floor exerts a horizontal force of magnitude $f = 125 \text{ N}$ that opposes the motion. Calculate the magnitude of the acceleration of the crate if (a) its mass is 310 kg and (b) its weight is 310 N .

78 In Fig. 5-64, a force \vec{F} of magnitude 12 N is applied to a FedEx box of mass $m_2 = 1.0 \text{ kg}$. The force is directed up a plane tilted by $\theta = 37^\circ$. The box is connected by a cord to a UPS box of mass $m_1 = 3.0 \text{ kg}$ on the floor. The floor, plane, and pulley are frictionless, and the masses of the pulley and cord are negligible. What is the tension in the cord?

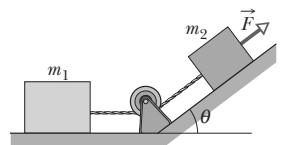


Figure 5-64 Problem 78.

79 A certain particle has a weight of 22 N at a point where $g = 9.8 \text{ m/s}^2$. What are its (a) weight and (b) mass at a point where $g = 4.9 \text{ m/s}^2$? What are its (c) weight and (d) mass if it is moved to a point in space where $g = 0$?

80 An 80 kg person is parachuting and experiencing a downward acceleration of 2.5 m/s^2 . The mass of the parachute is 5.0 kg . (a)

What is the upward force on the open parachute from the air? (b) What is the downward force on the parachute from the person?

81 A spaceship lifts off vertically from the Moon, where $g = 1.6 \text{ m/s}^2$. If the ship has an upward acceleration of 1.0 m/s^2 as it lifts off, what is the magnitude of the force exerted by the ship on its pilot, who weighs 735 N on Earth?

82 In the overhead view of Fig. 5-65, five forces pull on a box of mass $m = 4.0 \text{ kg}$. The force magnitudes are $F_1 = 11 \text{ N}$, $F_2 = 17 \text{ N}$, $F_3 = 3.0 \text{ N}$, $F_4 = 14 \text{ N}$, and $F_5 = 5.0 \text{ N}$, and angle θ_4 is 30° . Find the box's acceleration (a) in unit-vector notation and as (b) a magnitude and (c) an angle relative to the positive direction of the x axis.

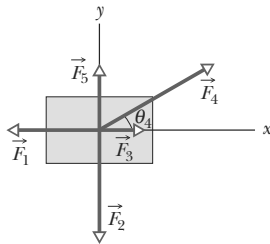


Figure 5-65 Problem 82.

83 SSM A certain force gives an object of mass m_1 an acceleration of 12.0 m/s^2 and an object of mass m_2 an acceleration of 3.30 m/s^2 . What acceleration would the force give to an object of mass (a) $m_2 - m_1$ and (b) $m_2 + m_1$?

84 You pull a short refrigerator with a constant force \vec{F} across a greased (frictionless) floor, either with \vec{F} horizontal (case 1) or with \vec{F} tilted upward at an angle θ (case 2). (a) What is the ratio of the refrigerator's speed in case 2 to its speed in case 1 if you pull for a certain time t ? (b) What is this ratio if you pull for a certain distance d ?

85 A 52 kg circus performer is to slide down a rope that will break if the tension exceeds 425 N. (a) What happens if the performer hangs stationary on the rope? (b) At what magnitude of acceleration does the performer just avoid breaking the rope?

86 Compute the weight of a 75 kg space ranger (a) on Earth, (b) on Mars, where $g = 3.7 \text{ m/s}^2$, and (c) in interplanetary space, where $g = 0$. (d) What is the ranger's mass at each location?

87 An object is hung from a spring balance attached to the ceiling of an elevator cab. The balance reads 65 N when the cab is standing still. What is the reading when the cab is moving upward (a) with a constant speed of 7.6 m/s and (b) with a speed of 7.6 m/s while decelerating at a rate of 2.4 m/s^2 ?

88 Imagine a landing craft approaching the surface of Callisto, one of Jupiter's moons. If the engine provides an upward force (thrust) of 3260 N, the craft descends at constant speed; if the engine provides only 2200 N, the craft accelerates downward at 0.39 m/s^2 . (a) What is the weight of the landing craft in the vicinity of Callisto's surface? (b) What is the mass of the craft? (c) What is the magnitude of the free-fall acceleration near the surface of Callisto?

89 A 1400 kg jet engine is fastened to the fuselage of a passenger jet by just three bolts (this is the usual practice). Assume that each bolt supports one-third of the load. (a) Calculate the force on each bolt as the plane waits in line for clearance to take off. (b) During flight, the plane encounters turbulence, which suddenly imparts an upward vertical acceleration of 2.6 m/s^2 to the plane. Calculate the force on each bolt now.

90 An interstellar ship has a mass of $1.20 \times 10^6 \text{ kg}$ and is initially at rest relative to a star system. (a) What constant acceleration is needed to bring the ship up to a speed of $0.10c$ (where c is the speed of light, $3.0 \times 10^8 \text{ m/s}$) relative to the star system in 3.0 days? (b) What is that

acceleration in g units? (c) What force is required for the acceleration? (d) If the engines are shut down when $0.10c$ is reached (the speed then remains constant), how long does the ship take (start to finish) to journey 5.0 light-months, the distance that light travels in 5.0 months?

91 SSM A motorcycle and 60.0 kg rider accelerate at 3.0 m/s^2 up a ramp inclined 10° above the horizontal. What are the magnitudes of (a) the net force on the rider and (b) the force on the rider from the motorcycle?

92 Compute the initial upward acceleration of a rocket of mass $1.3 \times 10^4 \text{ kg}$ if the initial upward force produced by its engine (the thrust) is $2.6 \times 10^5 \text{ N}$. Do not neglect the gravitational force on the rocket.

93 SSM Figure 5-66a shows a mobile hanging from a ceiling; it consists of two metal pieces ($m_1 = 3.5 \text{ kg}$ and $m_2 = 4.5 \text{ kg}$) that are strung together by cords of negligible mass. What is the tension in (a) the bottom cord and (b) the top cord? Figure 5-66b shows a mobile consisting of three metal pieces. Two of the masses are $m_3 = 4.8 \text{ kg}$ and $m_5 = 5.5 \text{ kg}$. The tension in the top cord is 199 N. What is the tension in (c) the lowest cord and (d) the middle cord?

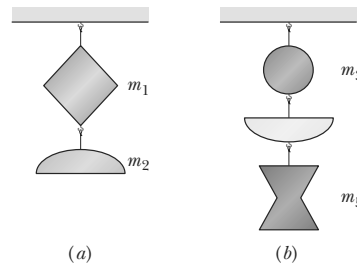


Figure 5-66 Problem 93.

94 For sport, a 12 kg armadillo runs onto a large pond of level, frictionless ice. The armadillo's initial velocity is 5.0 m/s along the positive direction of an x axis. Take its initial position on the ice as being the origin. It slips over the ice while being pushed by a wind with a force of 17 N in the positive direction of the y axis. In unit-vector notation, what are the animal's (a) velocity and (b) position vector when it has slid for 3.0 s?

95 Suppose that in Fig. 5-12, the masses of the blocks are 2.0 kg and 4.0 kg. (a) Which mass should the hanging block have if the magnitude of the acceleration is to be as large as possible? What then are (b) the magnitude of the acceleration and (c) the tension in the cord?

96 A nucleus that captures a stray neutron must bring the neutron to a stop within the diameter of the nucleus by means of the *strong force*. That force, which "glues" the nucleus together, is approximately zero outside the nucleus. Suppose that a stray neutron with an initial speed of $1.4 \times 10^7 \text{ m/s}$ is just barely captured by a nucleus with diameter $d = 1.0 \times 10^{-14} \text{ m}$. Assuming the strong force on the neutron is constant, find the magnitude of that force. The neutron's mass is $1.67 \times 10^{-27} \text{ kg}$.

97 If the 1 kg standard body is accelerated by only $\vec{F}_1 = (3.0 \text{ N})\hat{i} + (4.0 \text{ N})\hat{j}$ and $\vec{F}_2 = (-2.0 \text{ N})\hat{i} + (-6.0 \text{ N})\hat{j}$, then what is \vec{F}_{net} (a) in unit-vector notation and as (b) a magnitude and (c) an angle relative to the positive x direction? What are the (d) magnitude and (e) angle of \vec{a} ?

C H A P T E R 6

Force and Motion–II

6-1 FRICTION

Learning Objectives

After reading this module, you should be able to . . .

- 6.01** Distinguish between friction in a static situation and a kinetic situation.
6.02 Determine direction and magnitude of a frictional force.

- 6.03** For objects on horizontal, vertical, or inclined planes in situations involving friction, draw free-body diagrams and apply Newton's second law.

Key Ideas

- When a force \vec{F} tends to slide a body along a surface, a frictional force from the surface acts on the body. The frictional force is parallel to the surface and directed so as to oppose the sliding. It is due to bonding between the body and the surface.

If the body does not slide, the frictional force is a static frictional force \vec{f}_s . If there is sliding, the frictional force is a kinetic frictional force \vec{f}_k .

- If a body does not move, the static frictional force \vec{f}_s and the component of \vec{F} parallel to the surface are equal in magnitude, and \vec{f}_s is directed opposite that component. If the component increases, f_s also increases.

- The magnitude of \vec{f}_s has a maximum value $f_{s,\max}$ given by

$$f_{s,\max} = \mu_s F_N,$$

where μ_s is the coefficient of static friction and F_N is the magnitude of the normal force. If the component of \vec{F} parallel to the surface exceeds $f_{s,\max}$, the body slides on the surface.

- If the body begins to slide on the surface, the magnitude of the frictional force rapidly decreases to a constant value f_k given by

$$f_k = \mu_k F_N,$$

where μ_k is the coefficient of kinetic friction.

What Is Physics?

In this chapter we focus on the physics of three common types of force: frictional force, drag force, and centripetal force. An engineer preparing a car for the Indianapolis 500 must consider all three types. Frictional forces acting on the tires are crucial to the car's acceleration out of the pit and out of a curve (if the car hits an oil slick, the friction is lost and so is the car). Drag forces acting on the car from the passing air must be minimized or else the car will consume too much fuel and have to pit too early (even one 14 s pit stop can cost a driver the race). Centripetal forces are crucial in the turns (if there is insufficient centripetal force, the car slides into the wall). We start our discussion with frictional forces.

Friction

Frictional forces are unavoidable in our daily lives. If we were not able to counteract them, they would stop every moving object and bring to a halt every rotating shaft. About 20% of the gasoline used in an automobile is needed to counteract friction in the engine and in the drive train. On the other hand, if friction were totally absent, we could not get an automobile to go anywhere, and we could not walk or ride a bicycle. We could not hold a pencil, and, if we could, it would not write. Nails and screws would be useless, woven cloth would fall apart, and knots would untie.

Three Experiments. Here we deal with the frictional forces that exist between dry solid surfaces, either stationary relative to each other or moving across each other at slow speeds. Consider three simple thought experiments:

1. Send a book sliding across a long horizontal counter. As expected, the book slows and then stops. This means the book must have an acceleration parallel to the counter surface, in the direction opposite the book's velocity. From Newton's second law, then, a force must act on the book parallel to the counter surface, in the direction opposite its velocity. That force is a frictional force.
2. Push horizontally on the book to make it travel at constant velocity along the counter. Can the force from you be the only horizontal force on the book? No, because then the book would accelerate. From Newton's second law, there must be a second force, directed opposite your force but with the same magnitude, so that the two forces balance. That second force is a frictional force, directed parallel to the counter.
3. Push horizontally on a heavy crate. The crate does not move. From Newton's second law, a second force must also be acting on the crate to counteract your force. Moreover, this second force must be directed opposite your force and have the same magnitude as your force, so that the two forces balance. That second force is a frictional force. Push even harder. The crate still does not move. Apparently the frictional force can change in magnitude so that the two forces still balance. Now push with all your strength. The crate begins to slide. Evidently, there is a maximum magnitude of the frictional force. When you exceed that maximum magnitude, the crate slides.

Two Types of Friction. Figure 6-1 shows a similar situation. In Fig. 6-1a, a block rests on a tabletop, with the gravitational force \vec{F}_g balanced by a normal force \vec{F}_N . In Fig. 6-1b, you exert a force \vec{F} on the block, attempting to pull it to the left. In response, a frictional force \vec{f}_s is directed to the right, exactly balancing your force. The force \vec{f}_s is called the **static frictional force**. The block does not move.

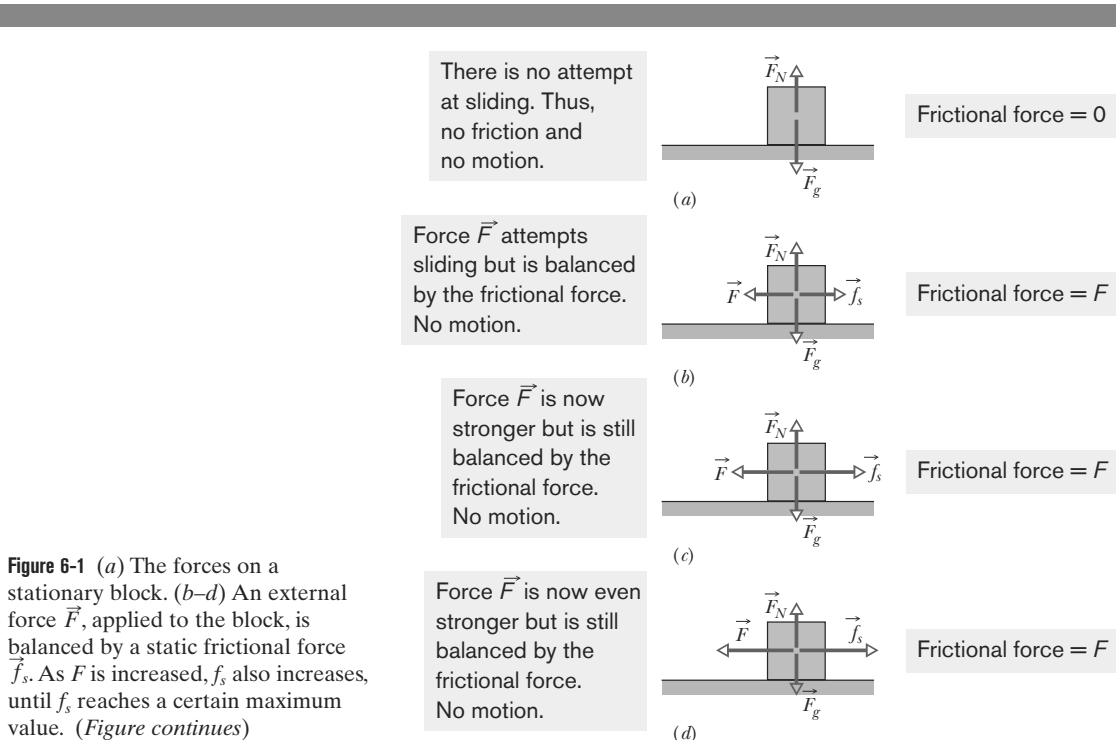
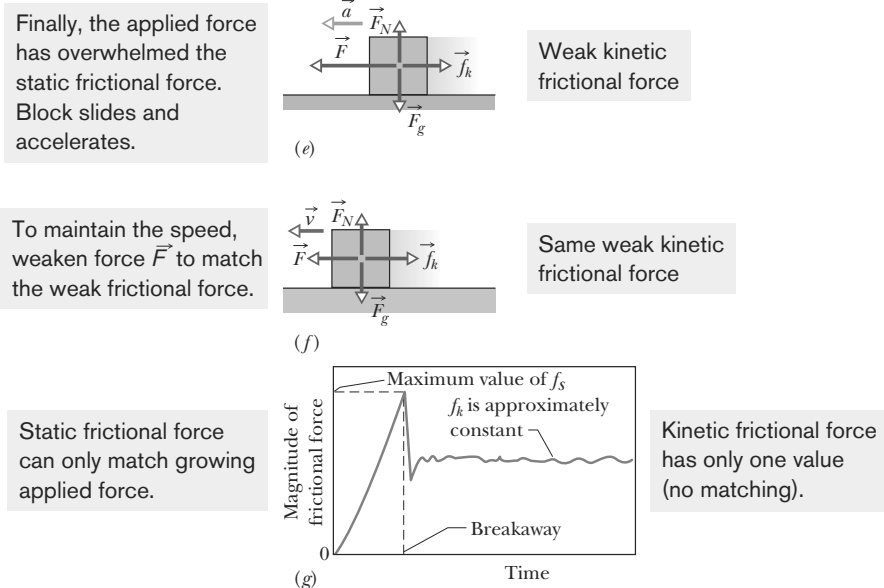


Figure 6-1 (a) The forces on a stationary block. (b–d) An external force \vec{F} , applied to the block, is balanced by a static frictional force \vec{f}_s . As F is increased, f_s also increases, until f_s reaches a certain maximum value. (Figure continues)

Figure 6-1 (Continued) (e) Once f_s reaches its maximum value, the block “breaks away,” accelerating suddenly in the direction of \vec{F} . (f) If the block is now to move with constant velocity, F must be reduced from the maximum value it had just before the block broke away. (g) Some experimental results for the sequence (a) through (f). In **WileyPLUS**, this figure is available as an animation with voiceover.



Figures 6-1c and 6-1d show that as you increase the magnitude of your applied force, the magnitude of the static frictional force \vec{f}_s also increases and the block remains at rest. When the applied force reaches a certain magnitude, however, the block “breaks away” from its intimate contact with the tabletop and accelerates leftward (Fig. 6-1e). The frictional force that then opposes the motion is called the **kinetic frictional force** \vec{f}_k .

Usually, the magnitude of the kinetic frictional force, which acts when there is motion, is less than the maximum magnitude of the static frictional force, which acts when there is no motion. Thus, if you wish the block to move across the surface with a constant speed, you must usually decrease the magnitude of the applied force once the block begins to move, as in Fig. 6-1f. As an example, Fig. 6-1g shows the results of an experiment in which the force on a block was slowly increased until breakaway occurred. Note the reduced force needed to keep the block moving at constant speed after breakaway.


Microscopic View. A frictional force is, in essence, the vector sum of many forces acting between the surface atoms of one body and those of another body. If two highly polished and carefully cleaned metal surfaces are brought together in a very good vacuum (to keep them clean), they cannot be made to slide over each other. Because the surfaces are so smooth, many atoms of one surface contact many atoms of the other surface, and the surfaces *cold-weld* together instantly, forming a single piece of metal. If a machinist’s specially polished gage blocks are brought together in air, there is less atom-to-atom contact, but the blocks stick firmly to each other and can be separated only by means of a wrenching motion. Usually, however, this much atom-to-atom contact is not possible. Even a highly polished metal surface is far from being flat on the atomic scale. Moreover, the surfaces of everyday objects have layers of oxides and other contaminants that reduce cold-welding.

When two ordinary surfaces are placed together, only the high points touch each other. (It is like having the Alps of Switzerland turned over and placed down on the Alps of Austria.) The actual *microscopic* area of contact is much less than the apparent *macroscopic* contact area, perhaps by a factor of 10^4 . Nonetheless,

many contact points do cold-weld together. These welds produce static friction when an applied force attempts to slide the surfaces relative to each other.

If the applied force is great enough to pull one surface across the other, there is first a tearing of welds (at breakaway) and then a continuous re-forming and tearing of welds as movement occurs and chance contacts are made (Fig. 6-2). The kinetic frictional force \vec{f}_k that opposes the motion is the vector sum of the forces at those many chance contacts.

If the two surfaces are pressed together harder, many more points cold-weld. Now getting the surfaces to slide relative to each other requires a greater applied force: The static frictional force \vec{f}_s has a greater maximum value. Once the surfaces are sliding, there are many more points of momentary cold-welding, so the kinetic frictional force \vec{f}_k also has a greater magnitude.

Often, the sliding motion of one surface over another is “jerky” because the two surfaces alternately stick together and then slip. Such repetitive *stick-and-slip* can produce squeaking or squealing, as when tires skid on dry pavement, fingernails scratch along a chalkboard, or a rusty hinge is opened. It can also produce beautiful and captivating sounds, as in music when a bow is drawn properly across a violin string. 

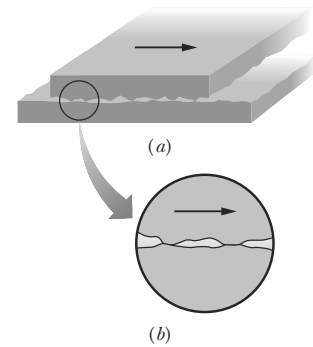


Figure 6-2 The mechanism of sliding friction. (a) The upper surface is sliding to the right over the lower surface in this enlarged view. (b) A detail, showing two spots where cold-welding has occurred. Force is required to break the welds and maintain the motion.

Properties of Friction

Experiment shows that when a dry and unlubricated body presses against a surface in the same condition and a force \vec{F} attempts to slide the body along the surface, the resulting frictional force has three properties:

Property 1. If the body does not move, then the static frictional force \vec{f}_s and the component of \vec{F} that is parallel to the surface balance each other. They are equal in magnitude, and \vec{f}_s is directed opposite that component of \vec{F} .

Property 2. The magnitude of \vec{f}_s has a maximum value $f_{s,\max}$ that is given by

$$f_{s,\max} = \mu_s F_N, \quad (6-1)$$

where μ_s is the **coefficient of static friction** and F_N is the magnitude of the normal force on the body from the surface. If the magnitude of the component of \vec{F} that is parallel to the surface exceeds $f_{s,\max}$, then the body begins to slide along the surface.

Property 3. If the body begins to slide along the surface, the magnitude of the frictional force rapidly decreases to a value f_k given by

$$f_k = \mu_k F_N, \quad (6-2)$$

where μ_k is the **coefficient of kinetic friction**. Thereafter, during the sliding, a kinetic frictional force \vec{f}_k with magnitude given by Eq. 6-2 opposes the motion.

The magnitude F_N of the normal force appears in properties 2 and 3 as a measure of how firmly the body presses against the surface. If the body presses harder, then, by Newton's third law, F_N is greater. Properties 1 and 2 are worded in terms of a single applied force \vec{F} , but they also hold for the net force of several applied forces acting on the body. Equations 6-1 and 6-2 are *not* vector equations; the direction of \vec{f}_s or \vec{f}_k is always parallel to the surface and opposed to the attempted sliding, and the normal force \vec{F}_N is perpendicular to the surface.

The coefficients μ_s and μ_k are dimensionless and must be determined experimentally. Their values depend on certain properties of both the body and the surface; hence, they are usually referred to with the preposition “between,” as in “the value of μ_s *between* an egg and a Teflon-coated skillet is 0.04, but that *between* rock-climbing shoes and rock is as much as 1.2.” We assume that the value of μ_k does not depend on the speed at which the body slides along the surface.

**Checkpoint 1**

A block lies on a floor. (a) What is the magnitude of the frictional force on it from the floor? (b) If a horizontal force of 5 N is now applied to the block, but the block does not move, what is the magnitude of the frictional force on it? (c) If the maximum value $f_{s,\max}$ of the static frictional force on the block is 10 N, will the block move if the magnitude of the horizontally applied force is 8 N? (d) If it is 12 N? (e) What is the magnitude of the frictional force in part (c)?

Sample Problem 6.01 Angled force applied to an initially stationary block

This sample problem involves a tilted applied force, which requires that we work with components to find a frictional force. The main challenge is to sort out all the components. Figure 6-3a shows a force of magnitude $F = 12.0$ N applied to an 8.00 kg block at a downward angle of $\theta = 30.0^\circ$. The coefficient of static friction between block and floor is $\mu_s = 0.700$; the coefficient of kinetic friction is $\mu_k = 0.400$. Does the block begin to slide or does it remain stationary? What is the magnitude of the frictional force on the block?

KEY IDEAS

(1) When the object is stationary on a surface, the static frictional force balances the force component that is attempting to slide the object along the surface. (2) The maximum possible magnitude of that force is given by Eq. 6-1 ($f_{s,\max} = \mu_s F_N$). (3) If the component of the applied force along the surface exceeds this limit on the static friction, the block begins to slide. (4) If the object slides, the kinetic frictional force is given by Eq. 6-2 ($f_k = \mu_k F_N$).

Calculations: To see if the block slides (and thus to calculate the magnitude of the frictional force), we must compare the applied force component F_x with the maximum magnitude $f_{s,\max}$ that the static friction can have. From the triangle of components and full force shown in Fig. 6-3b, we see that

$$\begin{aligned} F_x &= F \cos \theta \\ &= (12.0 \text{ N}) \cos 30^\circ = 10.39 \text{ N}. \end{aligned} \quad (6-3)$$

From Eq. 6-1, we know that $f_{s,\max} = \mu_s F_N$, but we need the magnitude F_N of the normal force to evaluate $f_{s,\max}$. Because the normal force is vertical, we need to write Newton's second law ($F_{\text{net},y} = ma_y$) for the vertical force components acting on the block, as displayed in Fig. 6-3c. The gravitational force with magnitude mg acts downward. The applied force has a downward component $F_y = F \sin \theta$. And the vertical acceleration a_y is just zero. Thus, we can write Newton's sec-

ond law as

$$F_N - mg - F \sin \theta = m(0), \quad (6-4)$$

which gives us

$$F_N = mg + F \sin \theta. \quad (6-5)$$

Now we can evaluate $f_{s,\max} = \mu_s F_N$:

$$\begin{aligned} f_{s,\max} &= \mu_s (mg + F \sin \theta) \\ &= (0.700)((8.00 \text{ kg})(9.8 \text{ m/s}^2) + (12.0 \text{ N})(\sin 30^\circ)) \\ &= 59.08 \text{ N}. \end{aligned} \quad (6-6)$$

Because the magnitude $F_x (= 10.39 \text{ N})$ of the force component attempting to slide the block is less than $f_{s,\max} (= 59.08 \text{ N})$, the block remains stationary. That means that the magnitude f_s of the frictional force matches F_x . From Fig. 6-3d, we can write Newton's second law for x components as

$$F_x - f_s = m(0), \quad (6-7)$$

and thus $f_s = F_x = 10.39 \text{ N} \approx 10.4 \text{ N}$. (Answer)

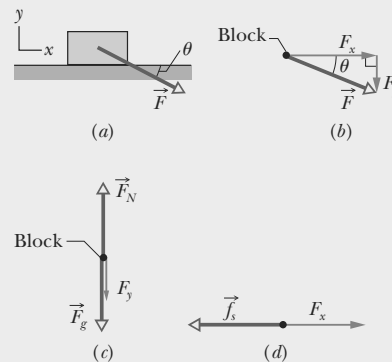


Figure 6-3 (a) A force is applied to an initially stationary block. (b) The components of the applied force. (c) The vertical force components. (d) The horizontal force components.



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Sample Problem 6.02 Sliding to a stop on icy roads, horizontal and inclined

Some of the funniest videos on the web involve motorists sliding uncontrollably on icy roads. Here let's compare the typical stopping distances for a car sliding to a stop from an initial speed of 10.0 m/s on a dry horizontal road, an icy horizontal road, and (everyone's favorite) an icy hill.

(a) How far does the car take to slide to a stop on a horizontal road (Fig. 6-4a) if the coefficient of kinetic friction is $\mu_k = 0.60$, which is typical of regular tires on dry pavement? Let's neglect any effect of the air on the car, assume that the wheels lock up and the tires slide, and extend an x axis in the car's direction of motion.

KEY IDEAS

(1) The car accelerates (its speed decreases) because a horizontal frictional force acts against the motion, in the negative direction of the x axis. (2) The frictional force is a kinetic frictional force with a magnitude given by Eq. 6-2 ($f_k = \mu_k F_N$), in which F_N is the magnitude of the normal force on the car from the road. (3) We can relate the frictional force to the resulting acceleration by writing Newton's second law ($F_{\text{net},x} = ma_x$) for motion along the road.

Calculations: Figure 6-4b shows the free-body diagram for the car. The normal force is upward, the gravitational force is downward, and the frictional force is horizontal. Because the frictional force is the only force with an x component, Newton's second law written for motion along the x axis becomes

$$-f_k = ma_x. \quad (6-8)$$

Substituting $f_k = \mu_k F_N$ gives us

$$-\mu_k F_N = ma_x. \quad (6-9)$$

From Fig. 6-4b we see that the upward normal force balances the downward gravitational force, so in Eq. 6-9 let's replace magnitude F_N with magnitude mg . Then we can cancel m (the stopping distance is thus independent of the car's mass—the car can be heavy or light, it does not matter). Solving for a_x we find

$$a_x = -\mu_k g. \quad (6-10)$$

Because this acceleration is constant, we can use the constant-acceleration equations of Table 2-1. The easiest choice for finding the sliding distance $x - x_0$ is Eq. 2-16 ($v^2 = v_0^2 + 2a(x - x_0)$), which gives us

$$x - x_0 = \frac{v^2 - v_0^2}{2a_x}. \quad (6-11)$$

Substituting from Eq. 6-10, we then have

$$x - x_0 = \frac{v^2 - v_0^2}{-2\mu_k g}. \quad (6-12)$$

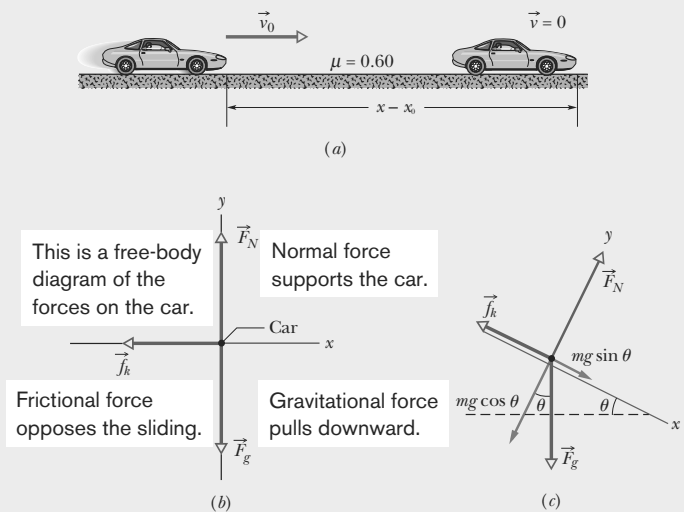


Figure 6-4 (a) A car sliding to the right and finally stopping after a displacement of 290 m. A free-body diagram for the car on (b) a horizontal road and (c) a hill.

Inserting the initial speed $v_0 = 10.0$ m/s, the final speed $v = 0$, and the coefficient of kinetic friction $\mu_k = 0.60$, we find that the car's stopping distance is

$$x - x_0 = 8.50 \text{ m} \approx 8.5 \text{ m}. \quad (\text{Answer})$$

(b) What is the stopping distance if the road is covered with ice with $\mu_k = 0.10$?

Calculation: Our solution is perfectly fine through Eq. 6-12 but now we substitute this new μ_k , finding

$$x - x_0 = 51 \text{ m}. \quad (\text{Answer})$$

Thus, a much longer clear path would be needed to avoid the car hitting something along the way.

(c) Now let's have the car sliding down an icy hill with an inclination of $\theta = 5.00^\circ$ (a mild incline, nothing like the hills of San Francisco). The free-body diagram shown in Fig. 6-4c is like the ramp in Sample Problem 5.04 except, to be consistent with Fig. 6-4b, the positive direction of the x axis is down the ramp. What now is the stopping distance?

Calculations: Switching from Fig. 6-4b to c involves two major changes. (1) Now a component of the gravitational force is along the tilted x axis, pulling the car down the hill. From Sample Problem 5.04 and Fig. 5-15, that down-the-hill component is $mg \sin \theta$, which is in the positive direction of the x axis in Fig. 6-4c. (2) The normal force (still perpendicular to the road) now balances only a component of the gravitational

force, not the full force. From Sample Problem 5.04 (see Fig. 5-15*i*), we write that balance as

$$F_N = mg \cos \theta.$$

In spite of these changes, we still want to write Newton's second law ($F_{\text{net},x} = ma_x$) for the motion along the (now tilted) x axis. We have

$$-f_k + mg \sin \theta = ma_x,$$

$$-\mu_k F_N + mg \sin \theta = ma_x,$$

and
$$-\mu_k mg \cos \theta + mg \sin \theta = ma_x.$$

Solving for the acceleration and substituting the given data

now give us

$$a_x = -\mu_k g \cos \theta + g \sin \theta$$

$$= -(0.10)(9.8 \text{ m/s}^2) \cos 5.00^\circ + (9.8 \text{ m/s}^2) \sin 5.00^\circ$$

$$= -0.122 \text{ m/s}^2. \quad (6-13)$$

Substituting this result into Eq. 6-11 gives us the stopping distance down the hill:

$$x - x_0 = 409 \text{ m} \approx 400 \text{ m}, \quad (\text{Answer})$$

which is about $\frac{1}{4}$ mi! Such icy hills separate people who can do this calculation (and thus know to stay home) from people who cannot (and thus end up in web videos).



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6-2 THE DRAG FORCE AND TERMINAL SPEED

Learning Objectives

After reading this module, you should be able to . . .

6.04 Apply the relationship between the drag force on an object moving through air and the speed of the object.

6.05 Determine the terminal speed of an object falling through air.

Key Ideas

● When there is relative motion between air (or some other fluid) and a body, the body experiences a drag force \vec{D} that opposes the relative motion and points in the direction in which the fluid flows relative to the body. The magnitude of \vec{D} is related to the relative speed v by an experimentally determined drag coefficient C according to

$$D = \frac{1}{2} C \rho A v^2,$$

where ρ is the fluid density (mass per unit volume) and A is the effective cross-sectional area of the body (the area

of a cross section taken perpendicular to the relative velocity \vec{v}).

● When a blunt object has fallen far enough through air, the magnitudes of the drag force \vec{D} and the gravitational force \vec{F}_g on the body become equal. The body then falls at a constant terminal speed v_t given by

$$v_t = \sqrt{\frac{2F_g}{C\rho A}}.$$

The Drag Force and Terminal Speed

A **fluid** is anything that can flow—generally either a gas or a liquid. When there is a relative velocity between a fluid and a body (either because the body moves through the fluid or because the fluid moves past the body), the body experiences a **drag force** \vec{D} that opposes the relative motion and points in the direction in which the fluid flows relative to the body.

Here we examine only cases in which air is the fluid, the body is blunt (like a baseball) rather than slender (like a javelin), and the relative motion is fast enough so that the air becomes turbulent (breaks up into swirls) behind the body. In such cases, the magnitude of the drag force \vec{D} is related to the relative speed v by an experimentally determined **drag coefficient** C according to

$$D = \frac{1}{2} C \rho A v^2, \quad (6-14)$$

Table 6-1 Some Terminal Speeds in Air

Object	Terminal Speed (m/s)	95% Distance ^a (m)
Shot (from shot put)	145	2500
Sky diver (typical)	60	430
Baseball	42	210
Tennis ball	31	115
Basketball	20	47
Ping-Pong ball	9	10
Raindrop (radius = 1.5 mm)	7	6
Parachutist (typical)	5	3

^aThis is the distance through which the body must fall from rest to reach 95% of its terminal speed. Based on Peter J. Brancazio, *Sport Science*, 1984, Simon & Schuster, New York.

where ρ is the air density (mass per volume) and A is the **effective cross-sectional area** of the body (the area of a cross section taken perpendicular to the velocity \vec{v}). The drag coefficient C (typical values range from 0.4 to 1.0) is not truly a constant for a given body because if v varies significantly, the value of C can vary as well. Here, we ignore such complications.

Downhill speed skiers know well that drag depends on A and v^2 . To reach high speeds a skier must reduce D as much as possible by, for example, riding the skis in the “egg position” (Fig. 6-5) to minimize A .

Falling. When a blunt body falls from rest through air, the drag force \vec{D} is directed upward; its magnitude gradually increases from zero as the speed of the body increases. This upward force \vec{D} opposes the downward gravitational force \vec{F}_g on the body. We can relate these forces to the body’s acceleration by writing Newton’s second law for a vertical y axis ($F_{\text{net},y} = ma_y$) as

$$D - F_g = ma, \quad (6-15)$$

where m is the mass of the body. As suggested in Fig. 6-6, if the body falls long enough, D eventually equals F_g . From Eq. 6-15, this means that $a = 0$, and so the body’s speed no longer increases. The body then falls at a constant speed, called the **terminal speed** v_t .

To find v_t , we set $a = 0$ in Eq. 6-15 and substitute for D from Eq. 6-14, obtaining

$$\frac{1}{2}C\rho A v_t^2 - F_g = 0,$$

which gives

$$v_t = \sqrt{\frac{2F_g}{C\rho A}}. \quad (6-16)$$

Table 6-1 gives values of v_t for some common objects.

According to calculations* based on Eq. 6-14, a cat must fall about six floors to reach terminal speed. Until it does so, $F_g > D$ and the cat accelerates downward because of the net downward force. Recall from Chapter 2 that your body is an accelerometer, not a speedometer. Because the cat also senses the acceleration, it is frightened and keeps its feet underneath its body, its head tucked in, and its spine bent upward, making A small, v_t large, and injury likely.

However, if the cat does reach v_t during a longer fall, the acceleration vanishes and the cat relaxes somewhat, stretching its legs and neck horizontally outward and

*W. O. Whitney and C. J. Mehlhaff, “High-Rise Syndrome in Cats.” *The Journal of the American Veterinary Medical Association*, 1987.



Karl-Josef Hildenbrand/dpa/Landov LLC

Figure 6-5 This skier crouches in an “egg position” so as to minimize her effective cross-sectional area and thus minimize the air drag acting on her.

As the cat’s speed increases, the upward drag force increases until it balances the gravitational force.

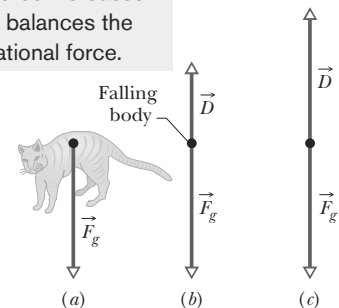


Figure 6-6 The forces that act on a body falling through air: (a) the body when it has just begun to fall and (b) the free-body diagram a little later, after a drag force has developed. (c) The drag force has increased until it balances the gravitational force on the body. The body now falls at its constant terminal speed.



Steve Fitchett/Taxi/Getty Images

Figure 6-7 Sky divers in a horizontal “spread eagle” maximize air drag.

straightening its spine (it then resembles a flying squirrel). These actions increase area A and thus also, by Eq. 6-14, the drag D . The cat begins to slow because now $D > F_g$ (the net force is upward), until a new, smaller v_t is reached. The decrease in v_t reduces the possibility of serious injury on landing. Just before the end of the fall, when it sees it is nearing the ground, the cat pulls its legs back beneath its body to prepare for the landing.

Humans often fall from great heights for the fun of skydiving. However, in April 1987, during a jump, sky diver Gregory Robertson noticed that fellow sky diver Debbie Williams had been knocked unconscious in a collision with a third sky diver and was unable to open her parachute. Robertson, who was well above Williams at the time and who had not yet opened his parachute for the 4 km plunge, reoriented his body head-down so as to minimize A and maximize his downward speed. Reaching an estimated v_t of 320 km/h, he caught up with Williams and then went into a horizontal “spread eagle” (as in Fig. 6-7) to increase D so that he could grab her. He opened her parachute and then, after releasing her, his own, a scant 10 s before impact. Williams received extensive internal injuries due to her lack of control on landing but survived.

Sample Problem 6.03 Terminal speed of falling raindrop

A raindrop with radius $R = 1.5$ mm falls from a cloud that is at height $h = 1200$ m above the ground. The drag coefficient C for the drop is 0.60. Assume that the drop is spherical throughout its fall. The density of water ρ_w is 1000 kg/m^3 , and the density of air ρ_a is 1.2 kg/m^3 .

(a) As Table 6-1 indicates, the raindrop reaches terminal speed after falling just a few meters. What is the terminal speed?

KEY IDEA

The drop reaches a terminal speed v_t when the gravitational force on it is balanced by the air drag force on it, so its acceleration is zero. We could then apply Newton’s second law and the drag force equation to find v_t , but Eq. 6-16 does all that for us.

Calculations: To use Eq. 6-16, we need the drop’s effective cross-sectional area A and the magnitude F_g of the gravitational force. Because the drop is spherical, A is the area of a circle (πR^2) that has the same radius as the sphere. To find F_g , we use three facts: (1) $F_g = mg$, where m is the drop’s mass; (2) the (spherical) drop’s volume is $V = \frac{4}{3}\pi R^3$; and (3) the density of the water in the drop is the mass per volume, or $\rho_w = m/V$. Thus, we find

$$F_g = V\rho_w g = \frac{4}{3}\pi R^3 \rho_w g.$$

We next substitute this, the expression for A , and the given data into Eq. 6-16. Being careful to distinguish between the air den-

sity ρ_a and the water density ρ_w , we obtain

$$\begin{aligned} v_t &= \sqrt{\frac{2F_g}{C\rho_a A}} = \sqrt{\frac{8\pi R^3 \rho_w g}{3C\rho_a \pi R^2}} = \sqrt{\frac{8R\rho_w g}{3C\rho_a}} \\ &= \sqrt{\frac{(8)(1.5 \times 10^{-3} \text{ m})(1000 \text{ kg/m}^3)(9.8 \text{ m/s}^2)}{(3)(0.60)(1.2 \text{ kg/m}^3)}} \\ &= 7.4 \text{ m/s} \approx 27 \text{ km/h.} \end{aligned} \quad (\text{Answer})$$

Note that the height of the cloud does not enter into the calculation.

(b) What would be the drop’s speed just before impact if there were no drag force?

KEY IDEA

With no drag force to reduce the drop’s speed during the fall, the drop would fall with the constant free-fall acceleration g , so the constant-acceleration equations of Table 2-1 apply.

Calculation: Because we know the acceleration is g , the initial velocity v_0 is 0, and the displacement $x - x_0$ is $-h$, we use Eq. 2-16 to find v :

$$\begin{aligned} v &= \sqrt{2gh} = \sqrt{(2)(9.8 \text{ m/s}^2)(1200 \text{ m})} \\ &= 153 \text{ m/s} \approx 550 \text{ km/h.} \end{aligned} \quad (\text{Answer})$$

Had he known this, Shakespeare would scarcely have written, “it droppeth as the gentle rain from heaven, upon the place beneath.” In fact, the speed is close to that of a bullet from a large-caliber handgun!

6-3 UNIFORM CIRCULAR MOTION

Learning Objectives

After reading this module, you should be able to. . .

- 6.06** Sketch the path taken in uniform circular motion and explain the velocity, acceleration, and force vectors (magnitudes and directions) during the motion.
- 6.07** Identify that unless there is a radially inward net force (a centripetal force), an object cannot move in circular motion.

- 6.08** For a particle in uniform circular motion, apply the relationship between the radius of the path, the particle's speed and mass, and the net force acting on the particle.

Key Ideas

- If a particle moves in a circle or a circular arc of radius R at constant speed v , the particle is said to be in uniform circular motion. It then has a centripetal acceleration \vec{a} with magnitude given by

$$a = \frac{v^2}{R}.$$

- This acceleration is due to a net centripetal force on the particle, with magnitude given by

$$F = \frac{mv^2}{R},$$

where m is the particle's mass. The vector quantities \vec{a} and \vec{F} are directed toward the center of curvature of the particle's path.

Uniform Circular Motion

From Module 4-5, recall that when a body moves in a circle (or a circular arc) at constant speed v , it is said to be in uniform circular motion. Also recall that the body has a centripetal acceleration (directed toward the center of the circle) of constant magnitude given by

$$a = \frac{v^2}{R} \quad (\text{centripetal acceleration}), \quad (6-17)$$

where R is the radius of the circle. Here are two examples:

- 1. Rounding a curve in a car.** You are sitting in the center of the rear seat of a car moving at a constant high speed along a flat road. When the driver suddenly turns left, rounding a corner in a circular arc, you slide across the seat toward the right and then jam against the car wall for the rest of the turn. What is going on?

While the car moves in the circular arc, it is in uniform circular motion; that is, it has an acceleration that is directed toward the center of the circle. By Newton's second law, a force must cause this acceleration. Moreover, the force must also be directed toward the center of the circle. Thus, it is a **centripetal force**, where the adjective indicates the direction. In this example, the centripetal force is a frictional force on the tires from the road; it makes the turn possible.

If you are to move in uniform circular motion along with the car, there must also be a centripetal force on you. However, apparently the frictional force on you from the seat was not great enough to make you go in a circle with the car. Thus, the seat slid beneath you, until the right wall of the car jammed into you. Then its push on you provided the needed centripetal force on you, and you joined the car's uniform circular motion.

- 2. Orbiting Earth.** This time you are a passenger in the space shuttle *Atlantis*. As it and you orbit Earth, you float through your cabin. What is going on?

Both you and the shuttle are in uniform circular motion and have accelerations directed toward the center of the circle. Again by Newton's second law, centripetal forces must cause these accelerations. This time the centripetal forces are gravitational pulls (the pull on you and the pull on the shuttle) exerted by Earth and directed radially inward, toward the center of Earth.

In both car and shuttle you are in uniform circular motion, acted on by a centripetal force—yet your sensations in the two situations are quite different. In the car, jammed up against the wall, you are aware of being compressed by the wall. In the orbiting shuttle, however, you are floating around with no sensation of any force acting on you. Why this difference?

The difference is due to the nature of the two centripetal forces. In the car, the centripetal force is the push on the part of your body touching the car wall. You can sense the compression on that part of your body. In the shuttle, the centripetal force is Earth's gravitational pull on every atom of your body. Thus, there is no compression (or pull) on any one part of your body and no sensation of a force acting on you. (The sensation is said to be one of “weightlessness,” but that description is tricky. The pull on you by Earth has certainly not disappeared and, in fact, is only a little less than it would be with you on the ground.)

Another example of a centripetal force is shown in Fig. 6-8. There a hockey puck moves around in a circle at constant speed v while tied to a string looped around a central peg. This time the centripetal force is the radially inward pull on the puck from the string. Without that force, the puck would slide off in a straight line instead of moving in a circle.

Note again that a centripetal force is not a new kind of force. The name merely indicates the direction of the force. It can, in fact, be a frictional force, a gravitational force, the force from a car wall or a string, or any other force. For any situation:



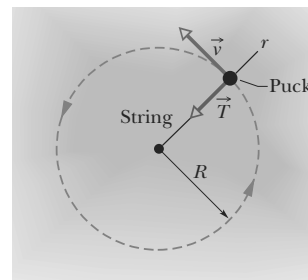
A centripetal force accelerates a body by changing the direction of the body's velocity without changing the body's speed.

From Newton's second law and Eq. 6-17 ($a = v^2/R$), we can write the magnitude F of a centripetal force (or a net centripetal force) as

$$F = m \frac{v^2}{R} \quad (\text{magnitude of centripetal force}). \quad (6-18)$$

Because the speed v here is constant, the magnitudes of the acceleration and the force are also constant.

However, the directions of the centripetal acceleration and force are not constant; they vary continuously so as to always point toward the center of the circle. For this reason, the force and acceleration vectors are sometimes drawn along a radial axis r that moves with the body and always extends from the center of the circle to the body, as in Fig. 6-8. The positive direction of the axis is radially outward, but the acceleration and force vectors point radially inward.



The puck moves in uniform circular motion only because of a toward-the-center force.

Figure 6-8 An overhead view of a hockey puck moving with constant speed v in a circular path of radius R on a horizontal frictionless surface. The centripetal force on the puck is \vec{T} , the pull from the string, directed inward along the radial axis r extending through the puck.



Checkpoint 2

As every amusement park fan knows, a Ferris wheel is a ride consisting of seats mounted on a tall ring that rotates around a horizontal axis. When you ride in a Ferris wheel at constant speed, what are the directions of your acceleration \vec{a} and the normal force \vec{F}_N on you (from the always upright seat) as you pass through (a) the highest point and (b) the lowest point of the ride? (c) How does the magnitude of the acceleration at the highest point compare with that at the lowest point? (d) How do the magnitudes of the normal force compare at those two points?

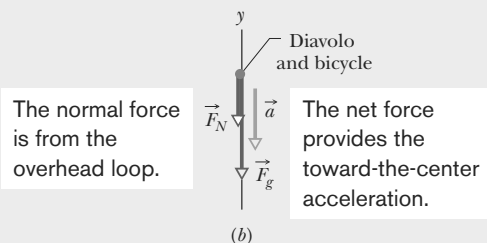
Sample Problem 6.04 Vertical circular loop, Diavolo

Largely because of riding in cars, you are used to horizontal circular motion. Vertical circular motion would be a novelty. In this sample problem, such motion seems to defy the gravitational force.

In a 1901 circus performance, Allo “Dare Devil” Diavolo introduced the stunt of riding a bicycle in a loop-the-loop (Fig. 6-9a). Assuming that the loop is a circle with radius $R = 2.7$ m, what is the least speed v that Diavolo and his bicycle could have at the top of the loop to remain in contact with it there?



(a)



(b)

Figure 6-9 (a) Contemporary advertisement for Diavolo and (b) free-body diagram for the performer at the top of the loop.

KEY IDEA

We can assume that Diavolo and his bicycle travel through the top of the loop as a single particle in uniform circular motion. Thus, at the top, the acceleration \vec{a} of this particle must have the magnitude $a = v^2/R$ given by Eq. 6-17 and be directed downward, toward the center of the circular loop.

Calculations: The forces on the particle when it is at the top of the loop are shown in the free-body diagram of Fig 6-9b. The gravitational force \vec{F}_g is downward along a y axis; so is the normal force \vec{F}_N on the particle from the loop (the loop can push down, not pull up); so also is the centripetal acceleration of the particle. Thus, Newton's second law for y components ($F_{\text{net},y} = ma_y$) gives us

$$-F_N - F_g = m(-a)$$

$$\text{and} \quad -F_N - mg = m\left(-\frac{v^2}{R}\right). \quad (6-19)$$

If the particle has the *least speed* v needed to remain in contact, then it is on the *verge of losing contact* with the loop (falling away from the loop), which means that $F_N = 0$ at the top of the loop (the particle and loop touch but without any normal force). Substituting 0 for F_N in Eq. 6-19, solving for v , and then substituting known values give us

$$\begin{aligned} v &= \sqrt{gR} = \sqrt{(9.8 \text{ m/s}^2)(2.7 \text{ m})} \\ &= 5.1 \text{ m/s.} \end{aligned} \quad (\text{Answer})$$

Comments: Diavolo made certain that his speed at the top of the loop was greater than 5.1 m/s so that he did not lose contact with the loop and fall away from it. Note that this speed requirement is independent of the mass of Diavolo and his bicycle. Had he feasted on, say, pierogies before his performance, he still would have had to exceed only 5.1 m/s to maintain contact as he passed through the top of the loop.



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Sample Problem 6.05 Car in flat circular turn

Upside-down racing: A modern race car is designed so that the passing air pushes down on it, allowing the car to travel much faster through a flat turn in a Grand Prix without friction failing. This downward push is called *negative lift*. Can a race car have so much negative lift that it could be driven upside down on a long ceiling, as done fictionally by a sedan in the first *Men in Black* movie?

Figure 6-10a represents a Grand Prix race car of mass $m = 600$ kg as it travels on a flat track in a circular arc of radius $R = 100$ m. Because of the shape of the car and the wings on it, the passing air exerts a negative lift \vec{F}_L downward on the car. The coefficient of static friction between the tires and the track is 0.75. (Assume that the forces on the four tires are identical.)

(a) If the car is on the verge of sliding out of the turn when its speed is 28.6 m/s, what is the magnitude of the negative lift \vec{F}_L acting downward on the car?

KEY IDEAS

1. A centripetal force must act on the car because the car is moving around a circular arc; that force must be directed toward the center of curvature of the arc (here, that is horizontally).
2. The only horizontal force acting on the car is a frictional force on the tires from the road. So the required centripetal force is a frictional force.
3. Because the car is not sliding, the frictional force must be a *static* frictional force \vec{f}_s (Fig. 6-10a).
4. Because the car is on the verge of sliding, the magnitude f_s is equal to the maximum value $f_{s,\max} = \mu_s F_N$, where F_N is the magnitude of the normal force \vec{F}_N acting on the car from the track.

Radial calculations: The frictional force \vec{f}_s is shown in the free-body diagram of Fig. 6-10b. It is in the negative direction of a radial axis r that always extends from the center of curvature through the car as the car moves. The force produces a centripetal acceleration of magnitude v^2/R . We can relate the force and acceleration by writing Newton's second law for components along the r axis ($F_{\text{net},r} = ma_r$) as

$$-f_s = m \left(-\frac{v^2}{R} \right). \quad (6-20)$$

Substituting $f_{s,\max} = \mu_s F_N$ for f_s leads us to

$$\mu_s F_N = m \left(\frac{v^2}{R} \right). \quad (6-21)$$

Vertical calculations: Next, let's consider the vertical forces on the car. The normal force \vec{F}_N is directed up, in the positive direction of the y axis in Fig. 6-10b. The gravitational force $\vec{F}_g = m\vec{g}$ and the negative lift \vec{F}_L are directed down. The acceleration of the car along the y axis is zero. Thus we can write Newton's second law for components along the y axis ($F_{\text{net},y} = ma_y$) as

$$F_N - mg - F_L = 0, \quad \text{or} \quad F_N = mg + F_L. \quad (6-22)$$

Combining results: Now we can combine our results along the two axes by substituting Eq. 6-22 for F_N in Eq. 6-21. Doing so and then solving for F_L lead to

$$\begin{aligned} F_L &= m \left(\frac{v^2}{\mu_s R} - g \right) \\ &= (600 \text{ kg}) \left(\frac{(28.6 \text{ m/s})^2}{(0.75)(100 \text{ m})} - 9.8 \text{ m/s}^2 \right) \\ &= 663.7 \text{ N} \approx 660 \text{ N}. \end{aligned} \quad (\text{Answer})$$

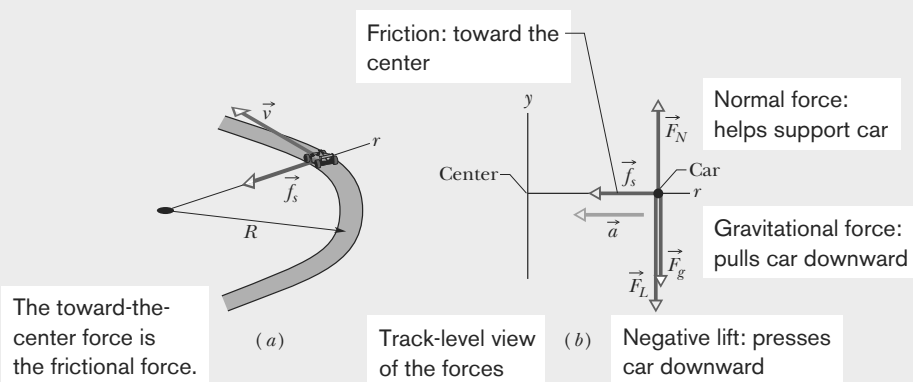


Figure 6-10 (a) A race car moves around a flat curved track at constant speed v . The frictional force \vec{f}_s provides the necessary centripetal force along a radial axis r . (b) A free-body diagram (not to scale) for the car, in the vertical plane containing r .

(b) The magnitude F_L of the negative lift on a car depends on the square of the car's speed v^2 , just as the drag force does (Eq. 6-14). Thus, the negative lift on the car here is greater when the car travels faster, as it does on a straight section of track. What is the magnitude of the negative lift for a speed of 90 m/s?

KEY IDEA

F_L is proportional to v^2 .

Calculations: Thus we can write a ratio of the negative lift $F_{L,90}$ at $v = 90$ m/s to our result for the negative lift F_L at $v = 28.6$ m/s as

$$\frac{F_{L,90}}{F_L} = \frac{(90 \text{ m/s})^2}{(28.6 \text{ m/s})^2}.$$

Sample Problem 6.06 Car in banked circular turn

This problem is quite challenging in setting up but takes only a few lines of algebra to solve. We deal with not only uniformly circular motion but also a ramp. However, we will not need a tilted coordinate system as with other ramps. Instead we can take a freeze-frame of the motion and work with simply horizontal and vertical axes. As always in this chapter, the starting point will be to apply Newton's second law, but that will require us to identify the force component that is responsible for the uniform circular motion.

Curved portions of highways are always banked (tilted) to prevent cars from sliding off the highway. When a highway is dry, the frictional force between the tires and the road surface may be enough to prevent sliding. When the highway is wet, however, the frictional force may be negligible, and banking is then essential. Figure 6-11a represents a car

Substituting our known negative lift of $F_L = 663.7$ N and solving for $F_{L,90}$ give us

$$F_{L,90} = 6572 \text{ N} \approx 6600 \text{ N.} \quad (\text{Answer})$$

Upside-down racing: The gravitational force is, of course, the force to beat if there is a chance of racing upside down:

$$\begin{aligned} F_g &= mg = (600 \text{ kg})(9.8 \text{ m/s}^2) \\ &= 5880 \text{ N.} \end{aligned}$$

With the car upside down, the negative lift is an *upward* force of 6600 N, which exceeds the downward 5880 N. Thus, the car could run on a long ceiling *provided* that it moves at about 90 m/s ($= 324 \text{ km/h} = 201 \text{ mi/h}$). However, moving that fast while right side up on a horizontal track is dangerous enough, so you are not likely to see upside-down racing except in the movies.

of mass m as it moves at a constant speed v of 20 m/s around a banked circular track of radius $R = 190$ m. (It is a normal car, rather than a race car, which means that any vertical force from the passing air is negligible.) If the frictional force from the track is negligible, what bank angle θ prevents sliding?

KEY IDEAS

Here the track is banked so as to tilt the normal force \vec{F}_N on the car toward the center of the circle (Fig. 6-11b). Thus, \vec{F}_N now has a centripetal component of magnitude F_{Nr} , directed inward along a radial axis r . We want to find the value of the bank angle θ such that this centripetal component keeps the car on the circular track without need of friction.

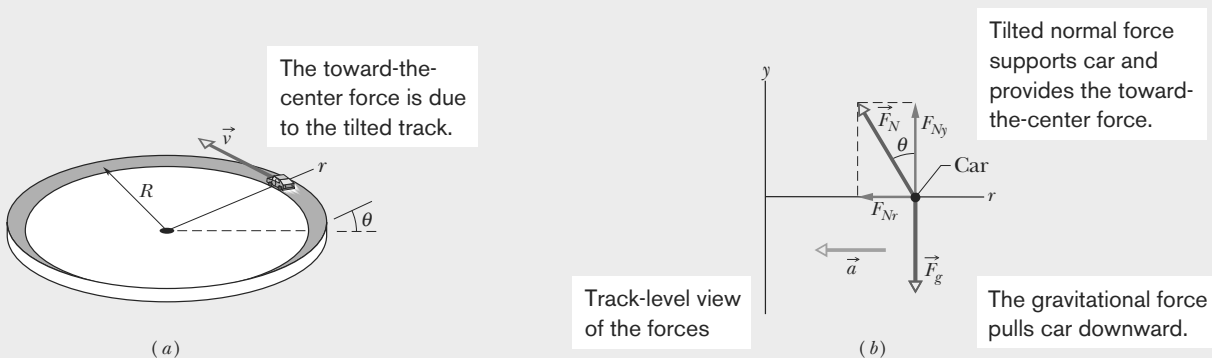


Figure 6-11 (a) A car moves around a curved banked road at constant speed v . The bank angle is exaggerated for clarity. (b) A free-body diagram for the car, assuming that friction between tires and road is zero and that the car lacks negative lift. The radially inward component F_{Nr} of the normal force (along radial axis r) provides the necessary centripetal force and radial acceleration.

Radial calculation: As Fig. 6-11b shows (and as you should verify), the angle that force \vec{F}_N makes with the vertical is equal to the bank angle θ of the track. Thus, the radial component F_{Nr} is equal to $F_N \sin \theta$. We can now write Newton's second law for components along the r axis ($F_{\text{net},r} = ma_r$) as

$$-F_N \sin \theta = m \left(-\frac{v^2}{R} \right). \quad (6-23)$$

We cannot solve this equation for the value of θ because it also contains the unknowns F_N and m .

Vertical calculations: We next consider the forces and acceleration along the y axis in Fig. 6-11b. The vertical component of the normal force is $F_{Ny} = F_N \cos \theta$, the gravitational force \vec{F}_g on the car has the magnitude mg , and the acceleration of the car along the y axis is zero. Thus we can

write Newton's second law for components along the y axis ($F_{\text{net},y} = ma_y$) as

$$F_N \cos \theta - mg = m(0),$$

from which

$$F_N \cos \theta = mg. \quad (6-24)$$

Combining results: Equation 6-24 also contains the unknowns F_N and m , but note that dividing Eq. 6-23 by Eq. 6-24 neatly eliminates both those unknowns. Doing so, replacing $(\sin \theta)/(\cos \theta)$ with $\tan \theta$, and solving for θ then yield

$$\begin{aligned} \theta &= \tan^{-1} \frac{v^2}{gR} \\ &= \tan^{-1} \frac{(20 \text{ m/s})^2}{(9.8 \text{ m/s}^2)(190 \text{ m})} = 12^\circ. \quad (\text{Answer}) \end{aligned}$$



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Review & Summary

Friction When a force \vec{F} tends to slide a body along a surface, a **frictional force** from the surface acts on the body. The frictional force is parallel to the surface and directed so as to oppose the sliding. It is due to bonding between the atoms on the body and the atoms on the surface, an effect called cold-welding.

If the body does not slide, the frictional force is a **static frictional force** \vec{f}_s . If there is sliding, the frictional force is a **kinetic frictional force** \vec{f}_k .

1. If a body does not move, the static frictional force \vec{f}_s and the component of \vec{F} parallel to the surface are equal in magnitude, and \vec{f}_s is directed opposite that component. If the component increases, f_s also increases.
2. The magnitude of \vec{f}_s has a maximum value $f_{s,\text{max}}$ given by

$$f_{s,\text{max}} = \mu_s F_N, \quad (6-1)$$

where μ_s is the **coefficient of static friction** and F_N is the magnitude of the normal force. If the component of \vec{F} parallel to the surface exceeds $f_{s,\text{max}}$, the static friction is overwhelmed and the body slides on the surface.

3. If the body begins to slide on the surface, the magnitude of the frictional force rapidly decreases to a constant value f_k given by

$$f_k = \mu_k F_N, \quad (6-2)$$

where μ_k is the **coefficient of kinetic friction**.

Drag Force When there is relative motion between air (or some other fluid) and a body, the body experiences a **drag force** \vec{D} that opposes the relative motion and points in the direction in which the fluid flows relative to the body. The magnitude of \vec{D} is

related to the relative speed v by an experimentally determined **drag coefficient** C according to

$$D = \frac{1}{2} C \rho A v^2, \quad (6-14)$$

where ρ is the fluid density (mass per unit volume) and A is the **effective cross-sectional area** of the body (the area of a cross section taken perpendicular to the relative velocity \vec{v}).

Terminal Speed When a blunt object has fallen far enough through air, the magnitudes of the drag force \vec{D} and the gravitational force \vec{F}_g on the body become equal. The body then falls at a constant **terminal speed** v_t given by

$$v_t = \sqrt{\frac{2F_g}{C\rho A}}. \quad (6-16)$$

Uniform Circular Motion If a particle moves in a circle or a circular arc of radius R at constant speed v , the particle is said to be in **uniform circular motion**. It then has a **centripetal acceleration** \vec{a} with magnitude given by

$$a = \frac{v^2}{R}. \quad (6-17)$$

This acceleration is due to a net **centripetal force** on the particle, with magnitude given by

$$F = \frac{mv^2}{R}, \quad (6-18)$$

where m is the particle's mass. The vector quantities \vec{a} and \vec{F} are directed toward the center of curvature of the particle's path. A particle can move in circular motion only if a net centripetal force acts on it.

Questions

1 In Fig. 6-12, if the box is stationary and the angle θ between the horizontal and force \vec{F} is increased somewhat, do the following quantities increase, decrease, or remain the same: (a) F_x ; (b) f_s ; (c) F_N ; (d) $f_{s,\max}$? (e) If, instead, the box is sliding and θ is increased, does the magnitude of the frictional force on the box increase, decrease, or remain the same?

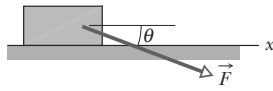


Figure 6-12 Question 1.

2 Repeat Question 1 for force \vec{F} angled upward instead of downward as drawn.

3 In Fig. 6-13, horizontal force \vec{F}_1 of magnitude 10 N is applied to a box on a floor, but the box does not slide. Then, as the magnitude of vertical force \vec{F}_2 is increased from zero, do the following quantities increase, decrease, or stay the same: (a) the magnitude of the frictional force \vec{f}_s on the box; (b) the magnitude of the normal force \vec{F}_N on the box from the floor; (c) the maximum value $f_{s,\max}$ of the magnitude of the static frictional force on the box; (d) Does the box eventually slide?

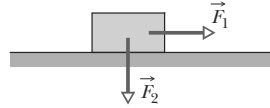


Figure 6-13 Question 3.

4 In three experiments, three different horizontal forces are applied to the same block lying on the same countertop. The force magnitudes are $F_1 = 12$ N, $F_2 = 8$ N, and $F_3 = 4$ N. In each experiment, the block remains stationary in spite of the applied force. Rank the forces according to (a) the magnitude f_s of the static frictional force on the block from the countertop and (b) the maximum value $f_{s,\max}$ of that force, greatest first.

5 If you press an apple crate against a wall so hard that the crate cannot slide down the wall, what is the direction of (a) the static frictional force \vec{f}_s on the crate from the wall and (b) the normal force \vec{F}_N on the crate from the wall? If you increase your push, what happens to (c) f_s , (d) F_N , and (e) $f_{s,\max}$?

6 In Fig. 6-14, a block of mass m is held stationary on a ramp by the frictional force on it from the ramp. A force \vec{F} , directed up the ramp, is then applied to the block and gradually increased in magnitude from zero. During the increase, what happens to the direction and magnitude of the frictional force on the block?

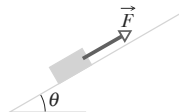


Figure 6-14 Question 6.

7 Reconsider Question 6 but with the force \vec{F} now directed down the ramp. As the magnitude of \vec{F} is increased from zero, what happens to the direction and magnitude of the frictional force on the block?

8 In Fig. 6-15, a horizontal force of 100 N is to be applied to a 10 kg slab that is initially stationary on a frictionless floor, to accelerate the slab. A 10 kg block lies on top of the slab; the coefficient of friction μ between the block and the slab is not known, and the

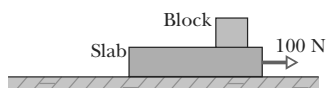


Figure 6-15 Question 8.

block might slip. In fact, the contact between the block and the slab might even be frictionless. (a) Considering that possibility, what is the possible range of values for the magnitude of the slab's acceleration a_{slab} ? (Hint: You don't need written calculations; just consider extreme values for μ .) (b) What is the possible range for the magnitude a_{block} of the block's acceleration?

9 Figure 6-16 shows the overhead view of the path of an amusement-park ride that travels at constant speed through five circular arcs of radii R_0 , $2R_0$, and $3R_0$. Rank the arcs according to the magnitude of the centripetal force on a rider traveling in the arcs, greatest first.

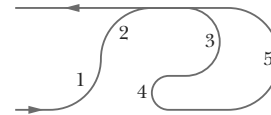


Figure 6-16 Question 9.

10 In 1987, as a Halloween stunt, two sky divers passed a pumpkin back and forth between them while they were in free fall just west of Chicago. The stunt was great fun until the last sky diver with the pumpkin opened his parachute. The pumpkin broke free from his grip, plummeted about 0.5 km, ripped through the roof of a house, slammed into the kitchen floor, and splattered all over the newly remodeled kitchen. From the sky diver's viewpoint and from the pumpkin's viewpoint, why did the sky diver lose control of the pumpkin?

11 A person riding a Ferris wheel moves through positions at (1) the top, (2) the bottom, and (3) midheight. If the wheel rotates at a constant rate, rank these three positions according to (a) the magnitude of the person's centripetal acceleration, (b) the magnitude of the net centripetal force on the person, and (c) the magnitude of the normal force on the person, greatest first.

12 During a routine flight in 1956, test pilot Tom Attridge put his jet fighter into a 20° dive for a test of the aircraft's 20 mm machine cannons. While traveling faster than sound at 4000 m altitude, he shot a burst of rounds. Then, after allowing the cannons to cool, he shot another burst at 2000 m; his speed was then 344 m/s, the speed of the rounds relative to him was 730 m/s, and he was still in a dive.

Almost immediately the canopy around him was shredded and his right air intake was damaged. With little flying capability left, the jet crashed into a wooded area, but Attridge managed to escape the resulting explosion. Explain what apparently happened just after the second burst of cannon rounds. (Attridge has been the only pilot who has managed to shoot himself down.)

13 A box is on a ramp that is at angle θ to the horizontal. As θ is increased from zero, and before the box slips, do the following increase, decrease, or remain the same: (a) the component of the gravitational force on the box, along the ramp, (b) the magnitude of the static frictional force on the box from the ramp, (c) the component of the gravitational force on the box, perpendicular to the ramp, (d) the magnitude of the normal force on the box from the ramp, and (e) the maximum value $f_{s,\max}$ of the static frictional force?

Problems



Tutoring problem available (at instructor's discretion) in WileyPLUS and WebAssign



Worked-out solution available in Student Solutions Manual

WWW Worked-out solution is at



Number of dots indicates level of problem difficulty



Interactive solution is at

<http://www.wiley.com/college/halliday>



Additional information available in *The Flying Circus of Physics* and at flyingcircusofphysics.com

Module 6-1 Friction

•1 The floor of a railroad flatcar is loaded with loose crates having a coefficient of static friction of 0.25 with the floor. If the train is initially moving at a speed of 48 km/h, in how short a distance can the train be stopped at constant acceleration without causing the crates to slide over the floor?

•2 In a pickup game of dorm shuffleboard, students crazed by final exams use a broom to propel a calculus book along the dorm hallway. If the 3.5 kg book is pushed from rest through a distance of 0.90 m by the horizontal 25 N force from the broom and then has a speed of 1.60 m/s, what is the coefficient of kinetic friction between the book and floor?

•3 **SSM WWW** A bedroom bureau with a mass of 45 kg, including drawers and clothing, rests on the floor. (a) If the coefficient of static friction between the bureau and the floor is 0.45, what is the magnitude of the minimum horizontal force that a person must apply to start the bureau moving? (b) If the drawers and clothing, with 17 kg mass, are removed before the bureau is pushed, what is the new minimum magnitude?

•4 A slide-loving pig slides down a certain 35° slide in twice the time it would take to slide down a frictionless 35° slide. What is the coefficient of kinetic friction between the pig and the slide?

•5 **GO** A 2.5 kg block is initially at rest on a horizontal surface. A horizontal force \vec{F} of magnitude 6.0 N and a vertical force \vec{P} are then applied to the block (Fig. 6-17). The coefficients of friction for the block and surface are $\mu_s = 0.40$ and $\mu_k = 0.25$. Determine the magnitude of the frictional force acting on the block if the magnitude of \vec{P} is (a) 8.0 N, (b) 10 N, and (c) 12 N.

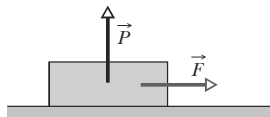


Figure 6-17 Problem 5.

•6 A baseball player with mass $m = 79$ kg, sliding into second base, is retarded by a frictional force of magnitude 470 N. What is the coefficient of kinetic friction μ_k between the player and the ground?

•7 **SSM ILW** A person pushes horizontally with a force of 220 N on a 55 kg crate to move it across a level floor. The coefficient of kinetic friction between the crate and the floor is 0.35. What is the magnitude of (a) the frictional force and (b) the acceleration of the crate?

•8 **The mysterious sliding stones.** Along the remote Racetrack Playa in Death Valley, California, stones sometimes gouge out prominent trails in the desert floor, as if the stones had been migrating (Fig. 6-18). For years curiosity mounted about why the stones moved. One explanation was that strong winds during occasional rainstorms would drag the rough stones

over ground softened by rain. When the desert dried out, the trails behind the stones were hard-baked in place. According to measurements, the coefficient of kinetic friction between the stones and the wet playa ground is about 0.80. What horizontal force must act on a 20 kg stone (a typical mass) to maintain the stone's motion once a gust has started it moving? (Story continues with Problem 37.)



Jerry Schad/Photo Researchers, Inc.

Figure 6-18 Problem 8. What moved the stone?

•9 **GO** A 3.5 kg block is pushed along a horizontal floor by a force \vec{F} of magnitude 15 N at an angle $\theta = 40^\circ$ with the horizontal (Fig. 6-19). The coefficient of kinetic friction between the block and the floor is 0.25. Calculate the magnitudes of (a) the frictional force on the block from the floor and (b) the block's acceleration.

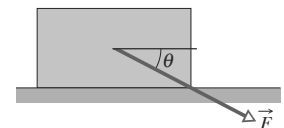


Figure 6-19 Problems 9 and 32.

•10 Figure 6-20 shows an initially stationary block of mass m on a floor. A force of magnitude $0.500mg$ is then applied at upward angle $\theta = 20^\circ$. What is the magnitude of the acceleration of the block across the floor if the friction coefficients are (a) $\mu_s = 0.600$ and $\mu_k = 0.500$ and (b) $\mu_s = 0.400$ and $\mu_k = 0.300$?

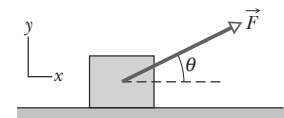


Figure 6-20 Problem 10.

•11 **SSM** A 68 kg crate is dragged across a floor by pulling on a rope attached to the crate and inclined 15° above the horizontal. (a) If the coefficient of static friction is 0.50, what minimum force magnitude is required from the rope to start the crate moving? (b) If $\mu_k = 0.35$, what is the magnitude of the initial acceleration of the crate?

•12 In about 1915, Henry Sincosky of Philadelphia suspended himself from a rafter by gripping the rafter with the thumb of each

hand on one side and the fingers on the opposite side (Fig. 6-21). Sincosky's mass was 79 kg. If the coefficient of static friction between hand and rafter was 0.70, what was the least magnitude of the normal force on the rafter from each thumb or opposite fingers? (After suspending himself, Sincosky chinned himself on the rafter and then moved hand-over-hand along the rafter. If you do not think Sincosky's grip was remarkable, try to repeat his stunt.)

•13 A worker pushes horizontally on a 35 kg crate with a force of magnitude 110 N. The coefficient of static friction between the crate and the floor is 0.37. (a) What is the value of $f_{s,\max}$ under the circumstances? (b) Does the crate move? (c) What is the frictional force on the crate from the floor? (d) Suppose, next, that a second worker pulls directly upward on the crate to help out. What is the least vertical pull that will allow the first worker's 110 N push to move the crate? (e) If, instead, the second worker pulls horizontally to help out, what is the least pull that will get the crate moving?

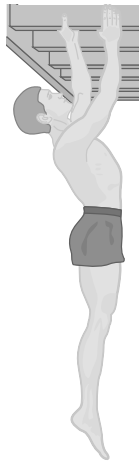


Figure 6-21
Problem 12.

•14 Figure 6-22 shows the cross section of a road cut into the side of a mountain. The solid line AA' represents a weak bedding plane along which sliding is possible. Block B directly above the highway is separated from uphill rock by a large crack (called a *joint*), so that only friction between the block and the bedding plane prevents sliding. The mass of the block is 1.8×10^7 kg, the *dip angle* θ of the bedding plane is 24° , and the coefficient of static friction between block and plane is 0.63. (a) Show that the block will not slide under these circumstances. (b) Next, water seeps into the joint and expands upon freezing, exerting on the block a force \vec{F} parallel to AA' . What minimum value of force magnitude F will trigger a slide down the plane?

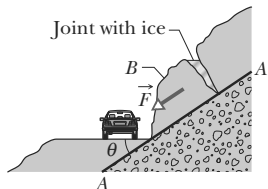


Figure 6-22 Problem 14.

•15 The coefficient of static friction between Teflon and scrambled eggs is about 0.04. What is the smallest angle from the horizontal that will cause the eggs to slide across the bottom of a Teflon-coated skillet?

•16 A loaded penguin sled weighing 80 N rests on a plane inclined at angle $\theta = 20^\circ$ to the horizontal (Fig. 6-23). Between the sled and the plane, the coefficient of static friction is 0.25, and the coefficient of kinetic friction is 0.15. (a) What is the least magnitude of the force \vec{F} , parallel to the plane, that will prevent the sled from slipping down the plane? (b) What is the minimum magnitude F that will start the sled moving up the plane? (c) What value of F is required to move the sled up the plane at constant velocity?

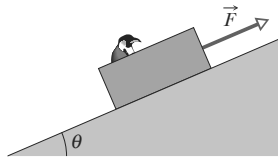


Figure 6-23
Problems 16 and 22.

•17 In Fig. 6-24, a force \vec{P} acts on a block weighing 45 N. The block is

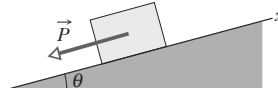


Figure 6-24 Problem 17.

initially at rest on a plane inclined at angle $\theta = 15^\circ$ to the horizontal. The positive direction of the x axis is up the plane. Between block and plane, the coefficient of static friction is $\mu_s = 0.50$ and the coefficient of kinetic friction is $\mu_k = 0.34$. In unit-vector notation, what is the frictional force on the block from the plane when \vec{P} is (a) $(-5.0 \text{ N})\hat{i}$, (b) $(-8.0 \text{ N})\hat{i}$, and (c) $(-15 \text{ N})\hat{i}$?

•18 GO You testify as an *expert witness* in a case involving an accident in which car A slid into the rear of car B , which was stopped at a red light along a road headed down a hill (Fig. 6-25). You find that the slope of the hill is $\theta = 12.0^\circ$, that the cars were separated by distance $d = 24.0$ m when the driver of car A put the car into a slide (it lacked any automatic anti-brake-lock system), and that the speed of car A at the onset of braking was $v_0 = 18.0$ m/s. With what speed did car A hit car B if the coefficient of kinetic friction was (a) 0.60 (dry road surface) and (b) 0.10 (road surface covered with wet leaves)?

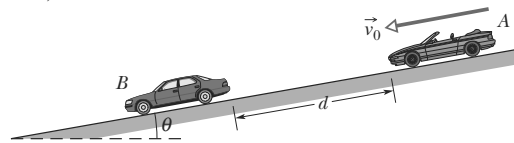


Figure 6-25 Problem 18.

•19 A 12 N horizontal force \vec{F} pushes a block weighing 5.0 N against a vertical wall (Fig. 6-26). The coefficient of static friction between the wall and the block is 0.60, and the coefficient of kinetic friction is 0.40. Assume that the block is not moving initially. (a) Will the block move? (b) In unit-vector notation, what is the force on the block from the wall?

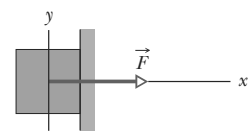


Figure 6-26 Problem 19.

•20 GO In Fig. 6-27, a box of Cheerios (mass $m_C = 1.0$ kg) and a box of Wheaties (mass $m_W = 3.0$ kg) are accelerated across a horizontal surface by a horizontal force \vec{F} applied to the Cheerios box. The magnitude of the frictional force on the Cheerios box is 2.0 N, and the magnitude of the frictional force on the Wheaties box is 4.0 N. If the magnitude of \vec{F} is 12 N, what is the magnitude of the force on the Wheaties box from the Cheerios box?

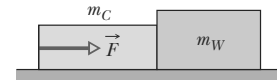


Figure 6-27 Problem 20.

•21 An initially stationary box of sand is to be pulled across a floor by means of a cable in which the tension should not exceed 1100 N. The coefficient of static friction between the box and the floor is 0.35. (a) What should be the angle between the cable and the horizontal in order to pull the greatest possible amount of sand, and (b) what is the weight of the sand and box in that situation?

•22 GO In Fig. 6-23, a sled is held on an inclined plane by a cord pulling directly up the plane. The sled is to be on the verge of moving up the plane. In Fig. 6-28, the magnitude F required of the cord's force on the sled is plotted versus a range of values for the coefficient of static friction μ_s between sled and plane: $F_1 = 2.0$ N, $F_2 = 5.0$ N, and $\mu_2 = 0.50$. At what angle θ is the plane inclined?

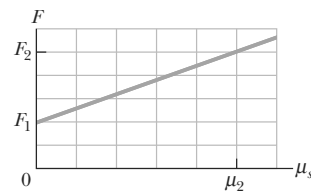


Figure 6-28 Problem 22.

••23 When the three blocks in Fig. 6-29 are released from rest, they accelerate with a magnitude of 0.500 m/s^2 . Block 1 has mass M , block 2 has $2M$, and block 3 has $2M$. What is the coefficient of kinetic friction between block 2 and the table?

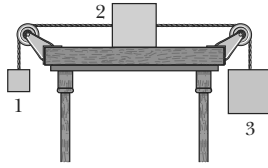


Figure 6-29 Problem 23.

••24 A 4.10 kg block is pushed along a floor by a constant applied force that is horizontal and has a magnitude of 40.0 N . Figure 6-30 gives the block's speed v versus time t as the block moves along an x axis on the floor. The scale of the figure's vertical axis is set by $v_s = 5.0 \text{ m/s}$. What is the coefficient of kinetic friction between the block and the floor?

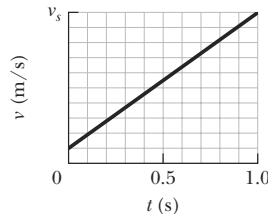


Figure 6-30 Problem 24.

••25 SSM WWW Block B in Fig. 6-31 weighs 711 N . The coefficient of static friction between block and table is 0.25 ; angle θ is 30° ; assume that the cord between B and the knot is horizontal. Find the maximum weight of block A for which the system will be stationary.

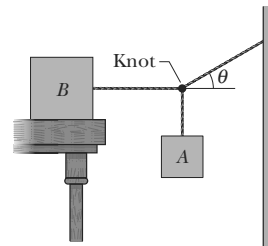


Figure 6-31 Problem 25.

••26 GO Figure 6-32 shows three crates being pushed over a concrete floor by a horizontal force \vec{F} of magnitude 440 N . The masses of the crates are $m_1 = 30.0 \text{ kg}$, $m_2 = 10.0 \text{ kg}$, and $m_3 = 20.0 \text{ kg}$. The coefficient of kinetic friction between the floor and each of the crates is 0.700 . (a) What is the magnitude F_{32} of the force on crate 3 from crate 2? (b) If the crates then slide onto a polished floor, where the coefficient of kinetic friction is less than 0.700 , is magnitude F_{32} more than, less than, or the same as it was when the coefficient was 0.700 ?

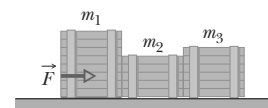


Figure 6-32 Problem 26.

••27 GO Body A in Fig. 6-33 weighs 102 N , and body B weighs 32 N . The coefficients of friction between A and the incline are $\mu_s = 0.56$ and $\mu_k = 0.25$. Angle θ is 40° . Let the positive direction of an x axis be up the incline. In unit-vector notation, what is the acceleration of A if A is initially (a) at rest, (b) moving up the incline, and (c) moving down the incline?

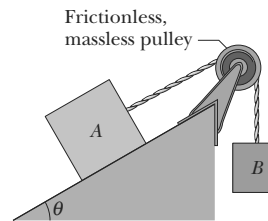


Figure 6-33 Problems 27 and 28.

••28 In Fig. 6-33, two blocks are connected over a pulley. The mass of block A is 10 kg , and the coefficient of kinetic friction between A and the incline is 0.20 . Angle θ of the incline is 30° . Block A slides down the incline at constant speed. What is the mass of block B ? Assume the connecting rope has negligible mass. (The pulley's function is only to redirect the rope.)

••29 GO In Fig. 6-34, blocks A and B have weights of 44 N and 22 N , respectively. (a) Determine the minimum weight of block C to keep A from sliding if μ_s between A and the table is 0.20 . (b) Block C suddenly is lifted off A . What is the acceleration of block A if μ_k between A and the table is 0.15 ?

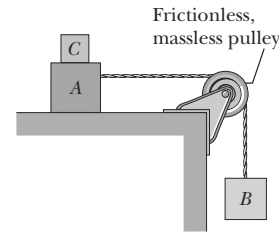


Figure 6-34 Problem 29.

••30 A toy chest and its contents have a combined weight of 180 N . The coefficient of static friction between toy chest and floor is 0.42 . The child in Fig. 6-35 attempts to move the chest across the floor by pulling on an attached rope. (a) If θ is 42° , what is the magnitude of the force \vec{F} that the child must exert on the rope to put the chest on the verge of moving? (b) Write an expression for the magnitude F required to put the chest on the verge of moving as a function of the angle θ . Determine (c) the value of θ for which F is a minimum and (d) that minimum magnitude.

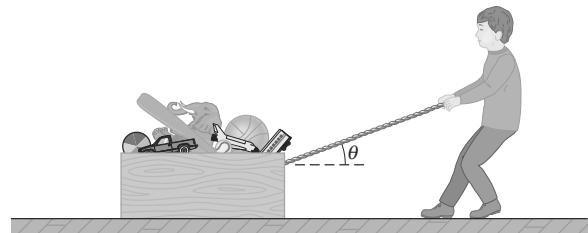


Figure 6-35 Problem 30.

••31 SSM Two blocks, of weights 3.6 N and 7.2 N , are connected by a massless string and slide down a 30° inclined plane. The coefficient of kinetic friction between the lighter block and the plane is 0.10 , and the coefficient between the heavier block and the plane is 0.20 . Assuming that the lighter block leads, find (a) the magnitude of the acceleration of the blocks and (b) the tension in the taut string.

••32 GO A block is pushed across a floor by a constant force that is applied at downward angle θ (Fig. 6-19). Figure 6-36 gives the acceleration magnitude a versus a range of values for the coefficient of kinetic friction μ_k between block and floor: $a_1 = 3.0 \text{ m/s}^2$, $\mu_{k2} = 0.20$, and $\mu_{k3} = 0.40$. What is the value of θ ?

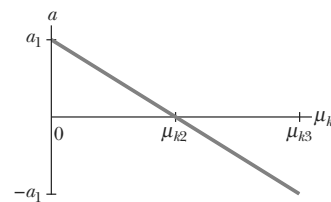


Figure 6-36 Problem 32.

••33 **SSM** A 1000 kg boat is traveling at 90 km/h when its engine is shut off. The magnitude of the frictional force \vec{f}_k between boat and water is proportional to the speed v of the boat: $f_k = 70v$, where v is in meters per second and f_k is in newtons. Find the time required for the boat to slow to 45 km/h.

••34 **GO** In Fig. 6-37, a slab of mass $m_1 = 40$ kg rests on a frictionless floor, and a block of mass $m_2 = 10$ kg rests on top of the slab. Between block and slab, the coefficient of static friction is 0.60, and the coefficient of kinetic friction is 0.40. A horizontal force \vec{F} of magnitude 100 N begins to pull directly on the block, as shown. In unit-vector notation, what are the resulting accelerations of (a) the block and (b) the slab?

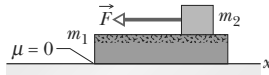


Figure 6-37 Problem 34.

••35 **ILW** The two blocks ($m = 16$ kg and $M = 88$ kg) in Fig. 6-38 are not attached to each other. The coefficient of static friction between the blocks is $\mu_s = 0.38$, but the surface beneath the larger block is frictionless. What is the minimum magnitude of the horizontal force \vec{F} required to keep the smaller block from slipping down the larger block?

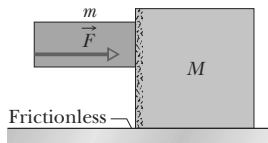


Figure 6-38 Problem 35.

Module 6-2 The Drag Force and Terminal Speed

•36 The terminal speed of a sky diver is 160 km/h in the spread-eagle position and 310 km/h in the nosedive position. Assuming that the diver's drag coefficient C does not change from one position to the other, find the ratio of the effective cross-sectional area A in the slower position to that in the faster position.

•37 **ILW** *Continuation of Problem 8.* Now assume that Eq. 6-14 gives the magnitude of the air drag force on the typical 20 kg stone, which presents to the wind a vertical cross-sectional area of 0.040 m² and has a drag coefficient C of 0.80. Take the air density to be 1.21 kg/m³, and the coefficient of kinetic friction to be 0.80. (a) In kilometers per hour, what wind speed V along the ground is needed to maintain the stone's motion once it has started moving? Because winds along the ground are retarded by the ground, the wind speeds reported for storms are often measured at a height of 10 m. Assume wind speeds are 2.00 times those along the ground. (b) For your answer to (a), what wind speed would be reported for the storm? (c) Is that value reasonable for a high-speed wind in a storm? (Story continues with Problem 65.)

•38 Assume Eq. 6-14 gives the drag force on a pilot plus ejection seat just after they are ejected from a plane traveling horizontally at 1300 km/h. Assume also that the mass of the seat is equal to the mass of the pilot and that the drag coefficient is that of a sky diver. Making a reasonable guess of the pilot's mass and using the appropriate v_t value from Table 6-1, estimate the magnitudes of (a) the drag force on the *pilot + seat* and (b) their horizontal deceleration (in terms of g), both just after ejection. (The result of (a) should indicate an engineering requirement: The seat must include a protective barrier to deflect the initial wind blast away from the pilot's head.)

•39 Calculate the ratio of the drag force on a jet flying at 1000 km/h at an altitude of 10 km to the drag force on a prop-driven transport flying at half that speed and altitude. The density

of air is 0.38 kg/m³ at 10 km and 0.67 kg/m³ at 5.0 km. Assume that the airplanes have the same effective cross-sectional area and drag coefficient C .

•40 **ILW** In downhill speed skiing a skier is retarded by both the air drag force on the body and the kinetic frictional force on the skis. (a) Suppose the slope angle is $\theta = 40.0^\circ$, the snow is dry snow with a coefficient of kinetic friction $\mu_k = 0.0400$, the mass of the skier and equipment is $m = 85.0$ kg, the cross-sectional area of the (tucked) skier is $A = 1.30$ m², the drag coefficient is $C = 0.150$, and the air density is 1.20 kg/m³. (a) What is the terminal speed? (b) If a skier can vary C by a slight amount dC by adjusting, say, the hand positions, what is the corresponding variation in the terminal speed?

Module 6-3 Uniform Circular Motion

•41 A cat dozes on a stationary merry-go-round in an amusement park, at a radius of 5.4 m from the center of the ride. Then the operator turns on the ride and brings it up to its proper turning rate of one complete rotation every 6.0 s. What is the least coefficient of static friction between the cat and the merry-go-round that will allow the cat to stay in place, without sliding (or the cat clinging with its claws)?

•42 Suppose the coefficient of static friction between the road and the tires on a car is 0.60 and the car has no negative lift. What speed will put the car on the verge of sliding as it rounds a level curve of 30.5 m radius?

•43 **ILW** What is the smallest radius of an unbanked (flat) track around which a bicyclist can travel if her speed is 29 km/h and the μ_s between tires and track is 0.32?


•44 During an Olympic bobsled run, the Jamaican team makes a turn of radius 7.6 m at a speed of 96.6 km/h. What is their acceleration in terms of g ?

•45 **SSM ILW** **ILW** A student of weight 667 N rides a steadily rotating Ferris wheel (the student sits upright). At the highest point, the magnitude of the normal force \vec{F}_N on the student from the seat is 556 N. (a) Does the student feel "light" or "heavy" there? (b) What is the magnitude of \vec{F}_N at the lowest point? If the wheel's speed is doubled, what is the magnitude F_N at the (c) highest and (d) lowest point?

•46 A police officer in hot pursuit drives her car through a circular turn of radius 300 m with a constant speed of 80.0 km/h. Her mass is 55.0 kg. What are (a) the magnitude and (b) the angle (relative to vertical) of the *net* force of the officer on the car seat? (*Hint:* Consider both horizontal and vertical forces.)

•47 **ILW** A circular-motion addict of mass 80 kg rides a Ferris wheel around in a vertical circle of radius 10 m at a constant speed of 6.1 m/s. (a) What is the period of the motion? What is the magnitude of the normal force on the addict from the seat when both go through (b) the highest point of the circular path and (c) the lowest point?

•48 **ILW** A roller-coaster car at an amusement park has a mass of 1200 kg when fully loaded with passengers. As the car passes over the top of a circular hill of radius 18 m, assume that its speed is not changing. At the top of the hill, what are the (a) magnitude F_N and (b) direction (up or down) of the normal force on the car from the track if the car's speed is $v = 11$ m/s? What are (c) F_N and (d) the direction if $v = 14$ m/s?

••49  In Fig. 6-39, a car is driven at constant speed over a circular hill and then into a circular valley with the same radius. At the top of the hill, the normal force on the driver from the car seat is 0. The driver's mass is 70.0 kg. What is the magnitude of the normal force on the driver from the seat when the car passes through the bottom of the valley?

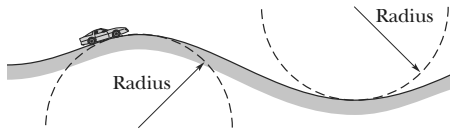


Figure 6-39 Problem 49.

••50 An 85.0 kg passenger is made to move along a circular path of radius $r = 3.50$ m in uniform circular motion. (a) Figure 6-40a is a plot of the required magnitude F of the net centripetal force for a range of possible values of the passenger's speed v . What is the plot's slope at $v = 8.30$ m/s? (b) Figure 6-40b is a plot of F for a range of possible values of T , the period of the motion. What is the plot's slope at $T = 2.50$ s?

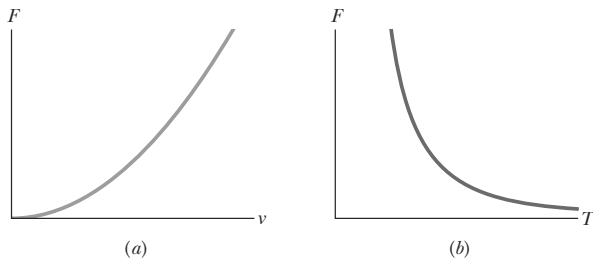




Figure 6-40 Problem 50.

••51   An airplane is flying in a horizontal circle at a speed of 480 km/h (Fig. 6-41). If its wings are tilted at angle $\theta = 40^\circ$ to the horizontal, what is the radius of the circle in which the plane is flying? Assume that the required force is provided entirely by an "aerodynamic lift" that is perpendicular to the wing surface.

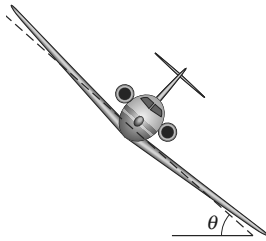




Figure 6-41 Problem 51.

••52  An amusement park ride consists of a car moving in a vertical circle on the end of a rigid boom of negligible mass. The combined weight of the car and riders is 5.0 kN, and the circle's radius is 10 m. At the top of the circle, what are the (a) magnitude F_B and (b) direction (up or down) of the force on the car from the boom if the car's speed is $v = 5.0$ m/s? What are (c) F_B and (d) the direction if $v = 12$ m/s?

••53 An old streetcar rounds a flat corner of radius 9.1 m, at 16 km/h. What angle with the vertical will be made by the loosely hanging hand straps?

••54  In designing circular rides for amusement parks, mechanical engineers must consider how small variations in certain parameters can alter the net force on a passenger. Consider a passenger of mass m riding around a horizontal circle of radius r at speed v . What is the variation dF in the net force magnitude for (a) a variation dr in the radius with v held constant, (b) a variation

dv in the speed with r held constant, and (c) a variation dT in the period with r held constant?

••55 A bolt is threaded onto one end of a thin horizontal rod, and the rod is then rotated horizontally about its other end. An engineer monitors the motion by flashing a strobe lamp onto the rod and bolt, adjusting the strobe rate until the bolt appears to be in the same eight places during each full rotation of the rod (Fig. 6-42). The strobe rate is 2000 flashes per second; the bolt has mass 30 g and is at radius 3.5 cm. What is the magnitude of the force on the bolt from the rod?

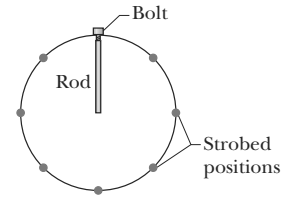




Figure 6-42 Problem 55.

••56  A banked circular highway curve is designed for traffic moving at 60 km/h. The radius of the curve is 200 m. Traffic is moving along the highway at 40 km/h on a rainy day. What is the minimum coefficient of friction between tires and road that will allow cars to take the turn without sliding off the road? (Assume the cars do not have negative lift.)

••57  A puck of mass $m = 1.50$ kg slides in a circle of radius $r = 20.0$ cm on a frictionless table while attached to a hanging cylinder of mass $M = 2.50$ kg by means of a cord that extends through a hole in the table (Fig. 6-43). What speed keeps the cylinder at rest?

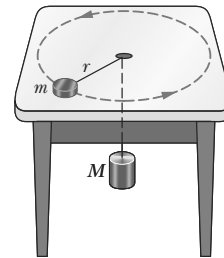



Figure 6-43 Problem 57.

••58  *Brake or turn?* Figure 6-44 depicts an overhead view of a car's path as the car travels toward a wall. Assume that the driver begins to brake the car when the distance to the wall is $d = 107$ m, and take the car's mass as $m = 1400$ kg, its initial speed as $v_0 = 35$ m/s, and the coefficient of static friction as $\mu_s = 0.50$. Assume that the car's weight is distributed evenly on the four wheels, even during braking. (a) What magnitude of static friction is needed (between tires and road) to stop the car just as it reaches the wall? (b) What is the maximum possible static friction $f_{s,\max}$? (c) If the coefficient of kinetic friction between the (sliding) tires and the road is $\mu_k = 0.40$, at what speed will the car hit the wall? To avoid the crash, a driver could elect to turn the car so that it just barely misses the wall, as shown in the figure. (d) What magnitude of frictional force would be required to keep the car in a circular path of radius d and at the given speed v_0 , so that the car moves in a quarter circle and then parallel to the wall? (e) Is the required force less than $f_{s,\max}$ so that a circular path is possible?

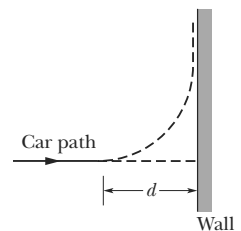


Figure 6-44 Problem 58.

•••59 **SSM** **ILW** In Fig. 6-45, a 1.34 kg ball is connected by means of two massless strings, each of length $L = 1.70$ m, to a vertical, rotating rod. The strings are tied to the rod with separation $d = 1.70$ m and are taut. The tension in the upper string is 35 N. What are the (a) tension in the lower string, (b) magnitude of the net force \vec{F}_{net} on the ball, and (c) speed of the ball? (d) What is the direction of \vec{F}_{net} ?

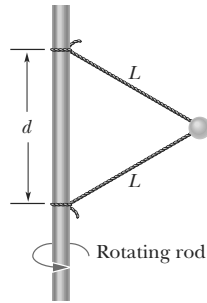


Figure 6-45
Problem 59.

Additional Problems

60 **GO** In Fig. 6-46, a box of ant aunts (total mass $m_1 = 1.65$ kg) and a box of ant uncles (total mass $m_2 = 3.30$ kg) slide down an inclined plane while attached by a massless rod parallel to the plane. The angle of incline is $\theta = 30.0^\circ$. The coefficient of kinetic friction between the aunt box and the incline is $\mu_1 = 0.226$; that between the uncle box and the incline is $\mu_2 = 0.113$. Compute (a) the tension in the rod and (b) the magnitude of the common acceleration of the two boxes. (c) How would the answers to (a) and (b) change if the uncles trailed the aunts?

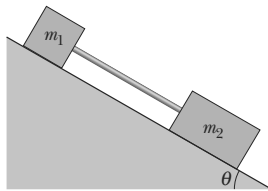


Figure 6-46 Problem 60.

61 **SSM** A block of mass $m_t = 4.0$ kg is put on top of a block of mass $m_b = 5.0$ kg. To cause the top block to slip on the bottom one while the bottom one is held fixed, a horizontal force of at least 12 N must be applied to the top block. The assembly of blocks is now placed on a horizontal, frictionless table (Fig. 6-47). Find the magnitudes of (a) the maximum horizontal force \vec{F} that can be applied to the lower block so that the blocks will move together and (b) the resulting acceleration of the blocks.

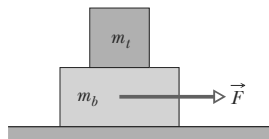


Figure 6-47 Problem 61.

62 A 5.00 kg stone is rubbed across the horizontal ceiling of a cave passageway (Fig. 6-48). If the coefficient of kinetic friction is 0.65 and the force applied to the stone is angled at $\theta = 70.0^\circ$, what must the magnitude of the force be for the stone to move at constant velocity?

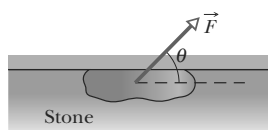


Figure 6-48 Problem 62.

63 **GO** In Fig. 6-49, a 49 kg rock climber is climbing a “chimney.” The coefficient of static friction between her shoes and the rock is 1.2; between her back and the rock is 0.80. She has reduced her push against the rock until her back and her shoes are on the verge of slipping. (a) Draw a free-body diagram of her. (b) What is the magnitude of her push against the rock? (c) What fraction of her weight is supported by the frictional force on her shoes?

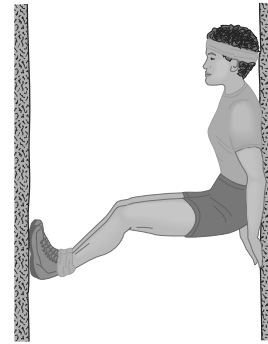


Figure 6-49 Problem 63.

64 A high-speed railway car goes around a flat, horizontal circle of radius 470 m at a constant speed. The magnitudes of the horizontal and vertical components of the force of the car on a 51.0 kg passenger are 210 N and 500 N, respectively. (a) What is the magnitude of the net force (of all the forces) on the passenger? (b) What is the speed of the car?

65 **GO** *Continuation of Problems 8 and 37.* Another explanation is that the stones move only when the water dumped on the playa during a storm freezes into a large, thin sheet of ice. The stones are trapped in place in the ice. Then, as air flows across the ice during a wind, the air-drag forces on the ice and stones move them both, with the stones gouging out the trails. The magnitude of the air-drag force on this horizontal “ice sail” is given by $D_{\text{ice}} = 4C_{\text{ice}}\rho A_{\text{ice}}v^2$, where C_{ice} is the drag coefficient (2.0×10^{-3}), ρ is the air density (1.21 kg/m^3), A_{ice} is the horizontal area of the ice, and v is the wind speed along the ice.

Assume the following: The ice sheet measures 400 m by 500 m by 4.0 mm and has a coefficient of kinetic friction of 0.10 with the ground and a density of 917 kg/m^3 . Also assume that 100 stones identical to the one in Problem 8 are trapped in the ice. To maintain the motion of the sheet, what are the required wind speeds (a) near the sheet and (b) at a height of 10 m? (c) Are these reasonable values for high-speed winds in a storm?

66 **GO** In Fig. 6-50, block 1 of mass $m_1 = 2.0$ kg and block 2 of mass $m_2 = 3.0$ kg are connected by a string of negligible mass and are initially held in place. Block 2 is on a frictionless surface tilted at $\theta = 30^\circ$. The coefficient of kinetic friction between block 1 and the horizontal surface is 0.25. The pulley has negligible mass and friction. Once they are released, the blocks move. What then is the tension in the string?

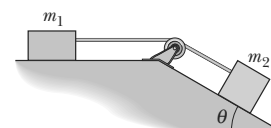


Figure 6-50 Problem 66.

67 In Fig. 6-51, a crate slides down an inclined right-angled trough. The coefficient of kinetic friction between the crate and the trough is μ_k . What is the acceleration of the crate in terms of μ_k , θ , and g ?

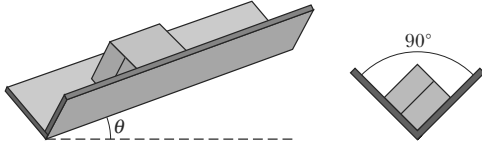


Figure 6-51 Problem 67.

68 *Engineering a highway curve.* If a car goes through a curve too fast, the car tends to slide out of the curve. For a banked curve with friction, a frictional force acts on a fast car to oppose the tendency to slide out of the curve; the force is directed down the bank (in the direction water would drain). Consider a circular curve of radius $R = 200$ m and bank angle θ , where the coefficient of static friction between tires and pavement is μ_s . A car (without negative lift) is driven around the curve as shown in Fig. 6-11. (a) Find an expression for the car speed v_{\max} that puts the car on the verge of sliding out. (b) On the same graph, plot v_{\max} versus angle θ for the range 0° to 50° , first for $\mu_s = 0.60$ (dry pavement) and then for $\mu_s = 0.050$ (wet or icy pavement). In kilometers per hour, evaluate v_{\max} for a bank angle of $\theta = 10^\circ$ and for (c) $\mu_s = 0.60$ and (d) $\mu_s = 0.050$. (Now you can see why accidents occur in highway curves when icy conditions are not obvious to drivers, who tend to drive at normal speeds.)

69 A student, crazed by final exams, uses a force \vec{P} of magnitude 80 N and angle $\theta = 70^\circ$ to push a 5.0 kg block across the ceiling of his room (Fig. 6-52). If the coefficient of kinetic friction between the block and the ceiling is 0.40, what is the magnitude of the block's acceleration?

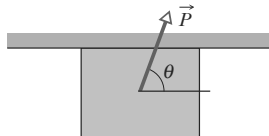



Figure 6-52 Problem 69.

70  Figure 6-53 shows a *conical pendulum*, in which the bob (the small object at the lower end of the cord) moves in a horizontal circle at constant speed. (The cord sweeps out a cone as the bob rotates.) The bob has a mass of 0.040 kg, the string has length $L = 0.90$ m and negligible mass, and the bob follows a circular path of circumference 0.94 m. What are (a) the tension in the string and (b) the period of the motion?

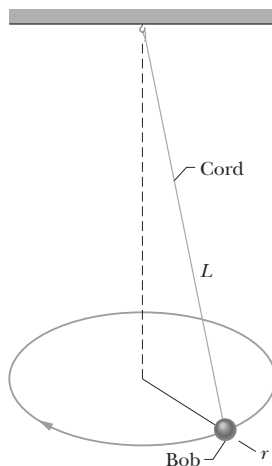


Figure 6-53 Problem 70.

71 An 8.00 kg block of steel is at rest on a horizontal table. The coefficient of static friction between the block and the table is 0.450. A force is to be applied to the block.

To three significant figures, what is the magnitude of that applied force if it puts the block on the verge of sliding when the force is directed (a) horizontally, (b) upward at 60.0° from the horizontal, and (c) downward at 60.0° from the horizontal?

72 A box of canned goods slides down a ramp from street level into the basement of a grocery store with acceleration 0.75 m/s^2 directed down the ramp. The ramp makes an angle of 40° with the horizontal. What is the coefficient of kinetic friction between the box and the ramp?

73 In Fig. 6-54, the coefficient of kinetic friction between the block and inclined plane is 0.20, and angle θ is 60° . What are the (a) magnitude a and (b) direction (up or down the plane) of the block's acceleration if the block is sliding down the plane? What are (c) a and (d) the direction if the block is sent sliding up the plane?

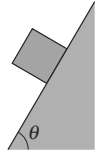


Figure 6-54 Problem 73.

74 A 110 g hockey puck sent sliding over ice is stopped in 15 m by the frictional force on it from the ice. (a) If its initial speed is 6.0 m/s, what is the magnitude of the frictional force? (b) What is the coefficient of friction between the puck and the ice?

75 A locomotive accelerates a 25-car train along a level track. Every car has a mass of 5.0×10^4 kg and is subject to a frictional force $f = 250v$, where the speed v is in meters per second and the force f is in newtons. At the instant when the speed of the train is 30 km/h, the magnitude of its acceleration is 0.20 m/s^2 . (a) What is the tension in the coupling between the first car and the locomotive? (b) If this tension is equal to the maximum force the locomotive can exert on the train, what is the steepest grade up which the locomotive can pull the train at 30 km/h?

76 A house is built on the top of a hill with a nearby slope at angle $\theta = 45^\circ$ (Fig. 6-55). An engineering study indicates that the slope angle should be reduced because the top layers of soil along the slope might slip past the lower layers. If the coefficient of static friction between two such layers is 0.5, what is the least angle ϕ through which the present slope should be reduced to prevent slippage?

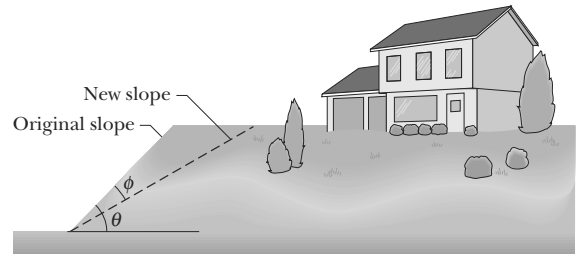


Figure 6-55 Problem 76.

77 What is the terminal speed of a 6.00 kg spherical ball that has a radius of 3.00 cm and a drag coefficient of 1.60? The density of the air through which the ball falls is 1.20 kg/m^3 .

78 A student wants to determine the coefficients of static friction and kinetic friction between a box and a plank. She places the box on the plank and gradually raises one end of the plank. When the angle of inclination with the horizontal reaches 30° , the box starts to slip, and it then slides 2.5 m down the plank in 4.0 s at constant acceleration. What are (a) the coefficient of static friction and (b) the coefficient of kinetic friction between the box and the plank?

79 SSM Block A in Fig. 6-56 has mass $m_A = 4.0$ kg, and block B has mass $m_B = 2.0$ kg. The coefficient of kinetic friction between block B and the horizontal plane is $\mu_k = 0.50$. The inclined plane is frictionless and at angle $\theta = 30^\circ$. The pulley serves only to change the direction of the cord connecting the blocks. The cord has negligible mass. Find (a) the tension in the cord and (b) the magnitude of the acceleration of the blocks.

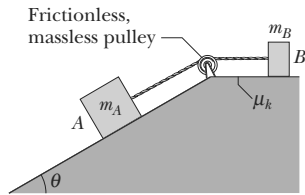


Figure 6-56 Problem 79.

80 Calculate the magnitude of the drag force on a missile 53 cm in diameter cruising at 250 m/s at low altitude, where the density of air is 1.2 kg/m^3 . Assume $C = 0.75$.

81 SSM A bicyclist travels in a circle of radius 25.0 m at a constant speed of 9.00 m/s. The bicycle-rider mass is 85.0 kg. Calculate the magnitudes of (a) the force of friction on the bicycle from the road and (b) the *net* force on the bicycle from the road.

82 In Fig. 6-57, a stuntman drives a car (without negative lift) over the top of a hill, the cross section of which can be approximated by a circle of radius $R = 250$ m. What is the greatest speed at which he can drive without the car leaving the road at the top of the hill?

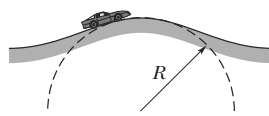


Figure 6-57 Problem 82.

83 You must push a crate across a floor to a docking bay. The crate weighs 165 N. The coefficient of static friction between crate and floor is 0.510, and the coefficient of kinetic friction is 0.32. Your force on the crate is directed horizontally. (a) What magnitude of your push puts the crate on the verge of sliding? (b) With what magnitude must you then push to keep the crate moving at a constant velocity? (c) If, instead, you then push with the same magnitude as the answer to (a), what is the magnitude of the crate's acceleration?

84 In Fig. 6-58, force \vec{F} is applied to a crate of mass m on a floor where the coefficient of static friction between crate and floor is μ_s . Angle θ is initially 0° but is gradually increased so that the force vector rotates clockwise in the figure. During the rotation, the magnitude F of the force is continuously adjusted so that the crate is always on the verge of sliding. For $\mu_s = 0.70$, (a) plot the ratio F/mg versus θ and (b) determine the angle θ_{inf} at which the ratio approaches an infinite value. (c) Does lubricating the floor increase or decrease θ_{inf} , or is the value unchanged? (d) What is θ_{inf} for $\mu_s = 0.60$?

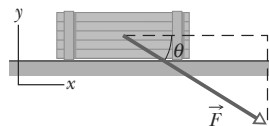


Figure 6-58 Problem 84.

85 In the early afternoon, a car is parked on a street that runs down a steep hill, at an angle of 35.0° relative to the horizontal. Just then the coefficient of static friction between the tires and the street surface is 0.725. Later, after nightfall, a sleet storm hits the area, and the coefficient decreases due to both the ice and a chemi-

cal change in the road surface because of the temperature decrease. By what percentage must the coefficient decrease if the car is to be in danger of sliding down the street?

86 A sling-thrower puts a stone (0.250 kg) in the sling's pouch (0.010 kg) and then begins to make the stone and pouch move in a vertical circle of radius 0.650 m. The cord between the pouch and the person's hand has negligible mass and will break when the tension in the cord is 33.0 N or more. Suppose the sling-thrower could gradually increase the speed of the stone. (a) Will the breaking occur at the lowest point of the circle or at the highest point? (b) At what speed of the stone will that breaking occur?

87 SSM A car weighing 10.7 kN and traveling at 13.4 m/s without negative lift attempts to round an unbanked curve with a radius of 61.0 m. (a) What magnitude of the frictional force on the tires is required to keep the car on its circular path? (b) If the coefficient of static friction between the tires and the road is 0.350, is the attempt at taking the curve successful?

88 In Fig. 6-59, block 1 of mass $m_1 = 2.0$ kg and block 2 of mass $m_2 = 1.0$ kg are connected by a string of negligible mass. Block 2 is pushed by force \vec{F} of magnitude 20 N and angle $\theta = 35^\circ$. The coefficient of kinetic friction between each block and the horizontal surface is 0.20. What is the tension in the string?

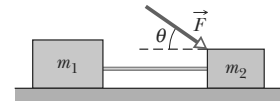


Figure 6-59 Problem 88.

89 SSM A filing cabinet weighing 556 N rests on the floor. The coefficient of static friction between it and the floor is 0.68, and the coefficient of kinetic friction is 0.56. In four different attempts to move it, it is pushed with horizontal forces of magnitudes (a) 222 N, (b) 334 N, (c) 445 N, and (d) 556 N. For each attempt, calculate the magnitude of the frictional force on it from the floor. (The cabinet is initially at rest.) (e) In which of the attempts does the cabinet move?

90 In Fig. 6-60, a block weighing 22 N is held at rest against a vertical wall by a horizontal force \vec{F} of magnitude 60 N. The coefficient of static friction between the wall and the block is 0.55, and the coefficient of kinetic friction between them is 0.38. In six experiments, a second force \vec{P} is applied to the block and directed parallel to the wall with these magnitudes and directions: (a) 34 N, up, (b) 12 N, up, (c) 48 N, up, (d) 62 N, up, (e) 10 N, down, and (f) 18 N, down. In each experiment, what is the magnitude of the frictional force on the block? In which does the block move (g) up the wall and (h) down the wall? (i) In which is the frictional force directed down the wall?

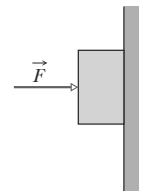


Figure 6-60 Problem 90.

91 SSM A block slides with constant velocity down an inclined plane that has slope angle θ . The block is then projected up the same plane with an initial speed v_0 . (a) How far up the plane will it move before coming to rest? (b) After the block comes to rest, will it slide down the plane again? Give an argument to back your answer.

92 A circular curve of highway is designed for traffic moving at 60 km/h. Assume the traffic consists of cars without negative lift. (a) If the radius of the curve is 150 m, what is the correct angle of banking of the road? (b) If the curve were not banked, what would be the minimum coefficient of friction between tires and road that would keep traffic from skidding out of the turn when traveling at 60 km/h?

93 A 1.5 kg box is initially at rest on a horizontal surface when at $t = 0$ a horizontal force $\vec{F} = (1.8t)\hat{i}$ N (with t in seconds) is applied to the box. The acceleration of the box as a function of time t is given by $\vec{a} = 0$ for $0 \leq t \leq 2.8$ s and $\vec{a} = (1.2t - 2.4)\hat{i}$ m/s² for $t > 2.8$ s. (a) What is the coefficient of static friction between the box and the surface? (b) What is the coefficient of kinetic friction between the box and the surface?

94 A child weighing 140 N sits at rest at the top of a playground slide that makes an angle of 25° with the horizontal. The child keeps from sliding by holding onto the sides of the slide. After letting go of the sides, the child has a constant acceleration of 0.86 m/s² (down the slide, of course). (a) What is the coefficient of kinetic friction between the child and the slide? (b) What maximum and minimum values for the coefficient of static friction between the child and the slide are consistent with the information given here?

95 In Fig. 6-61 a fastidious worker pushes directly along the handle of a mop with a force \vec{F} . The handle is at an angle θ with the vertical, and μ_s and μ_k are the coefficients of static and kinetic friction between the head of the mop and the floor. Ignore the mass of the handle and assume that all the mop's mass m is in its head. (a) If the mop head moves along the floor with a constant velocity, then what is F ? (b) Show that if θ is less than a certain value θ_0 , then \vec{F} (still directed along the handle) is unable to move the mop head. Find θ_0 .

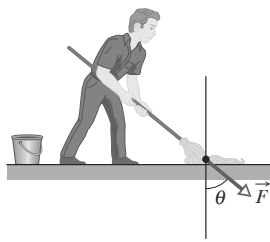


Figure 6-61 Problem 95.

96 A child places a picnic basket on the outer rim of a merry-go-round that has a radius of 4.6 m and revolves once every 30 s. (a) What is the speed of a point on that rim? (b) What is the lowest value of the coefficient of static friction between basket and merry-go-round that allows the basket to stay on the ride?

97 SSM A warehouse worker exerts a constant horizontal force of magnitude 85 N on a 40 kg box that is initially at rest on the horizontal floor of the warehouse. When the box has moved a distance of 1.4 m, its speed is 1.0 m/s. What is the coefficient of kinetic friction between the box and the floor?

98 In Fig. 6-62, a 5.0 kg block is sent sliding up a plane inclined at $\theta = 37^\circ$ while a horizontal force \vec{F} of magnitude 50 N acts on it. The coefficient of kinetic friction between block and plane is 0.30. What are the (a) magnitude and (b) direction (up or down the plane) of the block's acceleration? The block's initial speed is 4.0 m/s. (c) How far up the plane does the block go? (d) When it reaches its highest point, does it remain at rest or slide back down the plane?

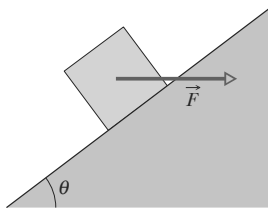


Figure 6-62 Problem 98.


99 An 11 kg block of steel is at rest on a horizontal table. The coefficient of static friction between block and table is 0.52. (a) What is the magnitude of the horizontal force that will put the block on the verge of moving? (b) What is the magnitude of a force acting upward 60° from the horizontal that will put the block on the verge of moving? (c) If the force acts downward at 60° from the horizontal, how large can its magnitude be without causing the block to move?

100 A ski that is placed on snow will stick to the snow. However, when the ski is moved along the snow, the rubbing warms and partially melts the snow, reducing the coefficient of kinetic friction and promoting sliding. Waxing the ski makes it water repellent and reduces friction with the resulting layer of water. A magazine reports that a new type of plastic ski is especially water repellent and that, on a gentle 200 m slope in the Alps, a skier reduced his top-to-bottom time from 61 s with standard skis to 42 s with the new skis. Determine the magnitude of his average acceleration with (a) the standard skis and (b) the new skis. Assuming a 3.0° slope, compute the coefficient of kinetic friction for (c) the standard skis and (d) the new skis.

101 Playing near a road construction site, a child falls over a barrier and down onto a dirt slope that is angled downward at 35° to the horizontal. As the child slides *down* the slope, he has an acceleration that has a magnitude of 0.50 m/s² and that is directed *up* the slope. What is the coefficient of kinetic friction between the child and the slope?

102 A 100 N force, directed at an angle θ above a horizontal floor, is applied to a 25.0 kg chair sitting on the floor. If $\theta = 0^\circ$, what are (a) the horizontal component F_h of the applied force and (b) the magnitude F_N of the normal force of the floor on the chair? If $\theta = 30.0^\circ$, what are (c) F_h and (d) F_N ? If $\theta = 60.0^\circ$, what are (e) F_h and (f) F_N ? Now assume that the coefficient of static friction between chair and floor is 0.420. Does the chair slide or remain at rest if θ is (g) 0° , (h) 30.0° , and (i) 60.0° ?

103 A certain string can withstand a maximum tension of 40 N without breaking. A child ties a 0.37 kg stone to one end and, holding the other end, whirls the stone in a vertical circle of radius 0.91 m, slowly increasing the speed until the string breaks. (a) Where is the stone on its path when the string breaks? (b) What is the speed of the stone as the string breaks?

104  A four-person bobsled (total mass = 630 kg) comes down a straightaway at the start of a bobsled run. The straightaway is 80.0 m long and is inclined at a constant angle of 10.2° with the horizontal. Assume that the combined effects of friction and air drag produce on the bobsled a constant force of 62.0 N that acts parallel to the incline and up the incline. Answer the following questions to three significant digits. (a) If the speed of the bobsled at the start of the run is 6.20 m/s, how long does the bobsled take to come down the straightaway? (b) Suppose the crew is able to reduce the effects of friction and air drag to 42.0 N. For the same initial velocity, how long does the bobsled now take to come down the straightaway?

105 As a 40 N block slides down a plane that is inclined at 25° to the horizontal, its acceleration is 0.80 m/s², directed up the plane. What is the coefficient of kinetic friction between the block and the plane?

C H A P T E R 7

Kinetic Energy and Work

7-1 KINETIC ENERGY

Learning Objectives

After reading this module, you should be able to . . .

7.01 Apply the relationship between a particle's kinetic energy, mass, and speed.

7.02 Identify that kinetic energy is a scalar quantity.

Key Idea

- The kinetic energy K associated with the motion of a particle of mass m and speed v , where v is well below the speed of light, is

$$K = \frac{1}{2}mv^2 \quad (\text{kinetic energy}).$$

What Is Physics?

One of the fundamental goals of physics is to investigate something that everyone talks about: energy. The topic is obviously important. Indeed, our civilization is based on acquiring and effectively using energy.

For example, everyone knows that any type of motion requires energy: Flying across the Pacific Ocean requires it. Lifting material to the top floor of an office building or to an orbiting space station requires it. Throwing a fastball requires it. We spend a tremendous amount of money to acquire and use energy. Wars have been started because of energy resources. Wars have been ended because of a sudden, overpowering use of energy by one side. Everyone knows many examples of energy and its use, but what does the term *energy* really mean?

What Is Energy?

The term *energy* is so broad that a clear definition is difficult to write. Technically, energy is a scalar quantity associated with the state (or condition) of one or more objects. However, this definition is too vague to be of help to us now.

A looser definition might at least get us started. Energy is a number that we associate with a system of one or more objects. If a force changes one of the objects by, say, making it move, then the energy number changes. After countless experiments, scientists and engineers realized that if the scheme by which we assign energy numbers is planned carefully, the numbers can be used to predict the outcomes of experiments and, even more important, to build machines, such as flying machines. This success is based on a wonderful property of our universe: Energy can be transformed from one type to another and transferred from one object to another, but the total amount is always the same (energy is *conserved*). No exception to this *principle of energy conservation* has ever been found.

Money. Think of the many types of energy as being numbers representing money in many types of bank accounts. Rules have been made about what such money numbers mean and how they can be changed. You can transfer money numbers from one account to another or from one system to another, perhaps

electronically with nothing material actually moving. However, the total amount (the total of all the money numbers) can always be accounted for: It is always conserved. In this chapter we focus on only one type of energy (*kinetic energy*) and on only one way in which energy can be transferred (*work*).

Kinetic Energy

Kinetic energy K is energy associated with the *state of motion* of an object. The faster the object moves, the greater is its kinetic energy. When the object is stationary, its kinetic energy is zero.

For an object of mass m whose speed v is well below the speed of light,

$$K = \frac{1}{2}mv^2 \quad (\text{kinetic energy}). \quad (7-1)$$


For example, a 3.0 kg duck flying past us at 2.0 m/s has a kinetic energy of $6.0 \text{ kg} \cdot \text{m}^2/\text{s}^2$; that is, we associate that number with the duck's motion.

The SI unit of kinetic energy (and all types of energy) is the **joule** (J), named for James Prescott Joule, an English scientist of the 1800s and defined as

$$1 \text{ joule} = 1 \text{ J} = 1 \text{ kg} \cdot \text{m}^2/\text{s}^2. \quad (7-2)$$

Thus, the flying duck has a kinetic energy of 6.0 J.

Sample Problem 7.01 Kinetic energy, train crash

In 1896 in Waco, Texas, William Crush parked two locomotives at opposite ends of a 6.4-km-long track, fired them up, tied their throttles open, and then allowed them to crash head-on at full speed (Fig. 7-1) in front of 30,000 spectators. Hundreds of people were hurt by flying debris; several were killed. Assuming each locomotive weighed $1.2 \times 10^6 \text{ N}$ and its acceleration was a constant 0.26 m/s^2 , what was the total kinetic energy of the two locomotives just before the collision? 

We can find the mass of each locomotive by dividing its given weight by g :

$$m = \frac{1.2 \times 10^6 \text{ N}}{9.8 \text{ m/s}^2} = 1.22 \times 10^5 \text{ kg}.$$

Now, using Eq. 7-1, we find the total kinetic energy of the two locomotives just before the collision as

$$\begin{aligned} K &= 2\left(\frac{1}{2}mv^2\right) = (1.22 \times 10^5 \text{ kg})(40.8 \text{ m/s})^2 \\ &= 2.0 \times 10^8 \text{ J}. \end{aligned} \quad (\text{Answer})$$

This collision was like an exploding bomb.

KEY IDEAS

(1) We need to find the kinetic energy of each locomotive with Eq. 7-1, but that means we need each locomotive's speed just before the collision and its mass. (2) Because we can assume each locomotive had constant acceleration, we can use the equations in Table 2-1 to find its speed v just before the collision.

Calculations: We choose Eq. 2-16 because we know values for all the variables except v :

$$v^2 = v_0^2 + 2a(x - x_0).$$

With $v_0 = 0$ and $x - x_0 = 3.2 \times 10^3 \text{ m}$ (half the initial separation), this yields

$$v^2 = 0 + 2(0.26 \text{ m/s}^2)(3.2 \times 10^3 \text{ m}),$$

$$\text{or} \quad v = 40.8 \text{ m/s} = 147 \text{ km/h}.$$



Courtesy Library of Congress

Figure 7-1 The aftermath of an 1896 crash of two locomotives.



Additional examples, video, and practice available at WileyPLUS

7-2 WORK AND KINETIC ENERGY

Learning Objectives

After reading this module, you should be able to . . .

7.03 Apply the relationship between a force (magnitude and direction) and the work done on a particle by the force when the particle undergoes a displacement.

7.04 Calculate work by taking a dot product of the force vector and the displacement vector, in either magnitude-angle or unit-vector notation.

7.05 If multiple forces act on a particle, calculate the net work done by them.

7.06 Apply the work–kinetic energy theorem to relate the work done by a force (or the net work done by multiple forces) and the resulting change in kinetic energy.

Key Ideas

- Work W is energy transferred to or from an object via a force acting on the object. Energy transferred to the object is positive work, and from the object, negative work.

- The work done on a particle by a constant force \vec{F} during displacement \vec{d} is

$$W = Fd \cos \phi = \vec{F} \cdot \vec{d} \quad (\text{work, constant force}),$$

in which ϕ is the constant angle between the directions of \vec{F} and \vec{d} .

- Only the component of \vec{F} that is along the displacement \vec{d} can do work on the object.

- When two or more forces act on an object, their net work is the sum of the individual works done by the forces, which is also equal to the work that would be done on the object by the net force \vec{F}_{net} of those forces.

- For a particle, a change ΔK in the kinetic energy equals the net work W done on the particle:

$$\Delta K = K_f - K_i = W \quad (\text{work–kinetic energy theorem}),$$

in which K_i is the initial kinetic energy of the particle and K_f is the kinetic energy after the work is done. The equation rearranged gives us

$$K_f = K_i + W.$$

Work

If you accelerate an object to a greater speed by applying a force to the object, you increase the kinetic energy $K (= \frac{1}{2}mv^2)$ of the object. Similarly, if you decelerate the object to a lesser speed by applying a force, you decrease the kinetic energy of the object. We account for these changes in kinetic energy by saying that your force has transferred energy *to* the object from yourself or *from* the object to yourself. In such a transfer of energy via a force, **work** W is said to be *done on the object by the force*. More formally, we define work as follows:



Work W is energy transferred to or from an object by means of a force acting on the object. Energy transferred to the object is positive work, and energy transferred from the object is negative work.

“Work,” then, is transferred energy; “doing work” is the act of transferring the energy. Work has the same units as energy and is a scalar quantity.

The term *transfer* can be misleading. It does not mean that anything material flows into or out of the object; that is, the transfer is not like a flow of water. Rather, it is like the electronic transfer of money between two bank accounts: The number in one account goes up while the number in the other account goes down, with nothing material passing between the two accounts.

Note that we are not concerned here with the common meaning of the word “work,” which implies that *any* physical or mental labor is work. For example, if you push hard against a wall, you tire because of the continuously repeated muscle contractions that are required, and you are, in the common sense, working. However, such effort does not cause an energy transfer to or from the wall and thus is not work done on the wall as defined here.

To avoid confusion in this chapter, we shall use the symbol W only for work and shall represent a weight with its equivalent mg .

Work and Kinetic Energy

Finding an Expression for Work

Let us find an expression for work by considering a bead that can slide along a frictionless wire that is stretched along a horizontal x axis (Fig. 7-2). A constant force \vec{F} , directed at an angle ϕ to the wire, accelerates the bead along the wire. We can relate the force and the acceleration with Newton's second law, written for components along the x axis:

$$F_x = ma_x, \quad (7-3)$$

where m is the bead's mass. As the bead moves through a displacement \vec{d} , the force changes the bead's velocity from an initial value \vec{v}_0 to some other value \vec{v} . Because the force is constant, we know that the acceleration is also constant. Thus, we can use Eq. 2-16 to write, for components along the x axis,

$$v^2 = v_0^2 + 2a_x d. \quad (7-4)$$

Solving this equation for a_x , substituting into Eq. 7-3, and rearranging then give us

$$\frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 = F_x d. \quad (7-5)$$

The first term is the kinetic energy K_f of the bead at the end of the displacement d , and the second term is the kinetic energy K_i of the bead at the start. Thus, the left side of Eq. 7-5 tells us the kinetic energy has been changed by the force, and the right side tells us the change is equal to $F_x d$. Therefore, the work W done on the bead by the force (the energy transfer due to the force) is

$$W = F_x d. \quad (7-6)$$

If we know values for F_x and d , we can use this equation to calculate the work W .



To calculate the work a force does on an object as the object moves through some displacement, we use only the force component along the object's displacement. The force component perpendicular to the displacement does zero work.

From Fig. 7-2, we see that we can write F_x as $F \cos \phi$, where ϕ is the angle between the directions of the displacement \vec{d} and the force \vec{F} . Thus,

$$W = Fd \cos \phi \quad (\text{work done by a constant force}). \quad (7-7)$$

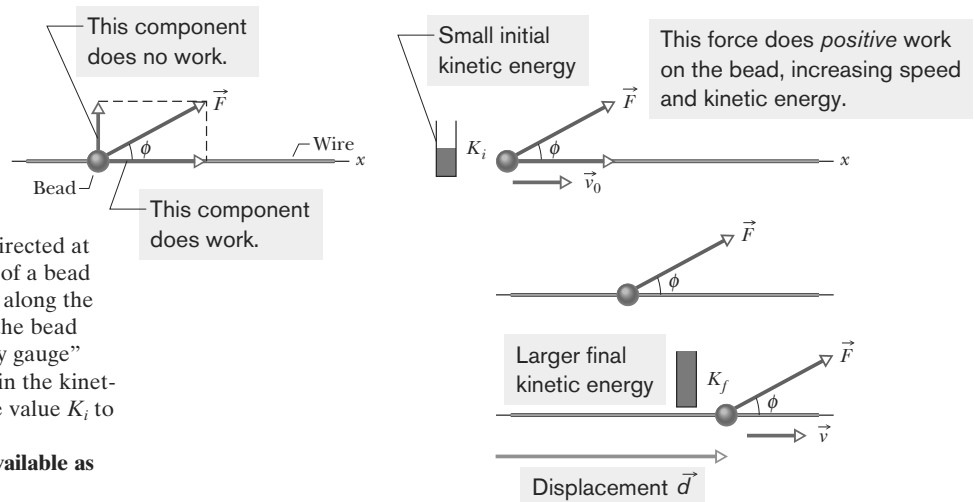


Figure 7-2 A constant force \vec{F} directed at angle ϕ to the displacement \vec{d} of a bead on a wire accelerates the bead along the wire, changing the velocity of the bead from \vec{v}_0 to \vec{v} . A “kinetic energy gauge” indicates the resulting change in the kinetic energy of the bead, from the value K_i to the value K_f .

In WileyPLUS, this figure is available as an animation with voiceover.

We can use the definition of the scalar (dot) product (Eq. 3-20) to write

$$W = \vec{F} \cdot \vec{d} \quad (\text{work done by a constant force}), \quad (7-8)$$

where F is the magnitude of \vec{F} . (You may wish to review the discussion of scalar products in Module 3-3.) Equation 7-8 is especially useful for calculating the work when \vec{F} and \vec{d} are given in unit-vector notation.

Cautions. There are two restrictions to using Eqs. 7-6 through 7-8 to calculate work done on an object by a force. First, the force must be a *constant force*; that is, it must not change in magnitude or direction as the object moves. (Later, we shall discuss what to do with a *variable force* that changes in magnitude.) Second, the object must be *particle-like*. This means that the object must be *rigid*; all parts of it must move together, in the same direction. In this chapter we consider only particle-like objects, such as the bed and its occupant being pushed in Fig. 7-3.

Signs for Work. The work done on an object by a force can be either positive work or negative work. For example, if angle ϕ in Eq. 7-7 is less than 90° , then $\cos \phi$ is positive and thus so is the work. However, if ϕ is greater than 90° (up to 180°), then $\cos \phi$ is negative and thus so is the work. (Can you see that the work is zero when $\phi = 90^\circ$?) These results lead to a simple rule. To find the sign of the work done by a force, consider the force vector component that is parallel to the displacement:



A force does positive work when it has a vector component in the same direction as the displacement, and it does negative work when it has a vector component in the opposite direction. It does zero work when it has no such vector component.

Units for Work. Work has the SI unit of the joule, the same as kinetic energy. However, from Eqs. 7-6 and 7-7 we can see that an equivalent unit is the newton-meter ($\text{N} \cdot \text{m}$). The corresponding unit in the British system is the foot-pound ($\text{ft} \cdot \text{lb}$). Extending Eq. 7-2, we have

$$1 \text{ J} = 1 \text{ kg} \cdot \text{m}^2/\text{s}^2 = 1 \text{ N} \cdot \text{m} = 0.738 \text{ ft} \cdot \text{lb}. \quad (7-9)$$

Net Work. When two or more forces act on an object, the **net work** done on the object is the sum of the works done by the individual forces. We can calculate the net work in two ways. (1) We can find the work done by each force and then sum those works. (2) Alternatively, we can first find the net force \vec{F}_{net} of those forces. Then we can use Eq. 7-7, substituting the magnitude F_{net} for F and also the angle between the directions of \vec{F}_{net} and \vec{d} for ϕ . Similarly, we can use Eq. 7-8 with \vec{F}_{net} substituted for \vec{F} .

Work-Kinetic Energy Theorem

Equation 7-5 relates the change in kinetic energy of the bead (from an initial $K_i = \frac{1}{2}mv_0^2$ to a later $K_f = \frac{1}{2}mv^2$) to the work $W (= F_x d)$ done on the bead. For such particle-like objects, we can generalize that equation. Let ΔK be the change in the kinetic energy of the object, and let W be the net work done on it. Then

$$\Delta K = K_f - K_i = W, \quad (7-10)$$

which says that

$$\left(\begin{array}{c} \text{change in the kinetic} \\ \text{energy of a particle} \end{array} \right) = \left(\begin{array}{c} \text{net work done on} \\ \text{the particle} \end{array} \right).$$

We can also write

$$K_f = K_i + W, \quad (7-11)$$

which says that

$$\left(\begin{array}{c} \text{kinetic energy after} \\ \text{the net work is done} \end{array} \right) = \left(\begin{array}{c} \text{kinetic energy} \\ \text{before the net work} \end{array} \right) + \left(\begin{array}{c} \text{the net} \\ \text{work done} \end{array} \right).$$

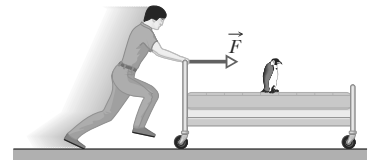


Figure 7-3 A contestant in a bed race. We can approximate the bed and its occupant as being a particle for the purpose of calculating the work done on them by the force applied by the contestant.

These statements are known traditionally as the **work–kinetic energy theorem** for particles. They hold for both positive and negative work: If the net work done on a particle is positive, then the particle’s kinetic energy increases by the amount of the work. If the net work done is negative, then the particle’s kinetic energy decreases by the amount of the work.

For example, if the kinetic energy of a particle is initially 5 J and there is a net transfer of 2 J to the particle (positive net work), the final kinetic energy is 7 J. If, instead, there is a net transfer of 2 J from the particle (negative net work), the final kinetic energy is 3 J.



Checkpoint 1

A particle moves along an x axis. Does the kinetic energy of the particle increase, decrease, or remain the same if the particle’s velocity changes (a) from -3 m/s to -2 m/s and (b) from -2 m/s to 2 m/s? (c) In each situation, is the work done on the particle positive, negative, or zero?

Sample Problem 7.02 Work done by two constant forces, industrial spies

Figure 7-4a shows two industrial spies sliding an initially stationary 225 kg floor safe a displacement \vec{d} of magnitude 8.50 m. The push \vec{F}_1 of spy 001 is 12.0 N at an angle of 30.0° downward from the horizontal; the pull \vec{F}_2 of spy 002 is 10.0 N at 40.0° above the horizontal. The magnitudes and directions of these forces do not change as the safe moves, and the floor and safe make frictionless contact.

(a) What is the net work done on the safe by forces \vec{F}_1 and \vec{F}_2 during the displacement \vec{d} ?

KEY IDEAS

(1) The net work W done on the safe by the two forces is the sum of the works they do individually. (2) Because we can treat the safe as a particle and the forces are constant in both magnitude and direction, we can use either Eq. 7-7 ($W = Fd \cos \phi$) or Eq. 7-8 ($W = \vec{F} \cdot \vec{d}$) to calculate those works. Let’s choose Eq. 7-7.

Calculations: From Eq. 7-7 and the free-body diagram for the safe in Fig. 7-4b, the work done by \vec{F}_1 is

$$W_1 = F_1 d \cos \phi_1 = (12.0 \text{ N})(8.50 \text{ m})(\cos 30.0^\circ) = 88.33 \text{ J},$$

and the work done by \vec{F}_2 is

$$W_2 = F_2 d \cos \phi_2 = (10.0 \text{ N})(8.50 \text{ m})(\cos 40.0^\circ) = 65.11 \text{ J}.$$

Thus, the net work W is

$$W = W_1 + W_2 = 88.33 \text{ J} + 65.11 \text{ J} = 153.4 \text{ J} \approx 153 \text{ J}. \quad (\text{Answer})$$

During the 8.50 m displacement, therefore, the spies transfer 153 J of energy to the kinetic energy of the safe.

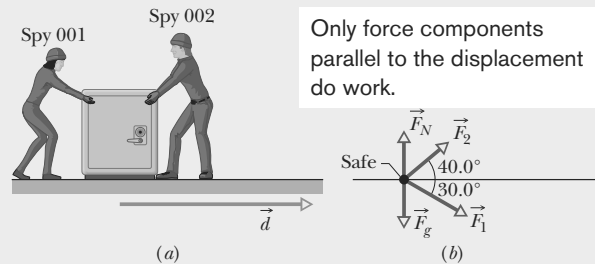


Figure 7-4 (a) Two spies move a floor safe through a displacement \vec{d} . (b) A free-body diagram for the safe.

(b) During the displacement, what is the work W_g done on the safe by the gravitational force \vec{F}_g and what is the work W_N done on the safe by the normal force \vec{F}_N from the floor?

KEY IDEA

Because these forces are constant in both magnitude and direction, we can find the work they do with Eq. 7-7.

Calculations: Thus, with mg as the magnitude of the gravitational force, we write

$$W_g = mgd \cos 90^\circ = mgd(0) = 0 \quad (\text{Answer})$$

$$\text{and} \quad W_N = F_N d \cos 90^\circ = F_N d(0) = 0. \quad (\text{Answer})$$

We should have known this result. Because these forces are perpendicular to the displacement of the safe, they do zero work on the safe and do not transfer any energy to or from it.

(c) The safe is initially stationary. What is its speed v_f at the end of the 8.50 m displacement?

KEY IDEA

The speed of the safe changes because its kinetic energy is changed when energy is transferred to it by \vec{F}_1 and \vec{F}_2 .

Calculations: We relate the speed to the work done by combining Eqs. 7-10 (the work–kinetic energy theorem) and 7-1 (the definition of kinetic energy):

$$W = K_f - K_i = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2.$$

The initial speed v_i is zero, and we now know that the work

done is 153.4 J. Solving for v_f and then substituting known data, we find that

$$v_f = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2(153.4 \text{ J})}{225 \text{ kg}}} = 1.17 \text{ m/s.} \quad (\text{Answer})$$

Sample Problem 7.03 Work done by a constant force in unit-vector notation

During a storm, a crate of crepe is sliding across a slick, oily parking lot through a displacement $\vec{d} = (-3.0 \text{ m})\hat{i}$ while a steady wind pushes against the crate with a force $\vec{F} = (2.0 \text{ N})\hat{i} + (-6.0 \text{ N})\hat{j}$. The situation and coordinate axes are shown in Fig. 7-5.

(a) How much work does this force do on the crate during the displacement?

KEY IDEA

Because we can treat the crate as a particle and because the wind force is constant (“steady”) in both magnitude and direction during the displacement, we can use either Eq. 7-7 ($W = Fd \cos \phi$) or Eq. 7-8 ($W = \vec{F} \cdot \vec{d}$) to calculate the work. Since we know \vec{F} and \vec{d} in unit-vector notation, we choose Eq. 7-8.

Calculations: We write

$$W = \vec{F} \cdot \vec{d} = [(2.0 \text{ N})\hat{i} + (-6.0 \text{ N})\hat{j}] \cdot [(-3.0 \text{ m})\hat{i}].$$

Of the possible unit-vector dot products, only $\hat{i} \cdot \hat{i}$, $\hat{j} \cdot \hat{j}$, and $\hat{k} \cdot \hat{k}$ are nonzero (see Appendix E). Here we obtain

$$\begin{aligned} W &= (2.0 \text{ N})(-3.0 \text{ m})\hat{i} \cdot \hat{i} + (-6.0 \text{ N})(-3.0 \text{ m})\hat{j} \cdot \hat{i} \\ &= (-6.0 \text{ J})(1) + 0 = -6.0 \text{ J.} \quad (\text{Answer}) \end{aligned}$$

The parallel force component does negative work, slowing the crate.

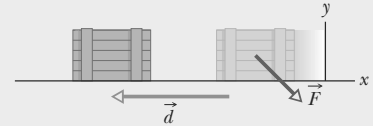


Figure 7-5 Force \vec{F} slows a crate during displacement \vec{d} .

Thus, the force does a negative 6.0 J of work on the crate, transferring 6.0 J of energy from the kinetic energy of the crate.

(b) If the crate has a kinetic energy of 10 J at the beginning of displacement \vec{d} , what is its kinetic energy at the end of \vec{d} ?

KEY IDEA

Because the force does negative work on the crate, it reduces the crate’s kinetic energy.

Calculation: Using the work–kinetic energy theorem in the form of Eq. 7-11, we have

$$K_f = K_i + W = 10 \text{ J} + (-6.0 \text{ J}) = 4.0 \text{ J.} \quad (\text{Answer})$$

Less kinetic energy means that the crate has been slowed.



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7-3 WORK DONE BY THE GRAVITATIONAL FORCE

Learning Objectives

After reading this module, you should be able to . . .

7.07 Calculate the work done by the gravitational force when an object is lifted or lowered.

7.08 Apply the work–kinetic energy theorem to situations where an object is lifted or lowered.

Key Ideas

● The work W_g done by the gravitational force \vec{F}_g on a particle-like object of mass m as the object moves through a displacement \vec{d} is given by

$$W_g = mgd \cos \phi,$$

in which ϕ is the angle between \vec{F}_g and \vec{d} .

● The work W_a done by an applied force as a particle-like object is either lifted or lowered is related to the work W_g

done by the gravitational force and the change ΔK in the object’s kinetic energy by

$$\Delta K = K_f - K_i = W_a + W_g.$$

If $K_f = K_i$, then the equation reduces to

$$W_a = -W_g,$$

which tells us that the applied force transfers as much energy to the object as the gravitational force transfers from it.

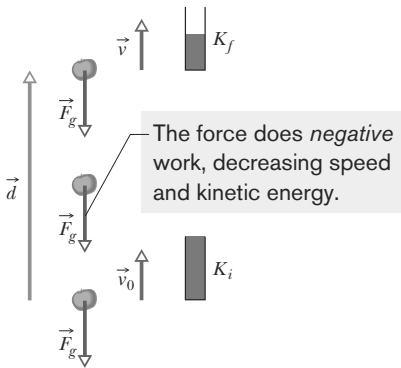


Figure 7-6 Because the gravitational force \vec{F}_g acts on it, a particle-like tomato of mass m thrown upward slows from velocity \vec{v}_0 to velocity \vec{v} during displacement \vec{d} . A kinetic energy gauge indicates the resulting change in the kinetic energy of the tomato, from $K_i (= \frac{1}{2}mv_0^2)$ to $K_f (= \frac{1}{2}mv^2)$.

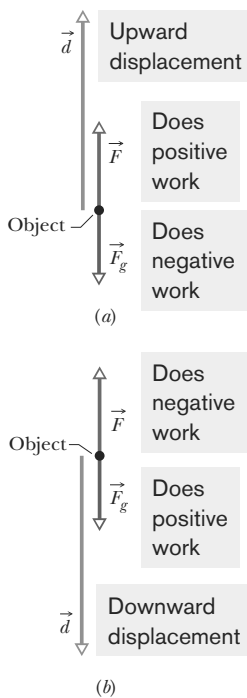


Figure 7-7 (a) An applied force \vec{F} lifts an object. The object's displacement \vec{d} makes an angle $\phi = 180^\circ$ with the gravitational force \vec{F}_g on the object. The applied force does positive work on the object. (b) An applied force \vec{F} lowers an object. The displacement \vec{d} of the object makes an angle $\phi = 0^\circ$ with the gravitational force \vec{F}_g . The applied force does negative work on the object.

Work Done by the Gravitational Force

We next examine the work done on an object by the gravitational force acting on it. Figure 7-6 shows a particle-like tomato of mass m that is thrown upward with initial speed v_0 and thus with initial kinetic energy $K_i = \frac{1}{2}mv_0^2$. As the tomato rises, it is slowed by a gravitational force \vec{F}_g ; that is, the tomato's kinetic energy decreases because \vec{F}_g does work on the tomato as it rises. Because we can treat the tomato as a particle, we can use Eq. 7-7 ($W = Fd \cos \phi$) to express the work done during a displacement \vec{d} . For the force magnitude F , we use mg as the magnitude of \vec{F}_g . Thus, the work W_g done by the gravitational force \vec{F}_g is

$$W_g = mgd \cos \phi \quad (\text{work done by gravitational force}). \quad (7-12)$$

For a rising object, force \vec{F}_g is directed opposite the displacement \vec{d} , as indicated in Fig. 7-6. Thus, $\phi = 180^\circ$ and

$$W_g = mgd \cos 180^\circ = mgd(-1) = -mgd. \quad (7-13)$$

The minus sign tells us that during the object's rise, the gravitational force acting on the object transfers energy in the amount mgd from the kinetic energy of the object. This is consistent with the slowing of the object as it rises.

After the object has reached its maximum height and is falling back down, the angle ϕ between force \vec{F}_g and displacement \vec{d} is zero. Thus,

$$W_g = mgd \cos 0^\circ = mgd(+1) = +mgd. \quad (7-14)$$

The plus sign tells us that the gravitational force now transfers energy in the amount mgd to the kinetic energy of the falling object (it speeds up, of course).

Work Done in Lifting and Lowering an Object

Now suppose we lift a particle-like object by applying a vertical force \vec{F} to it. During the upward displacement, our applied force does positive work W_a on the object while the gravitational force does negative work W_g on it. Our applied force tends to transfer energy to the object while the gravitational force tends to transfer energy from it. By Eq. 7-10, the change ΔK in the kinetic energy of the object due to these two energy transfers is

$$\Delta K = K_f - K_i = W_a + W_g, \quad (7-15)$$

in which K_f is the kinetic energy at the end of the displacement and K_i is that at the start of the displacement. This equation also applies if we lower the object, but then the gravitational force tends to transfer energy *to* the object while our force tends to transfer energy *from* it.

If an object is stationary before and after a lift (as when you lift a book from the floor to a shelf), then K_f and K_i are both zero, and Eq. 7-15 reduces to

$$W_a + W_g = 0$$

or

$$W_a = -W_g. \quad (7-16)$$

Note that we get the same result if K_f and K_i are not zero but are still equal. Either way, the result means that the work done by the applied force is the negative of the work done by the gravitational force; that is, the applied force transfers the same amount of energy to the object as the gravitational force transfers from the object. Using Eq. 7-12, we can rewrite Eq. 7-16 as

$$W_a = -mgd \cos \phi \quad (\text{work done in lifting and lowering; } K_f = K_i), \quad (7-17)$$

with ϕ being the angle between \vec{F}_g and \vec{d} . If the displacement is vertically upward (Fig. 7-7a), then $\phi = 180^\circ$ and the work done by the applied force equals mgd .

If the displacement is vertically downward (Fig. 7-7b), then $\phi = 0^\circ$ and the work done by the applied force equals $-mgd$.

Equations 7-16 and 7-17 apply to any situation in which an object is lifted or lowered, with the object stationary before and after the lift. They are independent of the magnitude of the force used. For example, if you lift a mug from the floor to over your head, your force on the mug varies considerably during the lift. Still, because the mug is stationary before and after the lift, the work your force does on the mug is given by Eqs. 7-16 and 7-17, where, in Eq. 7-17, mg is the weight of the mug and d is the distance you lift it.

Sample Problem 7.04 Work in pulling a sleigh up a snowy slope

In this problem an object is pulled along a ramp but the object starts and ends at rest and thus has no overall change in its kinetic energy (that is important). Figure 7-8a shows the situation. A rope pulls a 200 kg sleigh (which you may know) up a slope at incline angle $\theta = 30^\circ$, through distance $d = 20$ m. The sleigh and its contents have a total mass of 200 kg. The snowy slope is so slippery that we take it to be frictionless. How much work is done by each force acting on the sleigh?

KEY IDEAS

(1) During the motion, the forces are constant in magnitude and direction and thus we can calculate the work done by each with Eq. 7-7 ($W = Fd \cos \phi$) in which ϕ is the angle between the force and the displacement. We reach the same result with Eq. 7-8 ($W = \vec{F} \cdot \vec{d}$) in which we take a dot product of the force vector and displacement vector. (2) We can relate the net work done by the forces to the change in kinetic energy (or lack of a change, as here) with the work–kinetic energy theorem of Eq. 7-10 ($\Delta K = W$).

Calculations: The first thing to do with most physics problems involving forces is to draw a free-body diagram to organize our thoughts. For the sleigh, Fig. 7-8b is our free-body diagram, showing the gravitational force \vec{F}_g , the force \vec{T} from the rope, and the normal force \vec{F}_N from the slope.

Work W_N by the normal force. Let's start with this easy calculation. The normal force is perpendicular to the slope and thus also to the sleigh's displacement. Thus the normal force does not affect the sleigh's motion and does zero work. To be more formal, we can apply Eq. 7-7 to write

$$W_N = F_N d \cos 90^\circ = 0. \quad (\text{Answer})$$

Work W_g by the gravitational force. We can find the work done by the gravitational force in either of two ways (you pick the more appealing way). From an earlier discussion about ramps (Sample Problem 5.04 and Fig. 5-15), we know that the component of the gravitational force along the slope has magnitude $mg \sin \theta$ and is directed down the slope. Thus the magnitude is

$$\begin{aligned} F_{gx} &= mg \sin \theta = (200 \text{ kg})(9.8 \text{ m/s}^2) \sin 30^\circ \\ &= 980 \text{ N}. \end{aligned}$$

The angle ϕ between the displacement and this force component is 180° . So we can apply Eq. 7-7 to write

$$\begin{aligned} W_g &= F_{gx} d \cos 180^\circ = (980 \text{ N})(20 \text{ m})(-1) \\ &= -1.96 \times 10^4 \text{ J}. \end{aligned} \quad (\text{Answer})$$

The negative result means that the gravitational force removes energy from the sleigh.

The second (equivalent) way to get this result is to use the full gravitational force \vec{F}_g instead of a component. The angle between \vec{F}_g and \vec{d} is 120° (add the incline angle 30° to 90°). So, Eq. 7-7 gives us

$$\begin{aligned} W_g &= F_g d \cos 120^\circ = mgd \cos 120^\circ \\ &= (200 \text{ kg})(9.8 \text{ m/s}^2)(20 \text{ m}) \cos 120^\circ \\ &= -1.96 \times 10^4 \text{ J}. \end{aligned} \quad (\text{Answer})$$

Work W_T by the rope's force. We have two ways of calculating this work. The quickest way is to use the work–kinetic energy theorem of Eq. 7-10 ($\Delta K = W$), where the net work W done by the forces is $W_N + W_g + W_T$ and the change ΔK in the kinetic energy is just zero (because the initial and final kinetic energies are the same—namely, zero). So, Eq. 7-10 gives us

$$0 = W_N + W_g + W_T = 0 - 1.96 \times 10^4 \text{ J} + W_T$$

and

$$W_T = 1.96 \times 10^4 \text{ J}. \quad (\text{Answer})$$

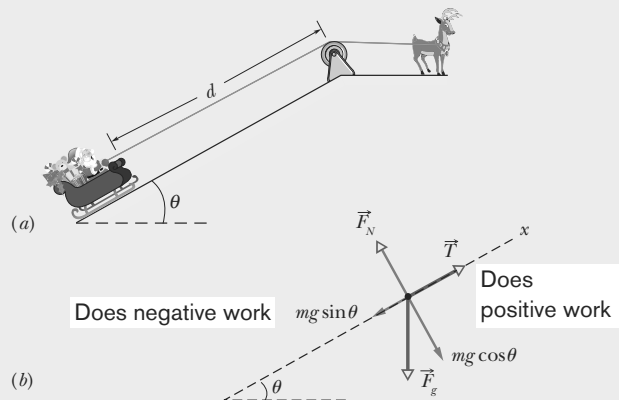


Figure 7-8 (a) A sleigh is pulled up a snowy slope. (b) The free-body diagram for the sleigh.

Instead of doing this, we can apply Newton's second law for motion along the x axis to find the magnitude F_T of the rope's force. Assuming that the acceleration along the slope is zero (except for the brief starting and stopping), we can write

$$F_{\text{net},x} = ma_x,$$

$$F_T - mg \sin 30^\circ = m(0),$$

to find

$$F_T = mg \sin 30^\circ.$$

This is the magnitude. Because the force and the displacement are both up the slope, the angle between those two vectors is zero. So, we can now write Eq. 7-7 to find the work done by the rope's force:

$$\begin{aligned} W_T &= F_T d \cos 0^\circ = (mg \sin 30^\circ) d \cos 0^\circ \\ &= (200 \text{ kg})(9.8 \text{ m/s}^2)(\sin 30^\circ)(20 \text{ m}) \cos 0^\circ \\ &= 1.96 \times 10^4 \text{ J.} \end{aligned} \quad (\text{Answer})$$

Sample Problem 7.05 Work done on an accelerating elevator cab

An elevator cab of mass $m = 500 \text{ kg}$ is descending with speed $v_i = 4.0 \text{ m/s}$ when its supporting cable begins to slip, allowing it to fall with constant acceleration $\vec{a} = \vec{g}/5$ (Fig. 7-9a).

(a) During the fall through a distance $d = 12 \text{ m}$, what is the work W_g done on the cab by the gravitational force \vec{F}_g ?

KEY IDEA

We can treat the cab as a particle and thus use Eq. 7-12 ($W_g = mgd \cos \phi$) to find the work W_g .

Calculation: From Fig. 7-9b, we see that the angle between the directions of \vec{F}_g and the cab's displacement \vec{d} is 0° . So,

$$\begin{aligned} W_g &= mgd \cos 0^\circ = (500 \text{ kg})(9.8 \text{ m/s}^2)(12 \text{ m})(1) \\ &= 5.88 \times 10^4 \text{ J} \approx 59 \text{ kJ.} \end{aligned} \quad (\text{Answer})$$

(b) During the 12 m fall, what is the work W_T done on the cab by the upward pull \vec{T} of the elevator cable?

KEY IDEA

We can calculate work W_T with Eq. 7-7 ($W = Fd \cos \phi$) by first writing $F_{\text{net},y} = ma_y$ for the components in Fig. 7-9b.

Calculations: We get

$$T - F_g = ma. \quad (7-18)$$

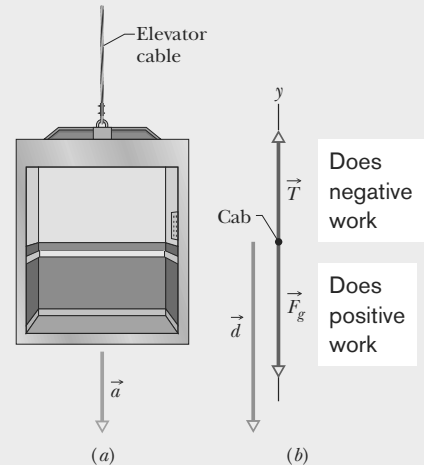
Solving for T , substituting mg for F_g , and then substituting the result in Eq. 7-7, we obtain

$$W_T = Td \cos \phi = m(a + g)d \cos \phi. \quad (7-19)$$

Next, substituting $-g/5$ for the (downward) acceleration a and then 180° for the angle ϕ between the directions of forces \vec{T} and $m\vec{g}$, we find

$$\begin{aligned} W_T &= m \left(-\frac{g}{5} + g \right) d \cos \phi = \frac{4}{5} mgd \cos \phi \\ &= \frac{4}{5} (500 \text{ kg})(9.8 \text{ m/s}^2)(12 \text{ m}) \cos 180^\circ \\ &= -4.70 \times 10^4 \text{ J} \approx -47 \text{ kJ.} \end{aligned} \quad (\text{Answer})$$

Figure 7-9 An elevator cab, descending with speed v_i , suddenly begins to accelerate downward. (a) It moves through a displacement \vec{d} with constant acceleration $\vec{a} = \vec{g}/5$. (b) A free-body diagram for the cab, displacement included.



Caution: Note that W_T is not simply the negative of W_g because the cab accelerates during the fall. Thus, Eq. 7-16 (which assumes that the initial and final kinetic energies are equal) does not apply here.

(c) What is the net work W done on the cab during the fall?

Calculation: The net work is the sum of the works done by the forces acting on the cab:

$$\begin{aligned} W &= W_g + W_T = 5.88 \times 10^4 \text{ J} - 4.70 \times 10^4 \text{ J} \\ &= 1.18 \times 10^4 \text{ J} \approx 12 \text{ kJ.} \end{aligned} \quad (\text{Answer})$$

(d) What is the cab's kinetic energy at the end of the 12 m fall?

KEY IDEA

The kinetic energy changes *because* of the net work done on the cab, according to Eq. 7-11 ($K_f = K_i + W$).

Calculation: From Eq. 7-1, we write the initial kinetic energy as $K_i = \frac{1}{2}mv_i^2$. We then write Eq. 7-11 as

$$\begin{aligned} K_f &= K_i + W = \frac{1}{2}mv_i^2 + W \\ &= \frac{1}{2}(500 \text{ kg})(4.0 \text{ m/s})^2 + 1.18 \times 10^4 \text{ J} \\ &= 1.58 \times 10^4 \text{ J} \approx 16 \text{ kJ.} \end{aligned} \quad (\text{Answer})$$



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7-4 WORK DONE BY A SPRING FORCE

Learning Objectives

After reading this module, you should be able to . . .

- 7.09** Apply the relationship (Hooke's law) between the force on an object due to a spring, the stretch or compression of the spring, and the spring constant of the spring.
- 7.10** Identify that a spring force is a variable force.
- 7.11** Calculate the work done on an object by a spring force by integrating the force from the initial position to the final

position of the object or by using the known generic result of that integration.

- 7.12** Calculate work by graphically integrating on a graph of force versus position of the object.

- 7.13** Apply the work–kinetic energy theorem to situations in which an object is moved by a spring force.

Key Ideas

- The force \vec{F}_s from a spring is

$$\vec{F}_s = -k\vec{d} \quad (\text{Hooke's law}),$$

where \vec{d} is the displacement of the spring's free end from its position when the spring is in its relaxed state (neither compressed nor extended), and k is the spring constant (a measure of the spring's stiffness). If an x axis lies along the spring, with the origin at the location of the spring's free end when the spring is in its relaxed state, we can write

$$F_x = -kx \quad (\text{Hooke's law}).$$

- A spring force is thus a variable force: It varies with the displacement of the spring's free end.

- If an object is attached to the spring's free end, the work W_s done on the object by the spring force when the object is moved from an initial position x_i to a final position x_f is

$$W_s = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2.$$

If $x_i = 0$ and $x_f = x$, then the equation becomes

$$W_s = -\frac{1}{2}kx^2.$$

Work Done by a Spring Force

We next want to examine the work done on a particle-like object by a particular type of *variable force*—namely, a **spring force**, the force from a spring. Many forces in nature have the same mathematical form as the spring force. Thus, by examining this one force, you can gain an understanding of many others.

The Spring Force

Figure 7-10a shows a spring in its **relaxed state**—that is, neither compressed nor extended. One end is fixed, and a particle-like object—a block, say—is attached to the other, free end. If we stretch the spring by pulling the block to the right as in Fig. 7-10b, the spring pulls on the block toward the left. (Because a spring force acts to restore the relaxed state, it is sometimes said to be a *restoring force*.) If we compress the spring by pushing the block to the left as in Fig. 7-10c, the spring now pushes on the block toward the right.

To a good approximation for many springs, the force \vec{F}_s from a spring is proportional to the displacement \vec{d} of the free end from its position when the spring is in the relaxed state. The *spring force* is given by

$$\vec{F}_s = -k\vec{d} \quad (\text{Hooke's law}), \quad (7-20)$$

which is known as **Hooke's law** after Robert Hooke, an English scientist of the late 1600s. The minus sign in Eq. 7-20 indicates that the direction of the spring force is always opposite the direction of the displacement of the spring's free end. The constant k is called the **spring constant** (or **force constant**) and is a measure of the stiffness of the spring. The larger k is, the stiffer the spring; that is, the larger k is, the stronger the spring's pull or push for a given displacement. The SI unit for k is the newton per meter.

In Fig. 7-10 an x axis has been placed parallel to the length of the spring, with the origin ($x = 0$) at the position of the free end when the spring is in its relaxed

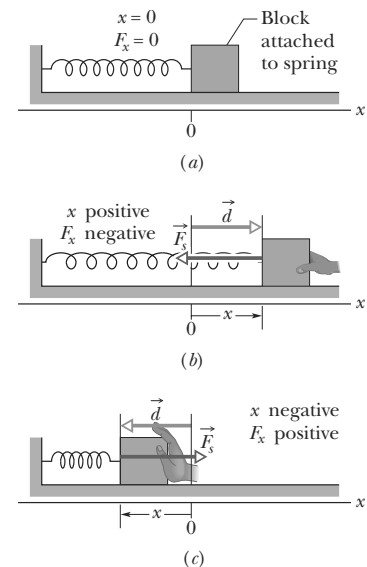


Figure 7-10 (a) A spring in its relaxed state. The origin of an x axis has been placed at the end of the spring that is attached to a block. (b) The block is displaced by \vec{d} , and the spring is stretched by a positive amount x . Note the restoring force \vec{F}_s exerted by the spring. (c) The spring is compressed by a negative amount x . Again, note the restoring force.

state. For this common arrangement, we can write Eq. 7-20 as

$$F_x = -kx \quad (\text{Hooke's law}), \quad (7-21)$$

where we have changed the subscript. If x is positive (the spring is stretched toward the right on the x axis), then F_x is negative (it is a pull toward the left). If x is negative (the spring is compressed toward the left), then F_x is positive (it is a push toward the right). Note that a spring force is a *variable force* because it is a function of x , the position of the free end. Thus F_x can be symbolized as $F(x)$. Also note that Hooke's law is a *linear* relationship between F_x and x .

The Work Done by a Spring Force

To find the work done by the spring force as the block in Fig. 7-10a moves, let us make two simplifying assumptions about the spring. (1) It is *massless*; that is, its mass is negligible relative to the block's mass. (2) It is an *ideal spring*; that is, it obeys Hooke's law exactly. Let us also assume that the contact between the block and the floor is frictionless and that the block is particle-like.

We give the block a rightward jerk to get it moving and then leave it alone. As the block moves rightward, the spring force F_x does work on the block, decreasing the kinetic energy and slowing the block. However, we *cannot* find this work by using Eq. 7-7 ($W = Fd \cos \phi$) because there is no one value of F to plug into that equation—the value of F increases as the block stretches the spring.

There is a neat way around this problem. (1) We break up the block's displacement into tiny segments that are so small that we can neglect the variation in F in each segment. (2) Then in each segment, the force has (approximately) a single value and thus we *can* use Eq. 7-7 to find the work in that segment. (3) Then we add up the work results for all the segments to get the total work. Well, that is our intent, but we don't really want to spend the next several days adding up a great many results and, besides, they would be only approximations. Instead, let's make the segments *infinitesimal* so that the error in each work result goes to zero. And then let's add up all the results by integration instead of by hand. Through the ease of calculus, we can do all this in minutes instead of days.

Let the block's initial position be x_i and its later position be x_f . Then divide the distance between those two positions into many segments, each of tiny length Δx . Label these segments, starting from x_i , as segments 1, 2, and so on. As the block moves through a segment, the spring force hardly varies because the segment is so short that x hardly varies. Thus, we can approximate the force magnitude as being constant within the segment. Label these magnitudes as F_{x1} in segment 1, F_{x2} in segment 2, and so on.

With the force now constant in each segment, we *can* find the work done within each segment by using Eq. 7-7. Here $\phi = 180^\circ$, and so $\cos \phi = -1$. Then the work done is $-F_{x1} \Delta x$ in segment 1, $-F_{x2} \Delta x$ in segment 2, and so on. The net work W_s done by the spring, from x_i to x_f , is the sum of all these works:

$$W_s = \sum -F_{xj} \Delta x, \quad (7-22)$$

where j labels the segments. In the limit as Δx goes to zero, Eq. 7-22 becomes

$$W_s = \int_{x_i}^{x_f} -F_x dx. \quad (7-23)$$

From Eq. 7-21, the force magnitude F_x is kx . Thus, substitution leads to

$$\begin{aligned} W_s &= \int_{x_i}^{x_f} -kx dx = -k \int_{x_i}^{x_f} x dx \\ &= \left(-\frac{1}{2}k\right)[x^2]_{x_i}^{x_f} = \left(-\frac{1}{2}k\right)(x_f^2 - x_i^2). \end{aligned} \quad (7-24)$$

Multiplied out, this yields

$$W_s = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2 \quad (\text{work by a spring force}). \quad (7-25)$$

This work W_s done by the spring force can have a positive or negative value, depending on whether the *net* transfer of energy is to or from the block as the block moves from x_i to x_f . **Caution:** The final position x_f appears in the *second* term on the right side of Eq. 7-25. Therefore, Eq. 7-25 tells us:



Work W_s is positive if the block ends up closer to the relaxed position ($x = 0$) than it was initially. It is negative if the block ends up farther away from $x = 0$. It is zero if the block ends up at the same distance from $x = 0$.

If $x_i = 0$ and if we call the final position x , then Eq. 7-25 becomes

$$W_s = -\frac{1}{2}kx^2 \quad (\text{work by a spring force}). \quad (7-26)$$

The Work Done by an Applied Force

Now suppose that we displace the block along the x axis while continuing to apply a force \vec{F}_a to it. During the displacement, our applied force does work W_a on the block while the spring force does work W_s . By Eq. 7-10, the change ΔK in the kinetic energy of the block due to these two energy transfers is

$$\Delta K = K_f - K_i = W_a + W_s, \quad (7-27)$$

in which K_f is the kinetic energy at the end of the displacement and K_i is that at the start of the displacement. If the block is stationary before and after the displacement, then K_f and K_i are both zero and Eq. 7-27 reduces to

$$W_a = -W_s. \quad (7-28)$$



If a block that is attached to a spring is stationary before and after a displacement, then the work done on it by the applied force displacing it is the negative of the work done on it by the spring force.

Caution: If the block is not stationary before and after the displacement, then this statement is *not* true.



Checkpoint 2

For three situations, the initial and final positions, respectively, along the x axis for the block in Fig. 7-10 are (a) -3 cm, 2 cm; (b) 2 cm, 3 cm; and (c) -2 cm, 2 cm. In each situation, is the work done by the spring force on the block positive, negative, or zero?

Sample Problem 7.06 Work done by a spring to change kinetic energy

When a spring does work on an object, we *cannot* find the work by simply multiplying the spring force by the object's displacement. The reason is that there is no one value for the force—it changes. However, we can split the displacement up into an infinite number of tiny parts and then approximate the force in each as being constant. Integration sums the work done in all those parts. Here we use the generic result of the integration.

In Fig. 7-11, a cumin canister of mass $m = 0.40$ kg slides across a horizontal frictionless counter with speed $v = 0.50$ m/s.

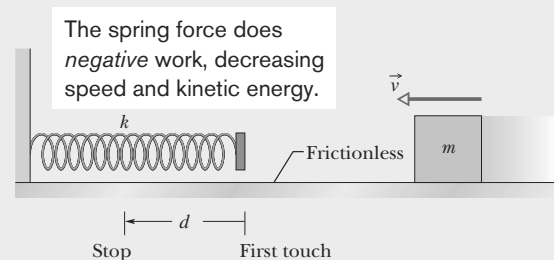


Figure 7-11 A canister moves toward a spring.

It then runs into and compresses a spring of spring constant $k = 750 \text{ N/m}$. When the canister is momentarily stopped by the spring, by what distance d is the spring compressed?

KEY IDEAS

1. The work W_s done on the canister by the spring force is related to the requested distance d by Eq. 7-26 ($W_s = -\frac{1}{2}kx^2$), with d replacing x .
2. The work W_s is also related to the kinetic energy of the canister by Eq. 7-10 ($K_f - K_i = W$).
3. The canister's kinetic energy has an initial value of $K = \frac{1}{2}mv^2$ and a value of zero when the canister is momentarily at rest.

Calculations: Putting the first two of these ideas together, we write the work–kinetic energy theorem for the canister as

$$K_f - K_i = -\frac{1}{2}kd^2.$$

Substituting according to the third key idea gives us this expression:

$$0 - \frac{1}{2}mv^2 = -\frac{1}{2}kd^2.$$

Simplifying, solving for d , and substituting known data then give us

$$\begin{aligned} d &= v \sqrt{\frac{m}{k}} = (0.50 \text{ m/s}) \sqrt{\frac{0.40 \text{ kg}}{750 \text{ N/m}}} \\ &= 1.2 \times 10^{-2} \text{ m} = 1.2 \text{ cm}. \end{aligned} \quad (\text{Answer})$$



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7-5 WORK DONE BY A GENERAL VARIABLE FORCE

Learning Objectives

After reading this module, you should be able to . . .

- 7.14 Given a variable force as a function of position, calculate the work done by it on an object by integrating the function from the initial to the final position of the object, in one or more dimensions.
- 7.15 Given a graph of force versus position, calculate the work done by graphically integrating from the initial position to the final position of the object.
- 7.16 Convert a graph of acceleration versus position to a graph of force versus position.
- 7.17 Apply the work–kinetic energy theorem to situations where an object is moved by a variable force.

Key Ideas

● When the force \vec{F} on a particle-like object depends on the position of the object, the work done by \vec{F} on the object while the object moves from an initial position r_i with coordinates (x_i, y_i, z_i) to a final position r_f with coordinates (x_f, y_f, z_f) must be found by integrating the force. If we assume that component F_x may depend on x but not on y or z , component F_y may depend on y but not on x or z , and component F_z may depend on z but not on x or y , then the

work is

$$W = \int_{x_i}^{x_f} F_x dx + \int_{y_i}^{y_f} F_y dy + \int_{z_i}^{z_f} F_z dz.$$

● If \vec{F} has only an x component, then this reduces to

$$W = \int_{x_i}^{x_f} F(x) dx.$$

Work Done by a General Variable Force

One-Dimensional Analysis

Let us return to the situation of Fig. 7-2 but now consider the force to be in the positive direction of the x axis and the force magnitude to vary with position x . Thus, as the bead (particle) moves, the magnitude $F(x)$ of the force doing work on it changes. Only the magnitude of this variable force changes, not its direction, and the magnitude at any position does not change with time.

Figure 7-12a shows a plot of such a *one-dimensional variable force*. We want an expression for the work done on the particle by this force as the particle moves from an initial point x_i to a final point x_f . However, we *cannot* use Eq. 7-7 ($W = Fd \cos \phi$) because it applies only for a constant force \vec{F} . Here, again, we shall use calculus. We divide the area under the curve of Fig. 7-12a into a number of narrow strips of width Δx (Fig. 7-12b). We choose Δx small enough to permit us to take the force $F(x)$ as being reasonably constant over that interval. We let $F_{j,\text{avg}}$ be the average value of $F(x)$ within the j th interval. Then in Fig. 7-12b, $F_{j,\text{avg}}$ is the height of the j th strip.

With $F_{j,\text{avg}}$ considered constant, the increment (small amount) of work ΔW_j done by the force in the j th interval is now approximately given by Eq. 7-7 and is

$$\Delta W_j = F_{j,\text{avg}} \Delta x. \quad (7-29)$$

In Fig. 7-12b, ΔW_j is then equal to the area of the j th rectangular, shaded strip.

To approximate the total work W done by the force as the particle moves from x_i to x_f , we add the areas of all the strips between x_i and x_f in Fig. 7-12b:

$$W = \sum \Delta W_j = \sum F_{j,\text{avg}} \Delta x. \quad (7-30)$$

Equation 7-30 is an approximation because the broken “skyline” formed by the tops of the rectangular strips in Fig. 7-12b only approximates the actual curve of $F(x)$.

We can make the approximation better by reducing the strip width Δx and using more strips (Fig. 7-12c). In the limit, we let the strip width approach zero; the number of strips then becomes infinitely large and we have, as an exact result,

$$W = \lim_{\Delta x \rightarrow 0} \sum F_{j,\text{avg}} \Delta x. \quad (7-31)$$

This limit is exactly what we mean by the integral of the function $F(x)$ between the limits x_i and x_f . Thus, Eq. 7-31 becomes

$$W = \int_{x_i}^{x_f} F(x) dx \quad (\text{work: variable force}). \quad (7-32)$$

If we know the function $F(x)$, we can substitute it into Eq. 7-32, introduce the proper limits of integration, carry out the integration, and thus find the work. (Appendix E contains a list of common integrals.) Geometrically, the work is equal to the area between the $F(x)$ curve and the x axis, between the limits x_i and x_f (shaded in Fig. 7-12d).

Three-Dimensional Analysis

Consider now a particle that is acted on by a three-dimensional force

$$\vec{F} = F_x \hat{i} + F_y \hat{j} + F_z \hat{k}, \quad (7-33)$$

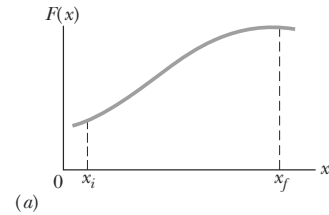
in which the components F_x , F_y , and F_z can depend on the position of the particle; that is, they can be functions of that position. However, we make three simplifications: F_x may depend on x but not on y or z , F_y may depend on y but not on x or z , and F_z may depend on z but not on x or y . Now let the particle move through an incremental displacement

$$d\vec{r} = dx \hat{i} + dy \hat{j} + dz \hat{k}. \quad (7-34)$$

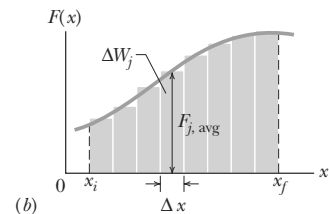
The increment of work dW done on the particle by \vec{F} during the displacement $d\vec{r}$ is, by Eq. 7-8,

$$dW = \vec{F} \cdot d\vec{r} = F_x dx + F_y dy + F_z dz. \quad (7-35)$$

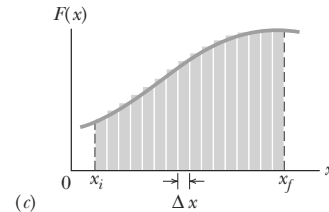
Work is equal to the area under the curve.



We can approximate that area with the area of these strips.



We can do better with more, narrower strips.



For the best, take the limit of strip widths going to zero.

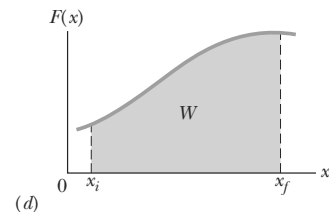


Figure 7-12 (a) A one-dimensional force $\vec{F}(x)$ plotted against the displacement x of a particle on which it acts. The particle moves from x_i to x_f . (b) Same as (a) but with the area under the curve divided into narrow strips. (c) Same as (b) but with the area divided into narrower strips. (d) The limiting case. The work done by the force is given by Eq. 7-32 and is represented by the shaded area between the curve and the x axis and between x_i and x_f .

The work W done by \vec{F} while the particle moves from an initial position r_i having coordinates (x_i, y_i, z_i) to a final position r_f having coordinates (x_f, y_f, z_f) is then

$$W = \int_{r_i}^{r_f} dW = \int_{x_i}^{x_f} F_x dx + \int_{y_i}^{y_f} F_y dy + \int_{z_i}^{z_f} F_z dz. \quad (7-36)$$

If \vec{F} has only an x component, then the y and z terms in Eq. 7-36 are zero and the equation reduces to Eq. 7-32.

Work-Kinetic Energy Theorem with a Variable Force

Equation 7-32 gives the work done by a variable force on a particle in a one-dimensional situation. Let us now make certain that the work is equal to the change in kinetic energy, as the work-kinetic energy theorem states.

Consider a particle of mass m , moving along an x axis and acted on by a net force $F(x)$ that is directed along that axis. The work done on the particle by this force as the particle moves from position x_i to position x_f is given by Eq. 7-32 as

$$W = \int_{x_i}^{x_f} F(x) dx = \int_{x_i}^{x_f} ma dx, \quad (7-37)$$

in which we use Newton's second law to replace $F(x)$ with ma . We can write the quantity $ma dx$ in Eq. 7-37 as

$$ma dx = m \frac{dv}{dt} dx. \quad (7-38)$$

From the chain rule of calculus, we have

$$\frac{dv}{dt} = \frac{dv}{dx} \frac{dx}{dt} = \frac{dv}{dx} v, \quad (7-39)$$

and Eq. 7-38 becomes

$$ma dx = m \frac{dv}{dx} v dx = mv dv. \quad (7-40)$$

Substituting Eq. 7-40 into Eq. 7-37 yields

$$\begin{aligned} W &= \int_{v_i}^{v_f} mv dv = m \int_{v_i}^{v_f} v dv \\ &= \frac{1}{2} mv_f^2 - \frac{1}{2} mv_i^2. \end{aligned} \quad (7-41)$$

Note that when we change the variable from x to v we are required to express the limits on the integral in terms of the new variable. Note also that because the mass m is a constant, we are able to move it outside the integral.

Recognizing the terms on the right side of Eq. 7-41 as kinetic energies allows us to write this equation as

$$W = K_f - K_i = \Delta K,$$

which is the work-kinetic energy theorem.



Sample Problem 7.07 Work calculated by graphical integration

In Fig. 7-13b, an 8.0 kg block slides along a frictionless floor as a force acts on it, starting at $x_1 = 0$ and ending at $x_3 = 6.5$ m. As the block moves, the magnitude and direction of the force varies according to the graph shown in Fig. 7-13a. For

example, from $x = 0$ to $x = 1$ m, the force is positive (in the positive direction of the x axis) and increases in magnitude from 0 to 40 N. And from $x = 4$ m to $x = 5$ m, the force is negative and increases in magnitude from 0 to 20 N.

(Note that this latter value is displayed as -20 N.) The block's kinetic energy at x_1 is $K_1 = 280$ J. What is the block's speed at $x_1 = 0$, $x_2 = 4.0$ m, and $x_3 = 6.5$ m?

KEY IDEAS

(1) At any point, we can relate the speed of the block to its kinetic energy with Eq. 7-1 ($K = \frac{1}{2}mv^2$). (2) We can relate the kinetic energy K_f at a later point to the initial kinetic K_i and the work W done on the block by using the work-kinetic energy theorem of Eq. 7-10 ($K_f - K_i = W$). (3) We can calculate the work W done by a variable force $F(x)$ by integrating the force versus position x . Equation 7-32 tells us that

$$W = \int_{x_i}^{x_f} F(x) dx.$$

We don't have a function $F(x)$ to carry out the integration, but we do have a graph of $F(x)$ where we can integrate by finding the area between the plotted line and the x axis. Where the plot is above the axis, the work (which is equal to the area) is positive. Where it is below the axis, the work is negative.

Calculations: The requested speed at $x = 0$ is easy because we already know the kinetic energy. So, we just plug the kinetic energy into the formula for kinetic energy:

$$K_1 = \frac{1}{2}mv_1^2,$$

$$280 \text{ J} = \frac{1}{2}(8.0 \text{ kg})v_1^2,$$

and then

$$v_1 = 8.37 \text{ m/s} \approx 8.4 \text{ m/s.} \quad (\text{Answer})$$

As the block moves from $x = 0$ to $x = 4.0$ m, the plot in Figure 7-13a is above the x axis, which means that positive work is being done on the block. We split the area under the plot into a triangle at the left, a rectangle in the center, and a triangle at the right. Their total area is

$$\frac{1}{2}(40 \text{ N})(1 \text{ m}) + (40 \text{ N})(2 \text{ m}) + \frac{1}{2}(40 \text{ N})(1 \text{ m}) = 120 \text{ N} \cdot \text{m}$$

$$= 120 \text{ J.}$$

This means that between $x = 0$ and $x = 4.0$ m, the force does 120 J of work on the block, increasing the kinetic energy and speed of the block. So, when the block reaches $x = 4.0$ m, the work-kinetic energy theorem tells us that the kinetic energy is

$$K_2 = K_1 + W$$

$$= 280 \text{ J} + 120 \text{ J} = 400 \text{ J.}$$

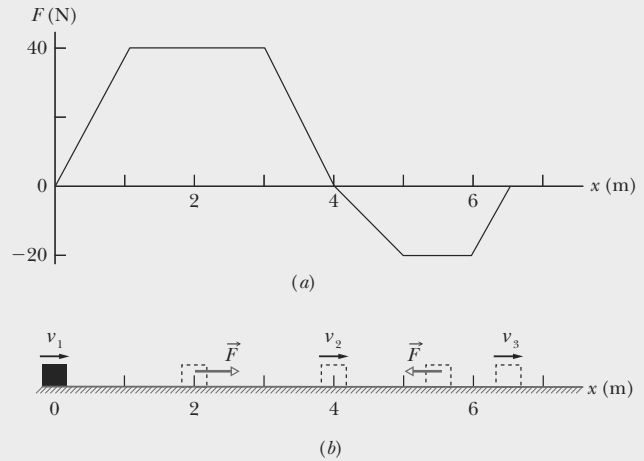


Figure 7-13 (a) A graph indicating the magnitude and direction of a variable force that acts on a block as it moves along an x axis on a floor, (b) The location of the block at several times.

Again using the definition of kinetic energy, we find

$$K_2 = \frac{1}{2}mv_2^2,$$

$$400 \text{ J} = \frac{1}{2}(8.0 \text{ kg})v_2^2,$$

and then

$$v_2 = 10 \text{ m/s.} \quad (\text{Answer})$$

This is the block's greatest speed because from $x = 4.0$ m to $x = 6.5$ m the force is negative, meaning that it opposes the block's motion, doing negative work on the block and thus decreasing the kinetic energy and speed. In that range, the area between the plot and the x axis is

$$\frac{1}{2}(20 \text{ N})(1 \text{ m}) + (20 \text{ N})(1 \text{ m}) + \frac{1}{2}(20 \text{ N})(0.5 \text{ m}) = 35 \text{ N} \cdot \text{m}$$

$$= 35 \text{ J.}$$

This means that the work done by the force in that range is -35 J. At $x = 4.0$, the block has $K = 400$ J. At $x = 6.5$ m, the work-kinetic energy theorem tells us that its kinetic energy is

$$K_3 = K_2 + W$$

$$= 400 \text{ J} - 35 \text{ J} = 365 \text{ J.}$$

Again using the definition of kinetic energy, we find

$$K_3 = \frac{1}{2}mv_3^2,$$

$$365 \text{ J} = \frac{1}{2}(8.0 \text{ kg})v_3^2,$$

and then

$$v_3 = 9.55 \text{ m/s} \approx 9.6 \text{ m/s.} \quad (\text{Answer})$$

The block is still moving in the positive direction of the x axis, a bit faster than initially.



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Sample Problem 7.08 Work, two-dimensional integration

When the force on an object depends on the position of the object, we *cannot* find the work done by it on the object by simply multiplying the force by the displacement. The reason is that there is no one value for the force—it changes. So, we must find the work in tiny little displacements and then add up all the work results. We effectively say, “Yes, the force varies over any given tiny little displacement, but the variation is so small we can approximate the force as being constant during the displacement.” Sure, it is not precise, but if we make the displacements infinitesimal, then our error becomes infinitesimal and the result becomes precise. But, to add an infinite number of work contributions by hand would take us forever, longer than a semester. So, we add them up via an integration, which allows us to do all this in minutes (much less than a semester).

Force $\vec{F} = (3x^2 \text{ N})\hat{i} + (4 \text{ N})\hat{j}$, with x in meters, acts on a particle, changing only the kinetic energy of the particle. How much work is done on the particle as it moves from coordinates (2 m, 3 m) to (3 m, 0 m)? Does the speed of the particle increase, decrease, or remain the same?

KEY IDEA

The force is a variable force because its x component depends on the value of x . Thus, we cannot use Eqs. 7-7 and 7-8 to find the work done. Instead, we must use Eq. 7-36 to integrate the force.

Calculation: We set up two integrals, one along each axis:

$$\begin{aligned} W &= \int_2^3 3x^2 dx + \int_3^0 4 dy = 3 \int_2^3 x^2 dx + 4 \int_3^0 dy \\ &= 3\left[\frac{1}{3}x^3\right]_2^3 + 4[y]_3^0 = [3^3 - 2^3] + 4[0 - 3] \\ &= 7.0 \text{ J.} \end{aligned} \quad (\text{Answer})$$

The positive result means that energy is transferred to the particle by force \vec{F} . Thus, the kinetic energy of the particle increases and, because $K = \frac{1}{2}mv^2$, its speed must also increase. If the work had come out negative, the kinetic energy and speed would have decreased.



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7-6 POWER

Learning Objectives

After reading this module, you should be able to . . .

- 7.18** Apply the relationship between average power, the work done by a force, and the time interval in which that work is done.
- 7.19** Given the work as a function of time, find the instantaneous power.

Key Ideas

- The power due to a force is the *rate* at which that force does work on an object.
- If the force does work W during a time interval Δt , the average power due to the force over that time interval is

$$P_{\text{avg}} = \frac{W}{\Delta t}.$$

- 7.20** Determine the instantaneous power by taking a dot product of the force vector and an object's velocity vector, in magnitude-angle and unit-vector notations.

- Instantaneous power is the instantaneous rate of doing work:

$$P = \frac{dW}{dt}.$$

- For a force \vec{F} at an angle ϕ to the direction of travel of the instantaneous velocity \vec{v} , the instantaneous power is

$$P = Fv \cos \phi = \vec{F} \cdot \vec{v}.$$

Power

The time rate at which work is done by a force is said to be the **power** due to the force. If a force does an amount of work W in an amount of time Δt , the **average power** due to the force during that time interval is

$$P_{\text{avg}} = \frac{W}{\Delta t} \quad (\text{average power}). \quad (7-42)$$

The **instantaneous power** P is the instantaneous time rate of doing work, which we can write as

$$P = \frac{dW}{dt} \quad (\text{instantaneous power}). \quad (7-43)$$

Suppose we know the work $W(t)$ done by a force as a function of time. Then to get the instantaneous power P at, say, time $t = 3.0$ s during the work, we would first take the time derivative of $W(t)$ and then evaluate the result for $t = 3.0$ s.

The SI unit of power is the joule per second. This unit is used so often that it has a special name, the **watt** (W), after James Watt, who greatly improved the rate at which steam engines could do work. In the British system, the unit of power is the foot-pound per second. Often the horsepower is used. These are related by

$$1 \text{ watt} = 1 \text{ W} = 1 \text{ J/s} = 0.738 \text{ ft} \cdot \text{lb/s} \quad (7-44)$$

$$\text{and} \quad 1 \text{ horsepower} = 1 \text{ hp} = 550 \text{ ft} \cdot \text{lb/s} = 746 \text{ W}. \quad (7-45)$$

Inspection of Eq. 7-42 shows that work can be expressed as power multiplied by time, as in the common unit kilowatt-hour. Thus,

$$\begin{aligned} 1 \text{ kilowatt-hour} &= 1 \text{ kW} \cdot \text{h} = (10^3 \text{ W})(3600 \text{ s}) \\ &= 3.60 \times 10^6 \text{ J} = 3.60 \text{ MJ}. \end{aligned} \quad (7-46)$$

Perhaps because they appear on our utility bills, the watt and the kilowatt-hour have become identified as electrical units. They can be used equally well as units for other examples of power and energy. Thus, if you pick up a book from the floor and put it on a tabletop, you are free to report the work that you have done as, say, $4 \times 10^{-6} \text{ kW} \cdot \text{h}$ (or more conveniently as $4 \text{ mW} \cdot \text{h}$).

We can also express the rate at which a force does work on a particle (or particle-like object) in terms of that force and the particle's velocity. For a particle that is moving along a straight line (say, an x axis) and is acted on by a constant force \vec{F} directed at some angle ϕ to that line, Eq. 7-43 becomes

$$P = \frac{dW}{dt} = \frac{F \cos \phi \, dx}{dt} = F \cos \phi \left(\frac{dx}{dt} \right),$$

$$\text{or} \quad P = Fv \cos \phi. \quad (7-47)$$

Reorganizing the right side of Eq. 7-47 as the dot product $\vec{F} \cdot \vec{v}$, we may also write the equation as

$$P = \vec{F} \cdot \vec{v} \quad (\text{instantaneous power}). \quad (7-48)$$

For example, the truck in Fig. 7-14 exerts a force \vec{F} on the trailing load, which has velocity \vec{v} at some instant. The instantaneous power due to \vec{F} is the rate at which \vec{F} does work on the load at that instant and is given by Eqs. 7-47 and 7-48. Saying that this power is “the power of the truck” is often acceptable, but keep in mind what is meant: Power is the rate at which the applied *force* does work.



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Figure 7-14 The power due to the truck's applied force on the trailing load is the rate at which that force does work on the load.



Checkpoint 3

A block moves with uniform circular motion because a cord tied to the block is anchored at the center of a circle. Is the power due to the force on the block from the cord positive, negative, or zero?

Sample Problem 7.09 Power, force, and velocity

Here we calculate an instantaneous work—that is, the rate at which work is being done at any given instant rather than averaged over a time interval. Figure 7-15 shows constant forces \vec{F}_1 and \vec{F}_2 acting on a box as the box slides rightward across a frictionless floor. Force \vec{F}_1 is horizontal, with magnitude 2.0 N; force \vec{F}_2 is angled upward by 60° to the floor and has magnitude 4.0 N. The speed v of the box at a certain instant is 3.0 m/s. What is the power due to each force acting on the box at that instant, and what is the net power? Is the net power changing at that instant?

KEY IDEA

We want an instantaneous power, not an average power over a time period. Also, we know the box's velocity (rather than the work done on it).

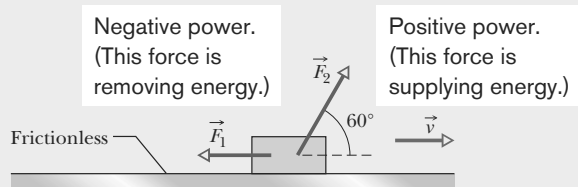


Figure 7-15 Two forces \vec{F}_1 and \vec{F}_2 act on a box that slides rightward across a frictionless floor. The velocity of the box is \vec{v} .

Calculation: We use Eq. 7-47 for each force. For force \vec{F}_1 , at angle $\phi_1 = 180^\circ$ to velocity \vec{v} , we have

$$P_1 = F_1 v \cos \phi_1 = (2.0 \text{ N})(3.0 \text{ m/s}) \cos 180^\circ = -6.0 \text{ W.} \quad (\text{Answer})$$

This negative result tells us that force \vec{F}_1 is transferring energy *from* the box at the rate of 6.0 J/s.

For force \vec{F}_2 , at angle $\phi_2 = 60^\circ$ to velocity \vec{v} , we have

$$P_2 = F_2 v \cos \phi_2 = (4.0 \text{ N})(3.0 \text{ m/s}) \cos 60^\circ = 6.0 \text{ W.} \quad (\text{Answer})$$

This positive result tells us that force \vec{F}_2 is transferring energy *to* the box at the rate of 6.0 J/s.

The net power is the sum of the individual powers (complete with their algebraic signs):

$$P_{\text{net}} = P_1 + P_2 = -6.0 \text{ W} + 6.0 \text{ W} = 0, \quad (\text{Answer})$$

which tells us that the net rate of transfer of energy to or from the box is zero. Thus, the kinetic energy ($K = \frac{1}{2}mv^2$) of the box is not changing, and so the speed of the box will remain at 3.0 m/s. With neither the forces \vec{F}_1 and \vec{F}_2 nor the velocity \vec{v} changing, we see from Eq. 7-48 that P_1 and P_2 are constant and thus so is P_{net} .



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Review & Summary

Kinetic Energy The **kinetic energy** K associated with the motion of a particle of mass m and speed v , where v is well below the speed of light, is

$$K = \frac{1}{2}mv^2 \quad (\text{kinetic energy}). \quad (7-1)$$

Work **Work** W is energy transferred to or from an object via a force acting on the object. Energy transferred to the object is positive work, and from the object, negative work.

Work Done by a Constant Force The work done on a particle by a constant force \vec{F} during displacement \vec{d} is

$$W = Fd \cos \phi = \vec{F} \cdot \vec{d} \quad (\text{work, constant force}), \quad (7-7, 7-8)$$

in which ϕ is the constant angle between the directions of \vec{F} and \vec{d} . Only the component of \vec{F} that is along the displacement \vec{d} can do work on the object. When two or more forces act on an object, their **net work** is the sum of the individual works done by the forces, which is also equal to the work that would be done on the object by the net force \vec{F}_{net} of those forces.

Work and Kinetic Energy For a particle, a change ΔK in the kinetic energy equals the net work W done on the particle:

$$\Delta K = K_f - K_i = W \quad (\text{work-kinetic energy theorem}), \quad (7-10)$$

in which K_i is the initial kinetic energy of the particle and K_f is the kinetic energy after the work is done. Equation 7-10 rearranged gives us

$$K_f = K_i + W. \quad (7-11)$$

Work Done by the Gravitational Force The work W_g done by the gravitational force \vec{F}_g on a particle-like object of mass m as the object moves through a displacement \vec{d} is given by

$$W_g = mgd \cos \phi, \quad (7-12)$$

in which ϕ is the angle between \vec{F}_g and \vec{d} .

Work Done in Lifting and Lowering an Object The work W_a done by an applied force as a particle-like object is either lifted or lowered is related to the work W_g done by the gravitational force and the change ΔK in the object's kinetic energy by

$$\Delta K = K_f - K_i = W_a + W_g. \quad (7-15)$$

If $K_f = K_i$, then Eq. 7-15 reduces to

$$W_a = -W_g, \quad (7-16)$$

which tells us that the applied force transfers as much energy to the object as the gravitational force transfers from it.

Spring Force The force \vec{F}_s from a spring is

$$\vec{F}_s = -k\vec{d} \quad (\text{Hooke's law}), \quad (7-20)$$

where \vec{d} is the displacement of the spring's free end from its position when the spring is in its **relaxed state** (neither compressed nor extended), and k is the **spring constant** (a measure of the spring's stiffness). If an x axis lies along the spring, with the origin at the location of the spring's free end when the spring is in its relaxed state, Eq. 7-20 can be written as

$$F_x = -kx \quad (\text{Hooke's law}). \quad (7-21)$$

A spring force is thus a variable force: It varies with the displacement of the spring's free end.

Work Done by a Spring Force If an object is attached to the spring's free end, the work W_s done on the object by the spring force when the object is moved from an initial position x_i to a final position x_f is

$$W_s = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2. \quad (7-25)$$

If $x_i = 0$ and $x_f = x$, then Eq. 7-25 becomes

$$W_s = -\frac{1}{2}kx^2. \quad (7-26)$$

Work Done by a Variable Force When the force \vec{F} on a particle-like object depends on the position of the object, the work done by \vec{F} on the object while the object moves from an initial position r_i with coordinates (x_i, y_i, z_i) to a final position r_f with coordinates (x_f, y_f, z_f)

must be found by integrating the force. If we assume that component F_x may depend on x but not on y or z , component F_y may depend on y but not on x or z , and component F_z may depend on z but not on x or y , then the work is

$$W = \int_{x_i}^{x_f} F_x dx + \int_{y_i}^{y_f} F_y dy + \int_{z_i}^{z_f} F_z dz. \quad (7-36)$$

If \vec{F} has only an x component, then Eq. 7-36 reduces to

$$W = \int_{x_i}^{x_f} F(x) dx. \quad (7-32)$$

Power The **power** due to a force is the **rate** at which that force does work on an object. If the force does work W during a time interval Δt , the **average power** due to the force over that time interval is

$$P_{\text{avg}} = \frac{W}{\Delta t}. \quad (7-42)$$

Instantaneous power is the instantaneous rate of doing work:

$$P = \frac{dW}{dt}. \quad (7-43)$$

For a force \vec{F} at an angle ϕ to the direction of travel of the instantaneous velocity \vec{v} , the instantaneous power is

$$P = Fv \cos \phi = \vec{F} \cdot \vec{v}. \quad (7-47, 7-48)$$

Questions

1 Rank the following velocities according to the kinetic energy a particle will have with each velocity, greatest first: (a) $\vec{v} = 4\hat{i} + 3\hat{j}$, (b) $\vec{v} = -4\hat{i} + 3\hat{j}$, (c) $\vec{v} = -3\hat{i} + 4\hat{j}$, (d) $\vec{v} = 3\hat{i} - 4\hat{j}$, (e) $\vec{v} = 5\hat{i}$, and (f) $v = 5 \text{ m/s}$ at 30° to the horizontal.

2 Figure 7-16a shows two horizontal forces that act on a block that is sliding to the right across a frictionless floor. Figure 7-16b shows three plots of the block's kinetic energy K versus time t . Which of the plots best corresponds to the following three situations: (a) $F_1 = F_2$, (b) $F_1 > F_2$, (c) $F_1 < F_2$?

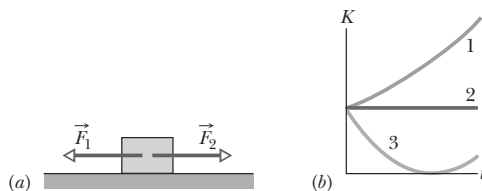


Figure 7-16 Question 2.

3 Is positive or negative work done by a constant force \vec{F} on a particle during a straight-line displacement \vec{d} if (a) the angle between \vec{F} and \vec{d} is 30° ; (b) the angle is 100° ; (c) $\vec{F} = 2\hat{i} - 3\hat{j}$ and $\vec{d} = -4\hat{i}$?

4 In three situations, a briefly applied horizontal force changes the velocity of a hockey puck that slides over frictionless ice. The overhead views of Fig. 7-17 indicate, for each situation, the puck's initial speed v_i , its final speed v_f , and the directions of the corresponding velocity vectors. Rank the situations according to the work done on the puck by the applied force, most positive first and most negative last.

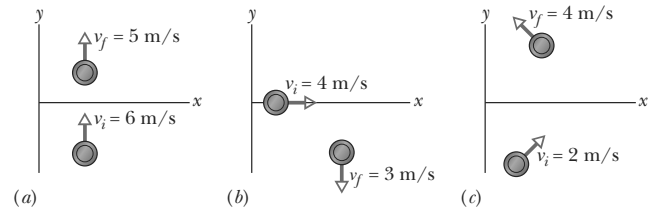


Figure 7-17 Question 4.

5 The graphs in Fig. 7-18 give the x component F_x of a force acting on a particle moving along an x axis. Rank them according to the work done by the force on the particle from $x = 0$ to $x = x_1$, from most positive work first to most negative work last.

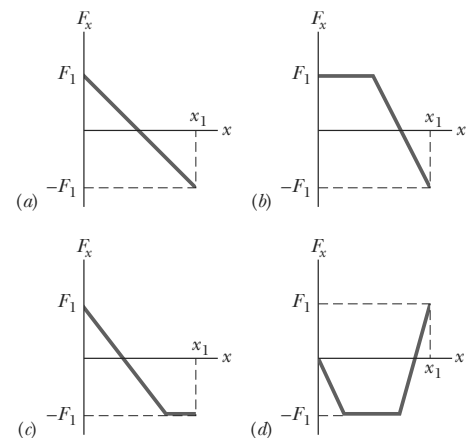


Figure 7-18 Question 5.

6 Figure 7-19 gives the x -component F_x of a force that can act on a particle. If the particle begins at rest at $x = 0$, what is its coordinate when it has (a) its greatest kinetic energy, (b) its greatest speed, and (c) zero speed? (d) What is the particle's direction of travel after it reaches $x = 6$ m?

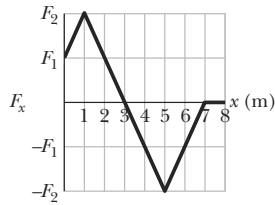


Figure 7-19 Question 6.

7 In Fig. 7-20, a greased pig has a choice of three frictionless slides along which to slide to the ground. Rank the slides according to how much work the gravitational force does on the pig during the descent, greatest first.

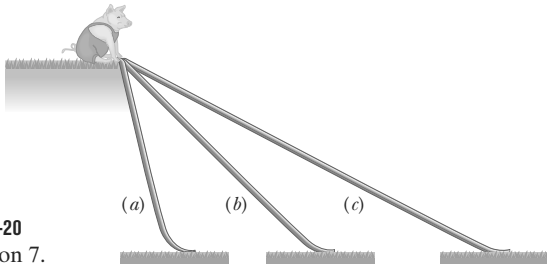


Figure 7-20 Question 7.

8 Figure 7-21a shows four situations in which a horizontal force acts on the same block, which is initially at rest. The force magnitudes are $F_2 = F_4 = 2F_1 = 2F_3$. The horizontal component v_x of the block's velocity is shown in Fig. 7-21b for the four situations. (a) Which plot in Fig. 7-21b best corresponds to which force in Fig. 7-21a? (b) Which

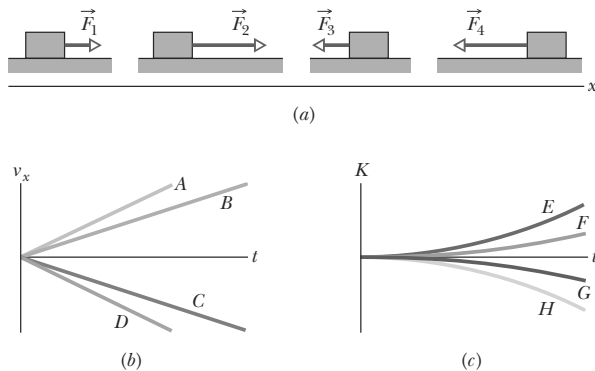


Figure 7-21 Question 8.

plot in Fig. 7-21c (for kinetic energy K versus time t) best corresponds to which plot in Fig. 7-21b?

9 Spring A is stiffer than spring B ($k_A > k_B$). The spring force of which spring does more work if the springs are compressed (a) the same distance and (b) by the same applied force?

10 A glob of slime is launched or dropped from the edge of a cliff. Which of the graphs in Fig. 7-22 could possibly show how the kinetic energy of the glob changes during its flight?

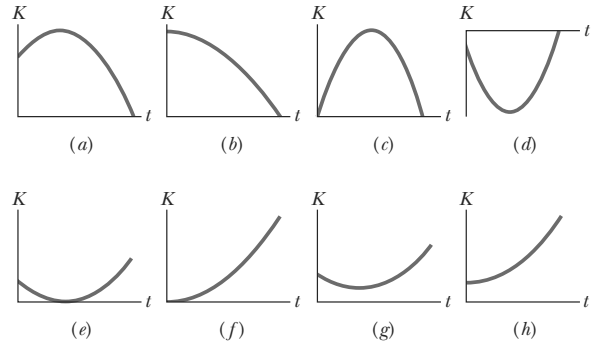


Figure 7-22 Question 10.

11 In three situations, a single force acts on a moving particle. Here are the velocities (at that instant) and the forces: (1) $\vec{v} = (-4\hat{i})$ m/s, $\vec{F} = (6\hat{i} - 20\hat{j})$ N; (2) $\vec{v} = (2\hat{i} - 3\hat{j})$ m/s, $\vec{F} = (-2\hat{j} + 7\hat{k})$ N; (3) $\vec{v} = (-3\hat{i} + \hat{j})$ m/s, $\vec{F} = (2\hat{i} + 6\hat{j})$ N. Rank the situations according to the rate at which energy is being transferred, greatest transfer to the particle ranked first, greatest transfer from the particle ranked last.

12 Figure 7-23 shows three arrangements of a block attached to identical springs that are in their relaxed state when the block is centered as shown. Rank the arrangements according to the magnitude of the net force on the block, largest first, when the block is displaced by distance d (a) to the right and (b) to the left. Rank the arrangements according to the work done on the block by the spring forces, greatest first, when the block is displaced by d (c) to the right and (d) to the left.

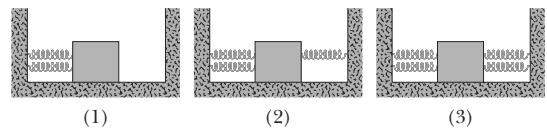


Figure 7-23 Question 12.

Problems



Tutoring problem available (at instructor's discretion) in WileyPLUS and WebAssign



Worked-out solution available in Student Solutions Manual

WWW Worked-out solution is at



Number of dots indicates level of problem difficulty

ILW Interactive solution is at



Additional information available in *The Flying Circus of Physics* and at flyingcircusofphysics.com

<http://www.wiley.com/college/halliday>


Module 7-1 Kinetic Energy

•1 SSM A proton (mass $m = 1.67 \times 10^{-27}$ kg) is being accelerated along a straight line at 3.6×10^{15} m/s² in a machine. If the proton has an initial speed of 2.4×10^7 m/s and travels 3.5 cm, what then is (a) its speed and (b) the increase in its kinetic energy?

•2 If a Saturn V rocket with an Apollo spacecraft attached had a combined mass of 2.9×10^5 kg and reached a speed of 11.2 km/s, how much kinetic energy would it then have?

•3 On August 10, 1972, a large meteorite skipped across the atmosphere above the western United States and western Canada,

much like a stone skipped across water. The accompanying fireball was so bright that it could be seen in the daytime sky and was brighter than the usual meteorite trail. The meteorite's mass was about 4×10^6 kg; its speed was about 15 km/s. Had it entered the atmosphere vertically, it would have hit Earth's surface with about the same speed. (a) Calculate the meteorite's loss of kinetic energy (in joules) that would have been associated with the vertical impact. (b) Express the energy as a multiple of the explosive energy of 1 megaton of TNT, which is 4.2×10^{15} J. (c) The energy associated with the atomic bomb explosion over Hiroshima was equivalent to 13 kilotons of TNT. To how many Hiroshima bombs would the meteorite impact have been equivalent?

•4  An explosion at ground level leaves a crater with a diameter that is proportional to the energy of the explosion raised to the $\frac{1}{3}$ power; an explosion of 1 megaton of TNT leaves a crater with a 1 km diameter. Below Lake Huron in Michigan there appears to be an ancient impact crater with a 50 km diameter. What was the kinetic energy associated with that impact, in terms of (a) megatons of TNT (1 megaton yields 4.2×10^{15} J) and (b) Hiroshima bomb equivalents (13 kilotons of TNT each)? (Ancient meteorite or comet impacts may have significantly altered the climate, killing off the dinosaurs and other life-forms.)

•5 A father racing his son has half the kinetic energy of the son, who has half the mass of the father. The father speeds up by 1.0 m/s and then has the same kinetic energy as the son. What are the original speeds of (a) the father and (b) the son?

•6 A bead with mass 1.8×10^{-2} kg is moving along a wire in the positive direction of an x axis. Beginning at time $t = 0$, when the bead passes through $x = 0$ with speed 12 m/s, a constant force acts on the bead. Figure 7-24 indicates the bead's position at these four times: $t_0 = 0$, $t_1 = 1.0$ s, $t_2 = 2.0$ s, and $t_3 = 3.0$ s. The bead momentarily stops at $t = 3.0$ s. What is the kinetic energy of the bead at $t = 10$ s?

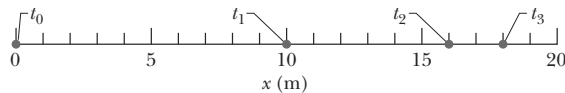


Figure 7-24 Problem 6.

Module 7-2 Work and Kinetic Energy

•7 A 3.0 kg body is at rest on a frictionless horizontal air track when a constant horizontal force \vec{F} acting in the positive direction of an x axis along the track is applied to the body. A stroboscopic graph of the position of the body as it slides to the right is shown in Fig. 7-25. The force \vec{F} is applied to the body at $t = 0$, and the graph records the position of the body at 0.50 s intervals. How much work is done on the body by the applied force \vec{F} between $t = 0$ and $t = 2.0$ s?

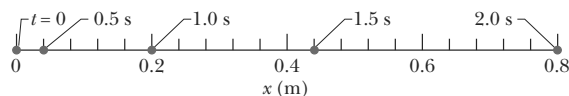


Figure 7-25 Problem 7.

•8 A ice block floating in a river is pushed through a displacement $\vec{d} = (15 \text{ m})\hat{i} - (12 \text{ m})\hat{j}$ along a straight embankment by rushing water, which exerts a force $\vec{F} = (210 \text{ N})\hat{i} - (150 \text{ N})\hat{j}$ on the block. How much work does the force do on the block during the displacement?

•9 The only force acting on a 2.0 kg canister that is moving in an xy plane has a magnitude of 5.0 N. The canister initially has a veloc-

ity of 4.0 m/s in the positive x direction and some time later has a velocity of 6.0 m/s in the positive y direction. How much work is done on the canister by the 5.0 N force during this time?

•10 A coin slides over a frictionless plane and across an xy coordinate system from the origin to a point with xy coordinates (3.0 m, 4.0 m) while a constant force acts on it. The force has magnitude 2.0 N and is directed at a counterclockwise angle of 100° from the positive direction of the x axis. How much work is done by the force on the coin during the displacement?

•11 A 12.0 N force with a fixed orientation does work on a particle as the particle moves through the three-dimensional displacement $\vec{d} = (2.00\hat{i} - 4.00\hat{j} + 3.00\hat{k})$ m. What is the angle between the force and the displacement if the change in the particle's kinetic energy is (a) +30.0 J and (b) -30.0 J?

•12 A can of bolts and nuts is pushed 2.00 m along an x axis by a broom along the greasy (frictionless) floor of a car repair shop in a version of shuffleboard. Figure 7-26 gives the work W done on the can by the constant horizontal force from the broom, versus the can's position x . The scale of the figure's vertical axis is set by $W_s = 6.0$ J. (a) What is the magnitude of that force? (b) If the can had an initial kinetic energy of 3.00 J, moving in the positive direction of the x axis, what is its kinetic energy at the end of the 2.00 m?

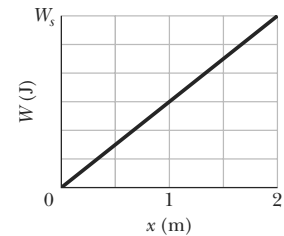



Figure 7-26 Problem 12.

•13 A luge and its rider, with a total mass of 85 kg, emerge from a downhill track onto a horizontal straight track with an initial speed of 37 m/s. If a force slows them to a stop at a constant rate of 2.0 m/s^2 , (a) what magnitude F is required for the force, (b) what distance d do they travel while slowing, and (c) what work W is done on them by the force? What are (d) F , (e) d , and (f) W if they, instead, slow at 4.0 m/s^2 ?

•14  Figure 7-27 shows an overhead view of three horizontal forces acting on a cargo canister that was initially stationary but now moves across a frictionless floor. The force magnitudes are $F_1 = 3.00$ N, $F_2 = 4.00$ N, and $F_3 = 10.0$ N, and the indicated angles are $\theta_2 = 50.0^\circ$ and $\theta_3 = 35.0^\circ$. What is the net work done on the canister by the three forces during the first 4.00 m of displacement?

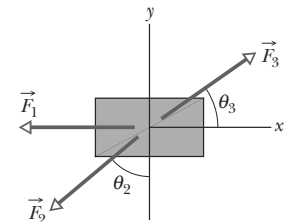



Figure 7-27 Problem 14.

•15  Figure 7-28 shows three forces applied to a trunk that moves leftward by 3.00 m over a frictionless floor. The force magnitudes are $F_1 = 5.00$ N, $F_2 = 9.00$ N, and $F_3 = 3.00$ N, and the indicated angle is $\theta = 60.0^\circ$. During the displacement, (a) what is the net work done on the trunk by the three forces and (b) does the kinetic energy of the trunk increase or decrease?

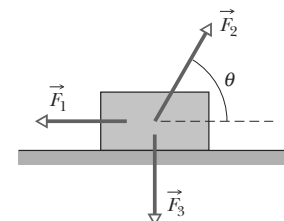



Figure 7-28 Problem 15.

•16  An 8.0 kg object is moving in the positive direction of an x axis. When it passes through $x = 0$, a constant force directed

along the axis begins to act on it. Figure 7-29 gives its kinetic energy K versus position x as it moves from $x = 0$ to $x = 5.0$ m; $K_0 = 30.0$ J. The force continues to act. What is v when the object moves back through $x = -3.0$ m?

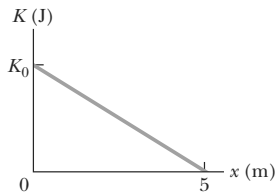


Figure 7-29 Problem 16.

Module 7-3 Work Done by the Gravitational Force

•17 **SSM WWW** A helicopter lifts a 72 kg astronaut 15 m vertically from the ocean by means of a cable. The acceleration of the astronaut is $g/10$. How much work is done on the astronaut by (a) the force from the helicopter and (b) the gravitational force on her? Just before she reaches the helicopter, what are her (c) kinetic energy and (d) speed?

•18 **GO** (a) In 1975 the roof of Montreal's Velodrome, with a weight of 360 kN, was lifted by 10 cm so that it could be centered. How much work was done on the roof by the forces making the lift? (b) In 1960 a Tampa, Florida, mother reportedly raised one end of a car that had fallen onto her son when a jack failed. If her panic lift effectively raised 4000 N (about $\frac{1}{4}$ of the car's weight) by 5.0 cm, how much work did her force do on the car?

•19 **GO** In Fig. 7-30, a block of ice slides down a frictionless ramp at angle $\theta = 50^\circ$ while an ice worker pulls on the block (via a rope) with a force \vec{F}_r that has a magnitude of 50 N and is directed up the ramp. As the block slides through distance $d = 0.50$ m along the ramp, its kinetic energy increases by 80 J. How much greater would its kinetic energy have been if the rope had not been attached to the block?

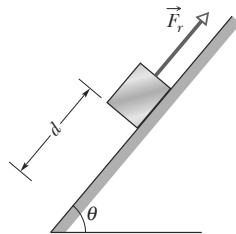


Figure 7-30 Problem 19.

•20 A block is sent up a frictionless ramp along which an x axis extends upward. Figure 7-31 gives the kinetic energy of the block as a function of position x ; the scale of the figure's vertical axis is set by $K_s = 40.0$ J. If the block's initial speed is 4.00 m/s, what is the normal force on the block?

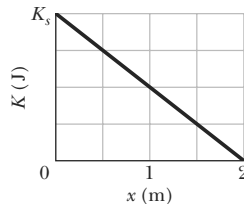


Figure 7-31 Problem 20.

•21 **SSM** A cord is used to vertically lower an initially stationary block of mass M at a constant downward acceleration of $g/4$. When the block has fallen a distance d , find (a) the work done by the cord's force on the block, (b) the work done by the gravitational force on the block, (c) the kinetic energy of the block, and (d) the speed of the block.

•22 A cave rescue team lifts an injured spelunker directly upward and out of a sinkhole by means of a motor-driven cable. The lift is performed in three stages, each requiring a vertical distance of 10.0 m: (a) the initially stationary spelunker is accelerated to a speed of 5.00 m/s; (b) he is then lifted at the constant speed of 5.00 m/s; (c) finally he is decelerated to zero speed. How much work is done on the 80.0 kg rescuee by the force lifting him during each stage?

•23 In Fig. 7-32, a constant force \vec{F}_a of magnitude 82.0 N is applied to a 3.00 kg shoe box at angle $\phi = 53.0^\circ$, causing

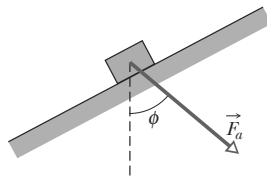


Figure 7-32 Problem 23.

the box to move up a frictionless ramp at constant speed. How much work is done on the box by \vec{F}_a when the box has moved through vertical distance $h = 0.150$ m?

•24 **GO** In Fig. 7-33, a horizontal force \vec{F}_a of magnitude 20.0 N is applied to a 3.00 kg psychology book as the book slides a distance $d = 0.500$ m up a frictionless ramp at angle $\theta = 30.0^\circ$. (a) During the displacement, what is the net work done on the book by \vec{F}_a , the gravitational force on the book, and the normal force on the book? (b) If the book has zero kinetic energy at the start of the displacement, what is its speed at the end of the displacement?

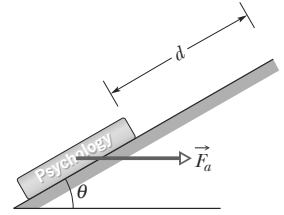


Figure 7-33 Problem 24.

•25 **GO** In Fig. 7-34, a 0.250 kg block of cheese lies on the floor of a 900 kg elevator cab that is being pulled upward by a cable through distance $d_1 = 2.40$ m and then through distance $d_2 = 10.5$ m. (a) Through d_1 , if the normal force on the block from the floor has constant magnitude $F_N = 3.00$ N, how much work is done on the cab by the force from the cable? (b) Through d_2 , if the work done on the cab by the (constant) force from the cable is 92.61 kJ, what is the magnitude of F_N ?

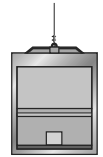


Figure 7-34 Problem 25.

Module 7-4 Work Done by a Spring Force

•26 In Fig. 7-10, we must apply a force of magnitude 80 N to hold the block stationary at $x = -2.0$ cm. From that position, we then slowly move the block so that our force does +4.0 J of work on the spring–block system; the block is then again stationary. What is the block's position? (*Hint:* There are two answers.)

•27 A spring and block are in the arrangement of Fig. 7-10. When the block is pulled out to $x = +4.0$ cm, we must apply a force of magnitude 360 N to hold it there. We pull the block to $x = 11$ cm and then release it. How much work does the spring do on the block as the block moves from $x_i = +5.0$ cm to (a) $x = +3.0$ cm, (b) $x = -3.0$ cm, (c) $x = -5.0$ cm, and (d) $x = -9.0$ cm?

•28 During spring semester at MIT, residents of the parallel buildings of the East Campus dorms battle one another with large catapults that are made with surgical hose mounted on a window frame. A balloon filled with dyed water is placed in a pouch attached to the hose, which is then stretched through the width of the room. Assume that the stretching of the hose obeys Hooke's law with a spring constant of 100 N/m. If the hose is stretched by 5.00 m and then released, how much work does the force from the hose do on the balloon in the pouch by the time the hose reaches its relaxed length?

•29 In the arrangement of Fig. 7-10, we gradually pull the block from $x = 0$ to $x = +3.0$ cm, where it is stationary. Figure 7-35 gives

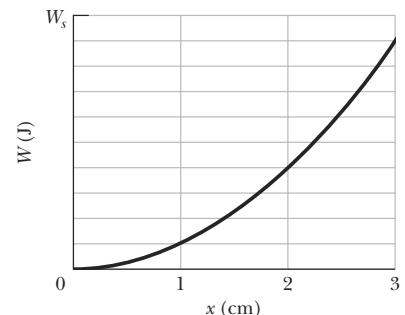


Figure 7-35 Problem 29.

the work that our force does on the block. The scale of the figure's vertical axis is set by $W_s = 1.0$ J. We then pull the block out to $x = +5.0$ cm and release it from rest. How much work does the spring do on the block when the block moves from $x_i = +5.0$ cm to (a) $x = +4.0$ cm, (b) $x = -2.0$ cm, and (c) $x = -5.0$ cm?

••30 In Fig. 7-10a, a block of mass m lies on a horizontal frictionless surface and is attached to one end of a horizontal spring (spring constant k) whose other end is fixed. The block is initially at rest at the position where the spring is unstretched ($x = 0$) when a constant horizontal force \vec{F} in the positive direction of the x axis is applied to it. A plot of the resulting kinetic energy of the block versus its position x is shown in Fig. 7-36. The scale of the figure's vertical axis is set by $K_s = 4.0$ J. (a) What is the magnitude of \vec{F} ? (b) What is the value of k ?

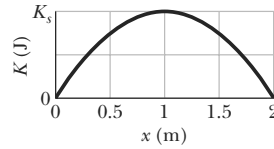


Figure 7-36 Problem 30.

••31 SSM WWW The only force acting on a 2.0 kg body as it moves along a positive x axis has an x component $F_x = -6x$ N, with x in meters. The velocity at $x = 3.0$ m is 8.0 m/s. (a) What is the velocity of the body at $x = 4.0$ m? (b) At what positive value of x will the body have a velocity of 5.0 m/s?

••32 Figure 7-37 gives spring force F_x versus position x for the spring-block arrangement of Fig. 7-10. The scale is set by $F_s = 160.0$ N. We release the block at $x = 12$ cm. How much work does the spring do on the block when the block moves from $x_i = +8.0$ cm to (a) $x = +5.0$ cm, (b) $x = -5.0$ cm, (c) $x = -8.0$ cm, and (d) $x = -10.0$ cm?

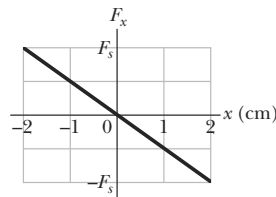


Figure 7-37 Problem 32.

•••33 GO The block in Fig. 7-10a lies on a horizontal frictionless surface, and the spring constant is 50 N/m. Initially, the spring is at its relaxed length and the block is stationary at position $x = 0$. Then an applied force with a constant magnitude of 3.0 N pulls the block in the positive direction of the x axis, stretching the spring until the block stops. When that stopping point is reached, what are (a) the position of the block, (b) the work that has been done on the block by the applied force, and (c) the work that has been done on the block by the spring force? During the block's displacement, what are (d) the block's position when its kinetic energy is maximum and (e) the value of that maximum kinetic energy?

Module 7-5 Work Done by a General Variable Force

•34 ILW A 10 kg brick moves along an x axis. Its acceleration as a function of its position is shown in Fig. 7-38. The scale of the figure's vertical axis is set by $a_s = 20.0$ m/s². What is the net work performed on the brick by the force causing the acceleration as the brick moves from $x = 0$ to $x = 8.0$ m?

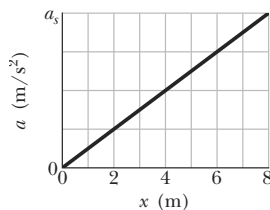


Figure 7-38 Problem 34.

•35 SSM WWW The force on a particle is directed along an x axis and given by $F = F_0(x/x_0 - 1)$. Find the work done by the force in moving the particle from $x = 0$ to $x = 2x_0$ by (a) plotting $F(x)$ and measuring the work from the graph and (b) integrating $F(x)$.

•36 GO A 5.0 kg block moves in a straight line on a horizontal frictionless surface under the influence of a force that varies with position as shown in Fig. 7-39. The scale of the figure's vertical axis is set by $F_s = 10.0$ N. How much work is done by the force as the block moves from the origin to $x = 8.0$ m?

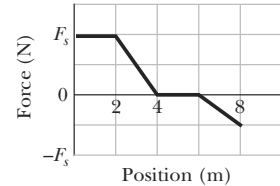


Figure 7-39 Problem 36.

••37 GO Figure 7-40 gives the acceleration of a 2.00 kg particle as an applied force \vec{F}_a moves it from rest along an x axis from $x = 0$ to $x = 9.0$ m. The scale of the figure's vertical axis is set by $a_s = 6.0$ m/s². How much work has the force done on the particle when the particle reaches (a) $x = 4.0$ m, (b) $x = 7.0$ m, and (c) $x = 9.0$ m? What is the particle's speed and direction of travel when it reaches (d) $x = 4.0$ m, (e) $x = 7.0$ m, and (f) $x = 9.0$ m?

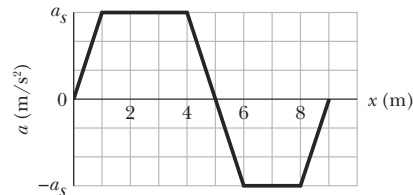


Figure 7-40 Problem 37.

••38 A 1.5 kg block is initially at rest on a horizontal frictionless surface when a horizontal force along an x axis is applied to the block. The force is given by $\vec{F}(x) = (2.5 - x^2)\hat{i}$ N, where x is in meters and the initial position of the block is $x = 0$. (a) What is the kinetic energy of the block as it passes through $x = 2.0$ m? (b) What is the maximum kinetic energy of the block between $x = 0$ and $x = 2.0$ m?

••39 GO A force $\vec{F} = (cx - 3.00x^2)\hat{i}$ acts on a particle as the particle moves along an x axis, with \vec{F} in newtons, x in meters, and c a constant. At $x = 0$, the particle's kinetic energy is 20.0 J; at $x = 3.00$ m, it is 11.0 J. Find c .

••40 A can of sardines is made to move along an x axis from $x = 0.25$ m to $x = 1.25$ m by a force with a magnitude given by $F = \exp(-4x^2)$, with x in meters and F in newtons. (Here \exp is the exponential function.) How much work is done on the can by the force?

••41 A single force acts on a 3.0 kg particle-like object whose position is given by $x = 3.0t - 4.0t^2 + 1.0t^3$, with x in meters and t in seconds. Find the work done by the force from $t = 0$ to $t = 4.0$ s.

••42 GO Figure 7-41 shows a cord attached to a cart that can slide along a frictionless horizontal rail aligned along an x axis. The left

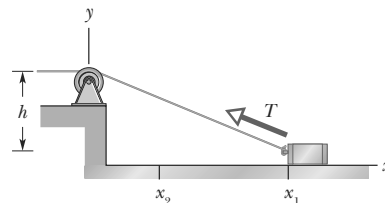


Figure 7-41 Problem 42.

end of the cord is pulled over a pulley, of negligible mass and friction and at cord height $h = 1.20$ m, so the cart slides from $x_1 = 3.00$ m to $x_2 = 1.00$ m. During the move, the tension in the cord is a constant 25.0 N. What is the change in the kinetic energy of the cart during the move?

Module 7-6 Power

•43 **SSM** A force of 5.0 N acts on a 15 kg body initially at rest. Compute the work done by the force in (a) the first, (b) the second, and (c) the third seconds and (d) the instantaneous power due to the force at the end of the third second.

•44 A skier is pulled by a towrope up a frictionless ski slope that makes an angle of 12° with the horizontal. The rope moves parallel to the slope with a constant speed of 1.0 m/s. The force of the rope does 900 J of work on the skier as the skier moves a distance of 8.0 m up the incline. (a) If the rope moved with a constant speed of 2.0 m/s, how much work would the force of the rope do on the skier as the skier moved a distance of 8.0 m up the incline? At what rate is the force of the rope doing work on the skier when the rope moves with a speed of (b) 1.0 m/s and (c) 2.0 m/s?

•45 **SSM ILW** A 100 kg block is pulled at a constant speed of 5.0 m/s across a horizontal floor by an applied force of 122 N directed 37° above the horizontal. What is the rate at which the force does work on the block?

•46 The loaded cab of an elevator has a mass of 3.0×10^3 kg and moves 210 m up the shaft in 23 s at constant speed. At what average rate does the force from the cable do work on the cab?

•47 A machine carries a 4.0 kg package from an initial position of $\vec{d}_i = (0.50 \text{ m})\hat{i} + (0.75 \text{ m})\hat{j} + (0.20 \text{ m})\hat{k}$ at $t = 0$ to a final position of $\vec{d}_f = (7.50 \text{ m})\hat{i} + (12.0 \text{ m})\hat{j} + (7.20 \text{ m})\hat{k}$ at $t = 12$ s. The constant force applied by the machine on the package is $\vec{F} = (2.00 \text{ N})\hat{i} + (4.00 \text{ N})\hat{j} + (6.00 \text{ N})\hat{k}$. For that displacement, find (a) the work done on the package by the machine's force and (b) the average power of the machine's force on the package.

•48 A 0.30 kg ladle sliding on a horizontal frictionless surface is attached to one end of a horizontal spring ($k = 500$ N/m) whose other end is fixed. The ladle has a kinetic energy of 10 J as it passes through its equilibrium position (the point at which the spring force is zero). (a) At what rate is the spring doing work on the ladle as the ladle passes through its equilibrium position? (b) At what rate is the spring doing work on the ladle when the spring is compressed 0.10 m and the ladle is moving away from the equilibrium position?

•49 **SSM** A fully loaded, slow-moving freight elevator has a cab with a total mass of 1200 kg, which is required to travel upward 54 m in 3.0 min, starting and ending at rest. The elevator's counterweight has a mass of only 950 kg, and so the elevator motor must help. What average power is required of the force the motor exerts on the cab via the cable?

•50 (a) At a certain instant, a particle-like object is acted on by a force $\vec{F} = (4.0 \text{ N})\hat{i} - (2.0 \text{ N})\hat{j} + (9.0 \text{ N})\hat{k}$ while the object's velocity is $\vec{v} = -(2.0 \text{ m/s})\hat{i} + (4.0 \text{ m/s})\hat{k}$. What is the instantaneous rate at which the force does work on the object? (b) At some other time, the velocity consists of only a y component. If the force is unchanged and the instantaneous power is -12 W, what is the velocity of the object?

•51 A force $\vec{F} = (3.00 \text{ N})\hat{i} + (7.00 \text{ N})\hat{j} + (7.00 \text{ N})\hat{k}$ acts on a 2.00 kg mobile object that moves from an initial position of

$\vec{d}_i = (3.00 \text{ m})\hat{i} - (2.00 \text{ m})\hat{j} + (5.00 \text{ m})\hat{k}$ to a final position of $\vec{d}_f = -(5.00 \text{ m})\hat{i} + (4.00 \text{ m})\hat{j} + (7.00 \text{ m})\hat{k}$ in 4.00 s. Find (a) the work done on the object by the force in the 4.00 s interval, (b) the average power due to the force during that interval, and (c) the angle between vectors \vec{d}_i and \vec{d}_f .

••52 A funny car accelerates from rest through a measured track distance in time T with the engine operating at a constant power P . If the track crew can increase the engine power by a differential amount dP , what is the change in the time required for the run?

Additional Problems

53 Figure 7-42 shows a cold package of hot dogs sliding rightward across a frictionless floor through a distance $d = 20.0$ cm while three forces act on the package. Two of them are horizontal and have the magnitudes $F_1 = 5.00$ N and $F_2 = 1.00$ N; the third is angled down by $\theta = 60.0^\circ$ and has the magnitude $F_3 = 4.00$ N. (a) For the 20.0 cm displacement, what is the *net* work done on the package by the three applied forces, the gravitational force on the package, and the normal force on the package? (b) If the package has a mass of 2.0 kg and an initial kinetic energy of 0, what is its speed at the end of the displacement?

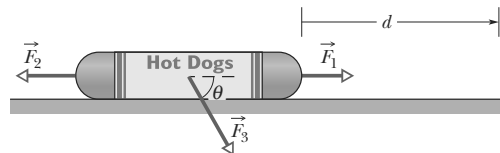


Figure 7-42 Problem 53.

54 **GO** The only force acting on a 2.0 kg body as the body moves along an x axis varies as shown in Fig. 7-43. The scale of the figure's vertical axis is set by $F_s = 4.0$ N. The velocity of the body at $x = 0$ is 4.0 m/s. (a) What is the kinetic energy of the body at $x = 3.0$ m? (b) At what value of x will the body have a kinetic energy of 8.0 J? (c) What is the maximum kinetic energy of the body between $x = 0$ and $x = 5.0$ m?

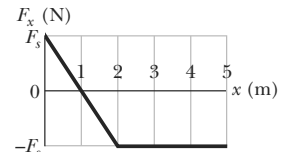


Figure 7-43 Problem 54.

55 **SSM** A horse pulls a cart with a force of 40 lb at an angle of 30° above the horizontal and moves along at a speed of 6.0 mi/h. (a) How much work does the force do in 10 min? (b) What is the average power (in horsepower) of the force?

56 An initially stationary 2.0 kg object accelerates horizontally and uniformly to a speed of 10 m/s in 3.0 s. (a) In that 3.0 s interval, how much work is done on the object by the force accelerating it? What is the instantaneous power due to that force (b) at the end of the interval and (c) at the end of the first half of the interval?

57 A 230 kg crate hangs from the end of a rope of length $L = 12.0$ m. You push horizontally on the crate with a varying force \vec{F} to move it distance $d = 4.00$ m to the side (Fig. 7-44). (a) What is the magnitude of \vec{F} when the crate is in this final position? During the crate's displacement, what are (b) the total

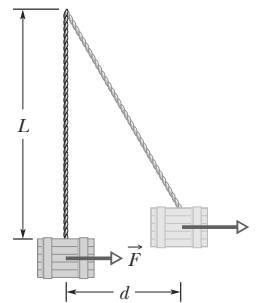


Figure 7-44 Problem 57.

work done on it, (c) the work done by the gravitational force on the crate, and (d) the work done by the pull on the crate from the rope? (e) Knowing that the crate is motionless before and after its displacement, use the answers to (b), (c), and (d) to find the work your force \vec{F} does on the crate. (f) Why is the work of your force not equal to the product of the horizontal displacement and the answer to (a)?

58 To pull a 50 kg crate across a horizontal frictionless floor, a worker applies a force of 210 N, directed 20° above the horizontal. As the crate moves 3.0 m, what work is done on the crate by (a) the worker's force, (b) the gravitational force, and (c) the normal force? (d) What is the total work?

59 A force \vec{F}_a is applied to a bead as the bead is moved along a straight wire through displacement $+5.0$ cm. The magnitude of \vec{F}_a is set at a certain value, but the angle ϕ between \vec{F}_a and the bead's displacement can be chosen. Figure 7-45 gives the work W done by \vec{F}_a on the bead for a range of ϕ values; $W_0 = 25$ J. How much work is done by \vec{F}_a if ϕ is (a) 64° and (b) 147° ?

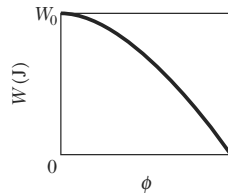


Figure 7-45
Problem 59.

60 A frightened child is restrained by her mother as the child slides down a frictionless playground slide. If the force on the child from the mother is 100 N up the slide, the child's kinetic energy increases by 30 J as she moves down the slide a distance of 1.8 m. (a) How much work is done on the child by the gravitational force during the 1.8 m descent? (b) If the child is not restrained by her mother, how much will the child's kinetic energy increase as she comes down the slide that same distance of 1.8 m?

61 How much work is done by a force $\vec{F} = (2x \text{ N})\hat{i} + (3 \text{ N})\hat{j}$, with x in meters, that moves a particle from a position $\vec{r}_i = (2 \text{ m})\hat{i} + (3 \text{ m})\hat{j}$ to a position $\vec{r}_f = -(4 \text{ m})\hat{i} - (3 \text{ m})\hat{j}$?

62 A 250 g block is dropped onto a relaxed vertical spring that has a spring constant of $k = 2.5 \text{ N/cm}$ (Fig. 7-46). The block becomes attached to the spring and compresses the spring 12 cm before momentarily stopping. While the spring is being compressed, what work is done on the block by (a) the gravitational force on it and (b) the spring force? (c) What is the speed of the block just before it hits the spring? (Assume that friction is negligible.) (d) If the speed at impact is doubled, what is the maximum compression of the spring?

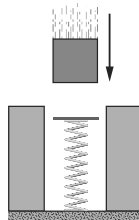


Figure 7-46
Problem 62.

63 SSM To push a 25.0 kg crate up a frictionless incline, angled at 25.0° to the horizontal, a worker exerts a force of 209 N parallel to the incline. As the crate slides 1.50 m, how much work is done on the crate by (a) the worker's applied force, (b) the gravitational force on the crate, and (c) the normal force exerted by the incline on the crate? (d) What is the total work done on the crate?

64 Boxes are transported from one location to another in a warehouse by means of a conveyor belt that moves with a constant speed of 0.50 m/s. At a certain location the conveyor belt moves for 2.0 m up an incline that makes an angle of 10° with the horizontal, then for 2.0 m horizontally, and finally for 2.0 m down an incline that makes an angle of 10° with the horizontal. Assume that a 2.0 kg box rides on the belt without slipping. At what rate is the force of the conveyor belt doing work on the box as the box moves (a) up the 10° incline, (b) horizontally, and (c) down the 10° incline?

65 In Fig. 7-47, a cord runs around two massless, frictionless pulleys. A canister with mass $m = 20$ kg hangs from one pulley, and you exert a force \vec{F} on the free end of the cord. (a) What must be the magnitude of \vec{F} if you are to lift the canister at a constant speed? (b) To lift the canister by 2.0 cm, how far must you pull the free end of the cord? During that lift, what is the work done on the canister by (c) your force (via the cord) and (d) the gravitational force? (Hint: When a cord loops around a pulley as shown, it pulls on the pulley with a net force that is twice the tension in the cord.)

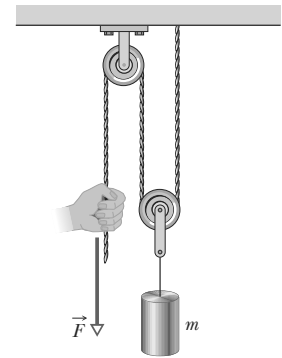


Figure 7-47 Problem 65.

66 If a car of mass 1200 kg is moving along a highway at 120 km/h, what is the car's kinetic energy as determined by someone standing alongside the highway?

67 SSM A spring with a pointer attached is hanging next to a scale marked in millimeters. Three different packages are hung from the spring, in turn, as shown in Fig. 7-48. (a) Which mark on the scale will the pointer indicate when no package is hung from the spring? (b) What is the weight W of the third package?

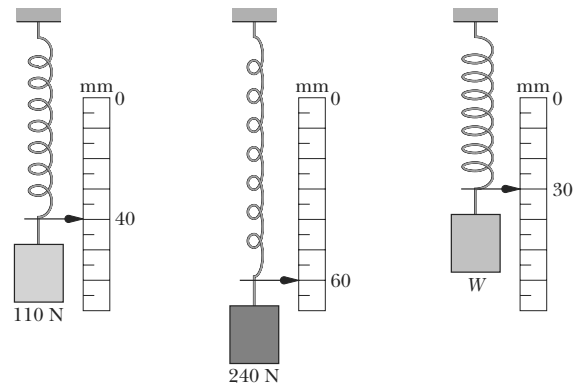


Figure 7-48 Problem 67.

68 An iceboat is at rest on a frictionless frozen lake when a sudden wind exerts a constant force of 200 N, toward the east, on the boat. Due to the angle of the sail, the wind causes the boat to slide in a straight line for a distance of 8.0 m in a direction 20° north of east. What is the kinetic energy of the iceboat at the end of that 8.0 m?

69 If a ski lift raises 100 passengers averaging 660 N in weight to a height of 150 m in 60.0 s, at constant speed, what average power is required of the force making the lift?

70 A force $\vec{F} = (4.0 \text{ N})\hat{i} + c\hat{j}$ acts on a particle as the particle goes through displacement $\vec{d} = (3.0 \text{ m})\hat{i} - (2.0 \text{ m})\hat{j}$. (Other forces also act on the particle.) What is c if the work done on the particle by force \vec{F} is (a) 0, (b) 17 J, and (c) -18 J?

71 A constant force of magnitude 10 N makes an angle of 150° (measured counterclockwise) with the positive x direction as it acts on a 2.0 kg object moving in an xy plane. How much work is done on the object by the force as the object moves from the origin to the point having position vector $(2.0 \text{ m})\hat{i} - (4.0 \text{ m})\hat{j}$?

72 In Fig. 7-49a, a 2.0 N force is applied to a 4.0 kg block at a downward angle θ as the block moves rightward through 1.0 m across a frictionless floor. Find an expression for the speed v_f of the block at the end of that distance if the block's initial velocity is (a) 0 and (b) 1.0 m/s to the right. (c) The situation in Fig. 7-49b is similar in that the block is initially moving at 1.0 m/s to the right, but now the 2.0 N force is directed downward to the left. Find an expression for the speed v_f of the block at the end of the 1.0 m distance. (d) Graph all three expressions for v_f versus downward angle θ for $\theta = 0^\circ$ to $\theta = 90^\circ$. Interpret the graphs.

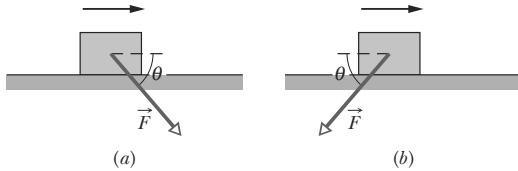


Figure 7-49 Problem 72.

73 A force \vec{F} in the positive direction of an x axis acts on an object moving along the axis. If the magnitude of the force is $F = 10e^{-x/2.0}$ N, with x in meters, find the work done by \vec{F} as the object moves from $x = 0$ to $x = 2.0$ m by (a) plotting $F(x)$ and estimating the area under the curve and (b) integrating to find the work analytically.

74 A particle moves along a straight path through displacement $\vec{d} = (8 \text{ m})\hat{i} + c\hat{j}$ while force $\vec{F} = (2 \text{ N})\hat{i} - (4 \text{ N})\hat{j}$ acts on it. (Other forces also act on the particle.) What is the value of c if the work done by \vec{F} on the particle is (a) zero, (b) positive, and (c) negative?

75 SSM What is the power of the force required to move a 4500 kg elevator cab with a load of 1800 kg upward at constant speed 3.80 m/s?

76 A 45 kg block of ice slides down a frictionless incline 1.5 m long and 0.91 m high. A worker pushes up against the ice, parallel to the incline, so that the block slides down at constant speed. (a) Find the magnitude of the worker's force. How much work is done on the block by (b) the worker's force, (c) the gravitational force on the block, (d) the normal force on the block from the surface of the incline, and (e) the net force on the block?

77 As a particle moves along an x axis, a force in the positive direction of the axis acts on it. Figure 7-50 shows the magnitude F of the force versus position x of the particle. The curve is given by $F = a/x^2$, with $a = 9.0 \text{ N} \cdot \text{m}^2$. Find the work done on the particle by the force as the particle moves from $x = 1.0$ m to $x = 3.0$ m by (a) estimating the work from the graph and (b) integrating the force function.

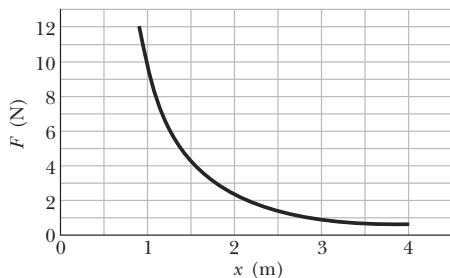


Figure 7-50 Problem 77.

78 A CD case slides along a floor in the positive direction of an x axis while an applied force \vec{F}_a acts on the case. The force is di-

rected along the x axis and has the x component $F_{ax} = 9x - 3x^2$, with x in meters and F_{ax} in newtons. The case starts at rest at the position $x = 0$, and it moves until it is again at rest. (a) Plot the work \vec{F}_a does on the case as a function of x . (b) At what position is the work maximum, and (c) what is that maximum value? (d) At what position has the work decreased to zero? (e) At what position is the case again at rest?

79 SSM A 2.0 kg lunchbox is sent sliding over a frictionless surface, in the positive direction of an x axis along the surface. Beginning at time $t = 0$, a steady wind pushes on the lunchbox in the negative direction of the x axis. Figure 7-51 shows the position x of the lunchbox as a function of time t as the wind pushes on the lunchbox. From the graph, estimate the kinetic energy of the lunchbox at (a) $t = 1.0$ s and (b) $t = 5.0$ s. (c) How much work does the force from the wind do on the lunchbox from $t = 1.0$ s to $t = 5.0$ s?

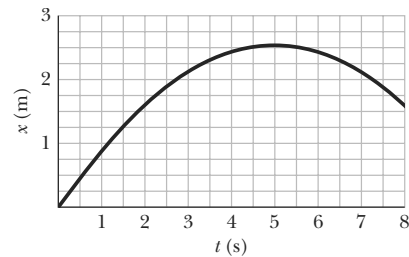


Figure 7-51 Problem 79.

80 Numerical integration. A breadbox is made to move along an x axis from $x = 0.15$ m to $x = 1.20$ m by a force with a magnitude given by $F = \exp(-2x^2)$, with x in meters and F in newtons. (Here \exp is the exponential function.) How much work is done on the breadbox by the force?

81 In the block-spring arrangement of Fig. 7-10, the block's mass is 4.00 kg and the spring constant is 500 N/m. The block is released from position $x_i = 0.300$ m. What are (a) the block's speed at $x = 0$, (b) the work done by the spring when the block reaches $x = 0$, (c) the instantaneous power due to the spring at the release point x_i , (d) the instantaneous power at $x = 0$, and (e) the block's position when the power is maximum?

82 A 4.00 kg block is pulled up a frictionless inclined plane by a 50.0 N force that is parallel to the plane, starting from rest. The normal force on the block from the plane has magnitude 13.41 N. What is the block's speed when its displacement up the ramp is 3.00 m?

83 A spring with a spring constant of 18.0 N/cm has a cage attached to its free end. (a) How much work does the spring force do on the cage when the spring is stretched from its relaxed length by 7.60 mm? (b) How much additional work is done by the spring force when the spring is stretched by an additional 7.60 mm?

84 A force $\vec{F} = (2.00\hat{i} + 9.00\hat{j} + 5.30\hat{k})$ N acts on a 2.90 kg object that moves in time interval 2.10 s from an initial position $\vec{r}_1 = (2.70\hat{i} - 2.90\hat{j} + 5.50\hat{k})$ m to a final position $\vec{r}_2 = (-4.10\hat{i} + 3.30\hat{j} + 5.40\hat{k})$ m. Find (a) the work done on the object by the force in that time interval, (b) the average power due to the force during that time interval, and (c) the angle between vectors \vec{r}_1 and \vec{r}_2 .

85 At $t = 0$, force $\vec{F} = (-5.00\hat{i} + 5.00\hat{j} + 4.00\hat{k})$ N begins to act on a 2.00 kg particle with an initial speed of 4.00 m/s. What is the particle's speed when its displacement from the initial point is $\vec{d} = (2.00\hat{i} + 2.00\hat{j} + 7.00\hat{k})$ m?

C H A P T E R 8

Potential Energy and Conservation of Energy

8-1 POTENTIAL ENERGY

Learning Objectives

After reading this module, you should be able to . . .

- 8.01** Distinguish a conservative force from a nonconservative force.
- 8.02** For a particle moving between two points, identify that the work done by a conservative force does not depend on which path the particle takes.

- 8.03** Calculate the gravitational potential energy of a particle (or, more properly, a particle–Earth system).
- 8.04** Calculate the elastic potential energy of a block–spring system.

Key Ideas

- A force is a conservative force if the net work it does on a particle moving around any closed path, from an initial point and then back to that point, is zero. Equivalently, a force is conservative if the net work it does on a particle moving between two points does not depend on the path taken by the particle. The gravitational force and the spring force are conservative forces; the kinetic frictional force is a nonconservative force.
- Potential energy is energy that is associated with the configuration of a system in which a conservative force acts. When the conservative force does work W on a particle within the system, the change ΔU in the potential energy of the system is

$$\Delta U = -W.$$

If the particle moves from point x_i to point x_f , the change in the potential energy of the system is

$$\Delta U = - \int_{x_i}^{x_f} F(x) dx.$$

- The potential energy associated with a system consisting of Earth and a nearby particle is gravitational potential energy. If the particle moves from height y_i to height y_f , the change in the gravitational potential energy of the particle–Earth system is

$$\Delta U = mg(y_f - y_i) = mg \Delta y.$$

- If the reference point of the particle is set as $y_i = 0$ and the corresponding gravitational potential energy of the system is set as $U_i = 0$, then the gravitational potential energy U when the particle is at any height y is

$$U(y) = mgy.$$

- Elastic potential energy is the energy associated with the state of compression or extension of an elastic object. For a spring that exerts a spring force $F = -kx$ when its free end has displacement x , the elastic potential energy is

$$U(x) = \frac{1}{2}kx^2.$$

- The reference configuration has the spring at its relaxed length, at which $x = 0$ and $U = 0$.

What Is Physics?

One job of physics is to identify the different types of energy in the world, especially those that are of common importance. One general type of energy is **potential energy** U . Technically, potential energy is energy that can be associated with the configuration (arrangement) of a system of objects that exert forces on one another.



Rough Guides/Greg Roden/Getty Images, Inc.

Figure 8-1 The kinetic energy of a bungee-cord jumper increases during the free fall, and then the cord begins to stretch, slowing the jumper.

This is a pretty formal definition of something that is actually familiar to you. An example might help better than the definition: A bungee-cord jumper plunges from a staging platform (Fig. 8-1). The system of objects consists of Earth and the jumper. The force between the objects is the gravitational force. The configuration of the system changes (the separation between the jumper and Earth decreases—that is, of course, the thrill of the jump). We can account for the jumper's motion and increase in kinetic energy by defining a **gravitational potential energy** U . This is the energy associated with the state of separation between two objects that attract each other by the gravitational force, here the jumper and Earth.

When the jumper begins to stretch the bungee cord near the end of the plunge, the system of objects consists of the cord and the jumper. The force between the objects is an elastic (spring-like) force. The configuration of the system changes (the cord stretches). We can account for the jumper's decrease in kinetic energy and the cord's increase in length by defining an **elastic potential energy** U . This is the energy associated with the state of compression or extension of an elastic object, here the bungee cord.

Physics determines how the potential energy of a system can be calculated so that energy might be stored or put to use. For example, before any particular bungee-cord jumper takes the plunge, someone (probably a mechanical engineer) must determine the correct cord to be used by calculating the gravitational and elastic potential energies that can be expected. Then the jump is only thrilling and not fatal.

Work and Potential Energy

In Chapter 7 we discussed the relation between work and a change in kinetic energy. Here we discuss the relation between work and a change in potential energy.

Let us throw a tomato upward (Fig. 8-2). We already know that as the tomato rises, the work W_g done on the tomato by the gravitational force is negative because the force transfers energy *from* the kinetic energy of the tomato. We can now finish the story by saying that this energy is transferred by the gravitational force *to* the gravitational potential energy of the tomato–Earth system.

The tomato slows, stops, and then begins to fall back down because of the gravitational force. During the fall, the transfer is reversed: The work W_g done on the tomato by the gravitational force is now positive—that force transfers energy *from* the gravitational potential energy of the tomato–Earth system *to* the kinetic energy of the tomato.

For either rise or fall, the change ΔU in gravitational potential energy is defined as being equal to the negative of the work done on the tomato by the gravitational force. Using the general symbol W for work, we write this as

$$\Delta U = -W. \quad (8-1)$$

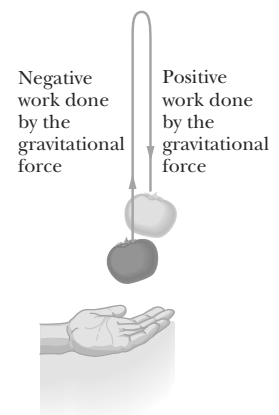


Figure 8-2 A tomato is thrown upward. As it rises, the gravitational force does negative work on it, decreasing its kinetic energy. As the tomato descends, the gravitational force does positive work on it, increasing its kinetic energy.

This equation also applies to a block–spring system, as in Fig. 8-3. If we abruptly shove the block to send it moving rightward, the spring force acts leftward and thus does negative work on the block, transferring energy from the kinetic energy of the block to the elastic potential energy of the spring–block system. The block slows and eventually stops, and then begins to move leftward because the spring force is still leftward. The transfer of energy is then reversed—it is from potential energy of the spring–block system to kinetic energy of the block.

Conservative and Nonconservative Forces

Let us list the key elements of the two situations we just discussed:

1. The *system* consists of two or more objects.
2. A *force* acts between a particle-like object (tomato or block) in the system and the rest of the system.
3. When the system configuration changes, the force does *work* (call it W_1) on the particle-like object, transferring energy between the kinetic energy K of the object and some other type of energy of the system.
4. When the configuration change is reversed, the force reverses the energy transfer, doing work W_2 in the process.

In a situation in which $W_1 = -W_2$ is always true, the other type of energy is a potential energy and the force is said to be a **conservative force**. As you might suspect, the gravitational force and the spring force are both conservative (since otherwise we could not have spoken of gravitational potential energy and elastic potential energy, as we did previously).

A force that is not conservative is called a **nonconservative force**. The kinetic frictional force and drag force are nonconservative. For an example, let us send a block sliding across a floor that is not frictionless. During the sliding, a kinetic frictional force from the floor slows the block by transferring energy from its kinetic energy to a type of energy called *thermal energy* (which has to do with the random motions of atoms and molecules). We know from experiment that this energy transfer cannot be reversed (thermal energy cannot be transferred back to kinetic energy of the block by the kinetic frictional force). Thus, although we have a system (made up of the block and the floor), a force that acts between parts of the system, and a transfer of energy by the force, the force is not conservative. Therefore, thermal energy is not a potential energy.

When only conservative forces act on a particle-like object, we can greatly simplify otherwise difficult problems involving motion of the object. Let's next develop a test for identifying conservative forces, which will provide one means for simplifying such problems.

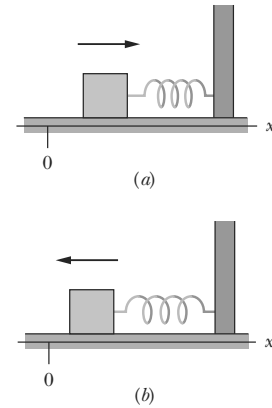


Figure 8-3 A block, attached to a spring and initially at rest at $x = 0$, is set in motion toward the right. (a) As the block moves rightward (as indicated by the arrow), the spring force does negative work on it. (b) Then, as the block moves back toward $x = 0$, the spring force does positive work on it.

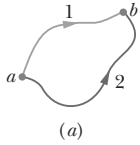
Path Independence of Conservative Forces

The primary test for determining whether a force is conservative or nonconservative is this: Let the force act on a particle that moves along any *closed path*, beginning at some initial position and eventually returning to that position (so that the particle makes a *round trip* beginning and ending at the initial position). The force is conservative only if the total energy it transfers to and from the particle during the round trip along this and any other closed path is zero. In other words:

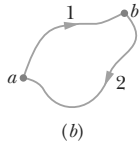


The net work done by a conservative force on a particle moving around any closed path is zero.

We know from experiment that the gravitational force passes this *closed-path test*. An example is the tossed tomato of Fig. 8-2. The tomato leaves the launch point with speed v_0 and kinetic energy $\frac{1}{2}mv_0^2$. The gravitational force acting



The force is conservative. Any choice of path between the points gives the same amount of work.



And a round trip gives a total work of zero.

Figure 8-4 (a) As a conservative force acts on it, a particle can move from point a to point b along either path 1 or path 2. (b) The particle moves in a round trip, from point a to point b along path 1 and then back to point a along path 2.

on the tomato slows it, stops it, and then causes it to fall back down. When the tomato returns to the launch point, it again has speed v_0 and kinetic energy $\frac{1}{2}mv_0^2$. Thus, the gravitational force transfers as much energy *from* the tomato during the ascent as it transfers *to* the tomato during the descent back to the launch point. The net work done on the tomato by the gravitational force during the round trip is zero.

An important result of the closed-path test is that:



The work done by a conservative force on a particle moving between two points does not depend on the path taken by the particle.

For example, suppose that a particle moves from point a to point b in Fig. 8-4a along either path 1 or path 2. If only a conservative force acts on the particle, then the work done on the particle is the same along the two paths. In symbols, we can write this result as

$$W_{ab,1} = W_{ab,2}, \quad (8-2)$$

where the subscript ab indicates the initial and final points, respectively, and the subscripts 1 and 2 indicate the path.

This result is powerful because it allows us to simplify difficult problems when only a conservative force is involved. Suppose you need to calculate the work done by a conservative force along a given path between two points, and the calculation is difficult or even impossible without additional information. You can find the work by substituting some other path between those two points for which the calculation is easier and possible.

Proof of Equation 8-2

Figure 8-4b shows an arbitrary round trip for a particle that is acted upon by a single force. The particle moves from an initial point a to point b along path 1 and then back to point a along path 2. The force does work on the particle as the particle moves along each path. Without worrying about where positive work is done and where negative work is done, let us just represent the work done from a to b along path 1 as $W_{ab,1}$ and the work done from b back to a along path 2 as $W_{ba,2}$. If the force is conservative, then the net work done during the round trip must be zero:

$$W_{ab,1} + W_{ba,2} = 0,$$

and thus

$$W_{ab,1} = -W_{ba,2}. \quad (8-3)$$

In words, the work done along the outward path must be the negative of the work done along the path back.

Let us now consider the work $W_{ab,2}$ done on the particle by the force when the particle moves from a to b along path 2, as indicated in Fig. 8-4a. If the force is conservative, that work is the negative of $W_{ba,2}$:

$$W_{ab,2} = -W_{ba,2}. \quad (8-4)$$

Substituting $W_{ab,2}$ for $-W_{ba,2}$ in Eq. 8-3, we obtain

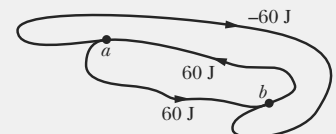
$$W_{ab,1} = W_{ab,2},$$

which is what we set out to prove.



Checkpoint 1

The figure shows three paths connecting points a and b . A single force \vec{F} does the indicated work on a particle moving along each path in the indicated direction. On the basis of this information, is force \vec{F} conservative?





Sample Problem 8.01 Equivalent paths for calculating work, slippery cheese

The main lesson of this sample problem is this: It is perfectly all right to choose an easy path instead of a hard path. Figure 8-5a shows a 2.0 kg block of slippery cheese that slides along a frictionless track from point *a* to point *b*. The cheese travels through a total distance of 2.0 m along the track, and a net vertical distance of 0.80 m. How much work is done on the cheese by the gravitational force during the slide?

KEY IDEAS

(1) We *cannot* calculate the work by using Eq. 7-12 ($W_g = mgd \cos \phi$). The reason is that the angle ϕ between the directions of the gravitational force \vec{F}_g and the displacement \vec{d} varies along the track in an unknown way. (Even if we did know the shape of the track and could calculate ϕ along it, the calculation could be very difficult.) (2) Because \vec{F}_g is a conservative force, we can find the work by choosing some other path between *a* and *b*—one that makes the calculation easy.

Calculations: Let us choose the dashed path in Fig. 8-5b; it consists of two straight segments. Along the horizontal segment, the angle ϕ is a constant 90° . Even though we do not know the displacement along that horizontal segment, Eq. 7-12 tells us that the work W_h done there is

$$W_h = mgd \cos 90^\circ = 0.$$

Along the vertical segment, the displacement d is 0.80 m and, with \vec{F}_g and \vec{d} both downward, the angle ϕ is a constant 0° . Thus, Eq. 7-12 gives us, for the work W_v done along the

The gravitational force is conservative. Any choice of path between the points gives the same amount of work.

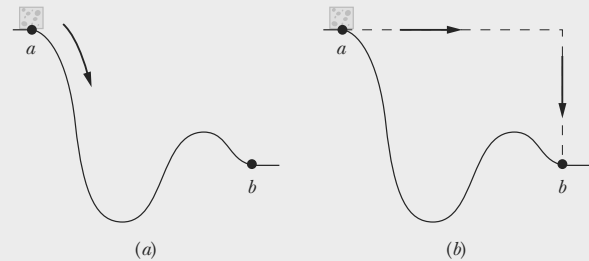


Figure 8-5 (a) A block of cheese slides along a frictionless track from point *a* to point *b*. (b) Finding the work done on the cheese by the gravitational force is easier along the dashed path than along the actual path taken by the cheese; the result is the same for both paths.

vertical part of the dashed path,

$$\begin{aligned} W_v &= mgd \cos 0^\circ \\ &= (2.0 \text{ kg})(9.8 \text{ m/s}^2)(0.80 \text{ m})(1) = 15.7 \text{ J.} \end{aligned}$$

The total work done on the cheese by \vec{F}_g as the cheese moves from point *a* to point *b* along the dashed path is then

$$W = W_h + W_v = 0 + 15.7 \text{ J} \approx 16 \text{ J.} \quad (\text{Answer})$$

This is also the work done as the cheese slides along the track from *a* to *b*.



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Determining Potential Energy Values

Here we find equations that give the value of the two types of potential energy discussed in this chapter: gravitational potential energy and elastic potential energy. However, first we must find a general relation between a conservative force and the associated potential energy.

Consider a particle-like object that is part of a system in which a conservative force \vec{F} acts. When that force does work W on the object, the change ΔU in the potential energy associated with the system is the negative of the work done. We wrote this fact as Eq. 8-1 ($\Delta U = -W$). For the most general case, in which the force may vary with position, we may write the work W as in Eq. 7-32:

$$W = \int_{x_i}^{x_f} F(x) dx. \quad (8-5)$$

This equation gives the work done by the force when the object moves from point x_i to point x_f , changing the configuration of the system. (Because the force is conservative, the work is the same for all paths between those two points.)

Substituting Eq. 8-5 into Eq. 8-1, we find that the change in potential energy due to the change in configuration is, in general notation,

$$\Delta U = - \int_{x_i}^{x_f} F(x) dx. \quad (8-6)$$

Gravitational Potential Energy

We first consider a particle with mass m moving vertically along a y axis (the positive direction is upward). As the particle moves from point y_i to point y_f , the gravitational force \vec{F}_g does work on it. To find the corresponding change in the gravitational potential energy of the particle–Earth system, we use Eq. 8-6 with two changes: (1) We integrate along the y axis instead of the x axis, because the gravitational force acts vertically. (2) We substitute $-mg$ for the force symbol F , because \vec{F}_g has the magnitude mg and is directed down the y axis. We then have

$$\Delta U = - \int_{y_i}^{y_f} (-mg) dy = mg \int_{y_i}^{y_f} dy = mg \left[y \right]_{y_i}^{y_f},$$

which yields

$$\Delta U = mg(y_f - y_i) = mg \Delta y. \quad (8-7)$$

Only *changes* ΔU in gravitational potential energy (or any other type of potential energy) are physically meaningful. However, to simplify a calculation or a discussion, we sometimes would like to say that a certain gravitational potential value U is associated with a certain particle–Earth system when the particle is at a certain height y . To do so, we rewrite Eq. 8-7 as

$$U - U_i = mg(y - y_i). \quad (8-8)$$

Then we take U_i to be the gravitational potential energy of the system when it is in a **reference configuration** in which the particle is at a **reference point** y_i . Usually we take $U_i = 0$ and $y_i = 0$. Doing this changes Eq. 8-8 to

$$U(y) = mgy \quad (\text{gravitational potential energy}). \quad (8-9)$$

This equation tells us:



The gravitational potential energy associated with a particle–Earth system depends only on the vertical position y (or height) of the particle relative to the reference position $y = 0$, not on the horizontal position.

Elastic Potential Energy

We next consider the block–spring system shown in Fig. 8-3, with the block moving on the end of a spring of spring constant k . As the block moves from point x_i to point x_f , the spring force $F_x = -kx$ does work on the block. To find the corresponding change in the elastic potential energy of the block–spring system, we substitute $-kx$ for $F(x)$ in Eq. 8-6. We then have

$$\Delta U = - \int_{x_i}^{x_f} (-kx) dx = k \int_{x_i}^{x_f} x dx = \frac{1}{2}k \left[x^2 \right]_{x_i}^{x_f},$$

or
$$\Delta U = \frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2. \quad (8-10)$$

To associate a potential energy value U with the block at position x , we choose the reference configuration to be when the spring is at its relaxed length and the block is at $x_i = 0$. Then the elastic potential energy U_i is 0, and Eq. 8-10

becomes

$$U - 0 = \frac{1}{2}kx^2 - 0,$$

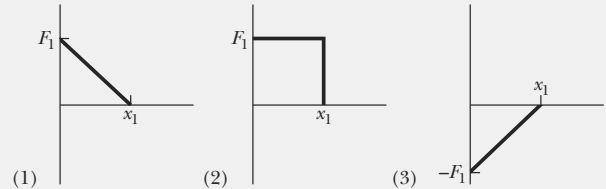
which gives us

$$U(x) = \frac{1}{2}kx^2 \quad (\text{elastic potential energy}). \quad (8-11)$$



Checkpoint 2

A particle is to move along an x axis from $x = 0$ to x_1 while a conservative force, directed along the x axis, acts on the particle. The figure shows three situations in which the x component of that force varies with x . The force has the same maximum magnitude F_1 in all three situations. Rank the situations according to the change in the associated potential energy during the particle's motion, most positive first.



Sample Problem 8.02 Choosing reference level for gravitational potential energy, sloth

Here is an example with this lesson plan: Generally you can choose any level to be the reference level, but once chosen, be consistent. A 2.0 kg sloth hangs 5.0 m above the ground (Fig. 8-6).

- (a) What is the gravitational potential energy U of the sloth–Earth system if we take the reference point $y = 0$ to be (1) at the ground, (2) at a balcony floor that is 3.0 m above

the ground, (3) at the limb, and (4) 1.0 m above the limb? Take the gravitational potential energy to be zero at $y = 0$.

KEY IDEA

Once we have chosen the reference point for $y = 0$, we can calculate the gravitational potential energy U of the system relative to that reference point with Eq. 8-9.

Calculations: For choice (1) the sloth is at $y = 5.0$ m, and

$$U = mgy = (2.0 \text{ kg})(9.8 \text{ m/s}^2)(5.0 \text{ m}) = 98 \text{ J.} \quad (\text{Answer})$$

For the other choices, the values of U are

$$\begin{aligned} (2) \quad U &= mgy = mg(2.0 \text{ m}) = 39 \text{ J,} \\ (3) \quad U &= mgy = mg(0) = 0 \text{ J,} \\ (4) \quad U &= mgy = mg(-1.0 \text{ m}) \\ &= -19.6 \text{ J} \approx -20 \text{ J.} \end{aligned} \quad (\text{Answer})$$

- (b) The sloth drops to the ground. For each choice of reference point, what is the change ΔU in the potential energy of the sloth–Earth system due to the fall?

KEY IDEA

The *change* in potential energy does not depend on the choice of the reference point for $y = 0$; instead, it depends on the change in height Δy .

Calculation: For all four situations, we have the same $\Delta y = -5.0$ m. Thus, for (1) to (4), Eq. 8-7 tells us that

$$\begin{aligned} \Delta U &= mg \Delta y = (2.0 \text{ kg})(9.8 \text{ m/s}^2)(-5.0 \text{ m}) \\ &= -98 \text{ J.} \end{aligned} \quad (\text{Answer})$$

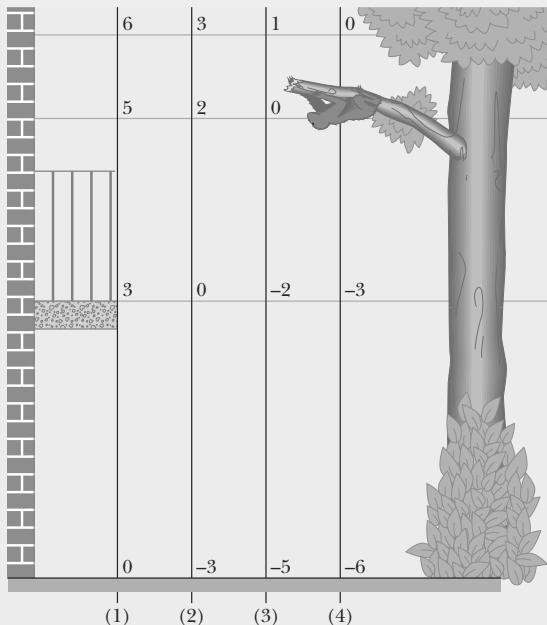


Figure 8-6 Four choices of reference point $y = 0$. Each y axis is marked in units of meters. The choice affects the value of the potential energy U of the sloth–Earth system. However, it does not affect the change ΔU in potential energy of the system if the sloth moves by, say, falling.



Additional examples, video, and practice available at WileyPLUS

8-2 CONSERVATION OF MECHANICAL ENERGY

Learning Objectives

After reading this module, you should be able to . . .

8.05 After first clearly defining which objects form a system, identify that the mechanical energy of the system is the sum of the kinetic energies and potential energies of those objects.

8.06 For an isolated system in which only conservative forces act, apply the conservation of mechanical energy to relate the initial potential and kinetic energies to the potential and kinetic energies at a later instant.

Key Ideas

● The mechanical energy E_{mec} of a system is the sum of its kinetic energy K and potential energy U :

$$E_{\text{mec}} = K + U.$$

● An isolated system is one in which no external force causes energy changes. If only conservative forces do work within an isolated system, then the mechanical energy E_{mec} of the

system cannot change. This principle of conservation of mechanical energy is written as

$$K_2 + U_2 = K_1 + U_1,$$

in which the subscripts refer to different instants during an energy transfer process. This conservation principle can also be written as

$$\Delta E_{\text{mec}} = \Delta K + \Delta U = 0.$$

Conservation of Mechanical Energy

The **mechanical energy** E_{mec} of a system is the sum of its potential energy U and the kinetic energy K of the objects within it:

$$E_{\text{mec}} = K + U \quad (\text{mechanical energy}). \quad (8-12)$$

In this module, we examine what happens to this mechanical energy when only conservative forces cause energy transfers within the system—that is, when frictional and drag forces do not act on the objects in the system. Also, we shall assume that the system is *isolated* from its environment; that is, no *external force* from an object outside the system causes energy changes inside the system.

When a conservative force does work W on an object within the system, that force transfers energy between kinetic energy K of the object and potential energy U of the system. From Eq. 7-10, the change ΔK in kinetic energy is

$$\Delta K = W \quad (8-13)$$

and from Eq. 8-1, the change ΔU in potential energy is

$$\Delta U = -W. \quad (8-14)$$

Combining Eqs. 8-13 and 8-14, we find that

$$\Delta K = -\Delta U. \quad (8-15)$$

In words, one of these energies increases exactly as much as the other decreases.

We can rewrite Eq. 8-15 as

$$K_2 - K_1 = -(U_2 - U_1), \quad (8-16)$$

where the subscripts refer to two different instants and thus to two different arrangements of the objects in the system. Rearranging Eq. 8-16 yields

$$K_2 + U_2 = K_1 + U_1 \quad (\text{conservation of mechanical energy}). \quad (8-17)$$

In words, this equation says:

$$\left(\begin{array}{c} \text{the sum of } K \text{ and } U \text{ for} \\ \text{any state of a system} \end{array} \right) = \left(\begin{array}{c} \text{the sum of } K \text{ and } U \text{ for} \\ \text{any other state of the system} \end{array} \right),$$



©AP/Wide World Photos

In olden days, a person would be tossed via a blanket to be able to see farther over the flat terrain. Nowadays, it is done just for fun. During the ascent of the person in the photograph, energy is transferred from kinetic energy to gravitational potential energy. The maximum height is reached when that transfer is complete. Then the transfer is reversed during the fall.

when the system is isolated and only conservative forces act on the objects in the system. In other words:



In an isolated system where only conservative forces cause energy changes, the kinetic energy and potential energy can change, but their sum, the mechanical energy E_{mec} of the system, cannot change.

This result is called the **principle of conservation of mechanical energy**. (Now you can see where *conservative* forces got their name.) With the aid of Eq. 8-15, we can write this principle in one more form, as

$$\Delta E_{\text{mec}} = \Delta K + \Delta U = 0. \quad (8-18)$$

The principle of conservation of mechanical energy allows us to solve problems that would be quite difficult to solve using only Newton's laws:



When the mechanical energy of a system is conserved, we can relate the sum of kinetic energy and potential energy at one instant to that at another instant *without considering the intermediate motion and without finding the work done by the forces involved*.

Figure 8-7 shows an example in which the principle of conservation of mechanical energy can be applied: As a pendulum swings, the energy of the

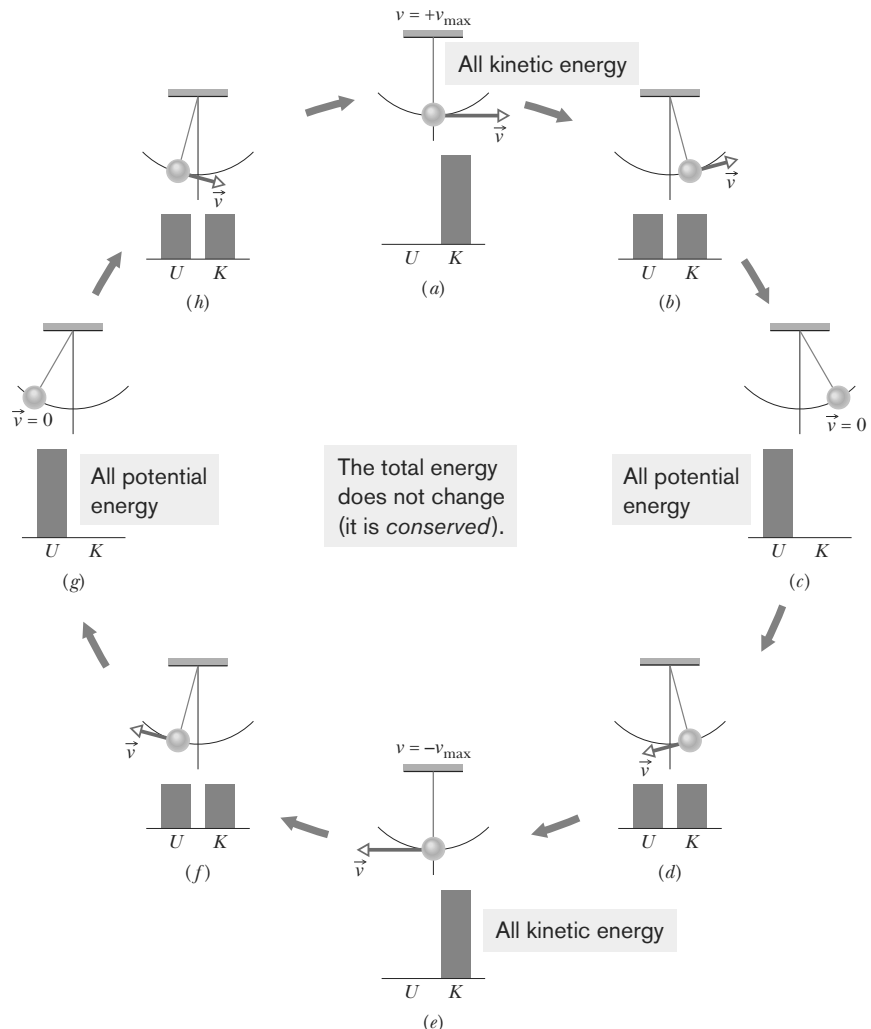


Figure 8-7 A pendulum, with its mass concentrated in a bob at the lower end, swings back and forth. One full cycle of the motion is shown. During the cycle the values of the potential and kinetic energies of the pendulum–Earth system vary as the bob rises and falls, but the mechanical energy E_{mec} of the system remains constant. The energy E_{mec} can be described as continuously shifting between the kinetic and potential forms. In stages (a) and (e), all the energy is kinetic energy. The bob then has its greatest speed and is at its lowest point. In stages (c) and (g), all the energy is potential energy. The bob then has zero speed and is at its highest point. In stages (b), (d), (f), and (h), half the energy is kinetic energy and half is potential energy. If the swinging involved a frictional force at the point where the pendulum is attached to the ceiling, or a drag force due to the air, then E_{mec} would not be conserved, and eventually the pendulum would stop.

pendulum–Earth system is transferred back and forth between kinetic energy K and gravitational potential energy U , with the sum $K + U$ being constant. If we know the gravitational potential energy when the pendulum bob is at its highest point (Fig. 8-7c), Eq. 8-17 gives us the kinetic energy of the bob at the lowest point (Fig. 8-7e).

For example, let us choose the lowest point as the reference point, with the gravitational potential energy $U_2 = 0$. Suppose then that the potential energy at the highest point is $U_1 = 20 \text{ J}$ relative to the reference point. Because the bob momentarily stops at its highest point, the kinetic energy there is $K_1 = 0$. Putting these values into Eq. 8-17 gives us the kinetic energy K_2 at the lowest point:

$$K_2 + 0 = 0 + 20 \text{ J} \quad \text{or} \quad K_2 = 20 \text{ J}.$$

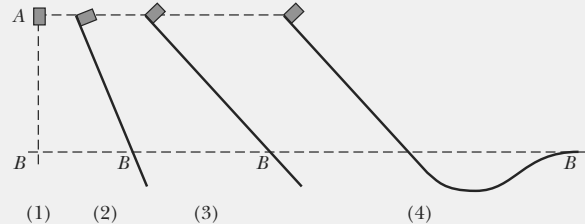
Note that we get this result without considering the motion between the highest and lowest points (such as in Fig. 8-7d) and without finding the work done by any forces involved in the motion.



Checkpoint 3

The figure shows four situations—one in which an initially stationary block is dropped and three in which the block is allowed to slide down frictionless ramps. (a) Rank the situations

according to the kinetic energy of the block at point B , greatest first. (b) Rank them according to the speed of the block at point B , greatest first.



Sample Problem 8.03 Conservation of mechanical energy, water slide

The huge advantage of using the conservation of energy instead of Newton's laws of motion is that we can jump from the initial state to the final state without considering all the intermediate motion. Here is an example. In Fig. 8-8, a child of mass m is released from rest at the top of a water slide, at height $h = 8.5 \text{ m}$ above the bottom of the slide. Assuming that the slide is frictionless because of the water on it, find the child's speed at the bottom of the slide.

KEY IDEAS

(1) We cannot find her speed at the bottom by using her acceleration along the slide as we might have in earlier chapters because we do not know the slope (angle) of the slide. However, because that speed is related to her kinetic energy, perhaps we can use the principle of conservation of mechanical energy to get the speed. Then we would not need to know the slope. (2) Mechanical energy is conserved in a system *if* the system is isolated and *if* only conservative forces cause energy transfers within it. Let's check.

Forces: Two forces act on the child. The *gravitational force*, a conservative force, does work on her. The *normal force* on her from the slide does no work because its direction at any point during the descent is always perpendicular to the direction in which the child moves.

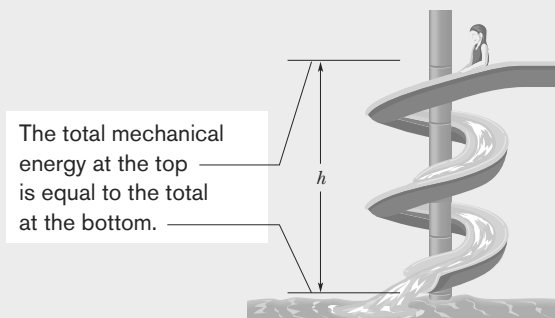


Figure 8-8 A child slides down a water slide as she descends a height h .

System: Because the only force doing work on the child is the gravitational force, we choose the child–Earth system as our system, which we can take to be isolated.

Thus, we have only a conservative force doing work in an isolated system, so we *can* use the principle of conservation of mechanical energy.

Calculations: Let the mechanical energy be $E_{\text{mec},t}$ when the child is at the top of the slide and $E_{\text{mec},b}$ when she is at the bottom. Then the conservation principle tells us

$$E_{\text{mec},b} = E_{\text{mec},t} \quad (8-19)$$

To show both kinds of mechanical energy, we have

$$K_b + U_b = K_t + U_t, \quad (8-20)$$

or $\frac{1}{2}mv_b^2 + mgy_b = \frac{1}{2}mv_t^2 + mgy_t.$

Dividing by m and rearranging yield

$$v_b^2 = v_t^2 + 2g(y_t - y_b).$$

Putting $v_t = 0$ and $y_t - y_b = h$ leads to

$$\begin{aligned} v_b &= \sqrt{2gh} = \sqrt{(2)(9.8 \text{ m/s}^2)(8.5 \text{ m})} \\ &= 13 \text{ m/s.} \end{aligned} \quad (\text{Answer})$$

This is the same speed that the child would reach if she fell 8.5 m vertically. On an actual slide, some frictional forces would act and the child would not be moving quite so fast.

Comments: Although this problem is hard to solve directly with Newton's laws, using conservation of mechanical energy makes the solution much easier. However, if we were asked to find the time taken for the child to reach the bottom of the slide, energy methods would be of no use; we would need to know the shape of the slide, and we would have a difficult problem.



Additional examples, video, and practice available at WileyPLUS



8-3 READING A POTENTIAL ENERGY CURVE

Learning Objectives

After reading this module, you should be able to . . .

- 8.07** Given a particle's potential energy as a function of its position x , determine the force on the particle.
- 8.08** Given a graph of potential energy versus x , determine the force on a particle.
- 8.09** On a graph of potential energy versus x , superimpose a line for a particle's mechanical energy and determine the particle's kinetic energy for any given value of x .

- 8.10** If a particle moves along an x axis, use a potential-energy graph for that axis and the conservation of mechanical energy to relate the energy values at one position to those at another position.
- 8.11** On a potential-energy graph, identify any turning points and any regions where the particle is not allowed because of energy requirements.
- 8.12** Explain neutral equilibrium, stable equilibrium, and unstable equilibrium.

Key Ideas

- If we know the potential energy function $U(x)$ for a system in which a one-dimensional force $F(x)$ acts on a particle, we can find the force as

$$F(x) = -\frac{dU(x)}{dx}.$$

- If $U(x)$ is given on a graph, then at any value of x , the force $F(x)$ is the negative of the slope of the curve there and the

kinetic energy of the particle is given by

$$K(x) = E_{\text{mec}} - U(x),$$

where E_{mec} is the mechanical energy of the system.

- A turning point is a point x at which the particle reverses its motion (there, $K = 0$).
- The particle is in equilibrium at points where the slope of the $U(x)$ curve is zero (there, $F(x) = 0$).

Reading a Potential Energy Curve

Once again we consider a particle that is part of a system in which a conservative force acts. This time suppose that the particle is constrained to move along an x axis while the conservative force does work on it. We want to plot the potential energy $U(x)$ that is associated with that force and the work that it does, and then we want to consider how we can relate the plot back to the force and to the kinetic energy of the particle. However, before we discuss such plots, we need one more relationship between the force and the potential energy.

Finding the Force Analytically

Equation 8-6 tells us how to find the change ΔU in potential energy between two points in a one-dimensional situation if we know the force $F(x)$. Now we want to

go the other way; that is, we know the potential energy function $U(x)$ and want to find the force.

For one-dimensional motion, the work W done by a force that acts on a particle as the particle moves through a distance Δx is $F(x) \Delta x$. We can then write Eq. 8-1 as

$$\Delta U(x) = -W = -F(x) \Delta x. \quad (8-21)$$

Solving for $F(x)$ and passing to the differential limit yield

$$F(x) = -\frac{dU(x)}{dx} \quad (\text{one-dimensional motion}), \quad (8-22)$$

which is the relation we sought.

We can check this result by putting $U(x) = \frac{1}{2}kx^2$, which is the elastic potential energy function for a spring force. Equation 8-22 then yields, as expected, $F(x) = -kx$, which is Hooke's law. Similarly, we can substitute $U(x) = mgx$, which is the gravitational potential energy function for a particle–Earth system, with a particle of mass m at height x above Earth's surface. Equation 8-22 then yields $F = -mg$, which is the gravitational force on the particle.

The Potential Energy Curve

Figure 8-9a is a plot of a potential energy function $U(x)$ for a system in which a particle is in one-dimensional motion while a conservative force $F(x)$ does work on it. We can easily find $F(x)$ by (graphically) taking the slope of the $U(x)$ curve at various points. (Equation 8-22 tells us that $F(x)$ is the negative of the slope of the $U(x)$ curve.) Figure 8-9b is a plot of $F(x)$ found in this way.

Turning Points

In the absence of a nonconservative force, the mechanical energy E of a system has a constant value given by

$$U(x) + K(x) = E_{\text{mec}}. \quad (8-23)$$

Here $K(x)$ is the *kinetic energy function* of a particle in the system (this $K(x)$ gives the kinetic energy as a function of the particle's location x). We may rewrite Eq. 8-23 as

$$K(x) = E_{\text{mec}} - U(x). \quad (8-24)$$

Suppose that E_{mec} (which has a constant value, remember) happens to be 5.0 J. It would be represented in Fig. 8-9c by a horizontal line that runs through the value 5.0 J on the energy axis. (It is, in fact, shown there.)

Equation 8-24 and Fig. 8-9d tell us how to determine the kinetic energy K for any location x of the particle: On the $U(x)$ curve, find U for that location x and then subtract U from E_{mec} . In Fig. 8-9e for example, if the particle is at any point to the right of x_5 , then $K = 1.0$ J. The value of K is greatest (5.0 J) when the particle is at x_2 and least (0 J) when the particle is at x_1 .

Since K can never be negative (because v^2 is always positive), the particle can never move to the left of x_1 , where $E_{\text{mec}} - U$ is negative. Instead, as the particle moves toward x_1 from x_2 , K decreases (the particle slows) until $K = 0$ at x_1 (the particle stops there).

Note that when the particle reaches x_1 , the force on the particle, given by Eq. 8-22, is positive (because the slope dU/dx is negative). This means that the particle does not remain at x_1 but instead begins to move to the right, opposite its earlier motion. Hence x_1 is a **turning point**, a place where $K = 0$ (because $U = E$) and the particle changes direction. There is no turning point (where $K = 0$) on the right side of the graph. When the particle heads to the right, it will continue indefinitely.

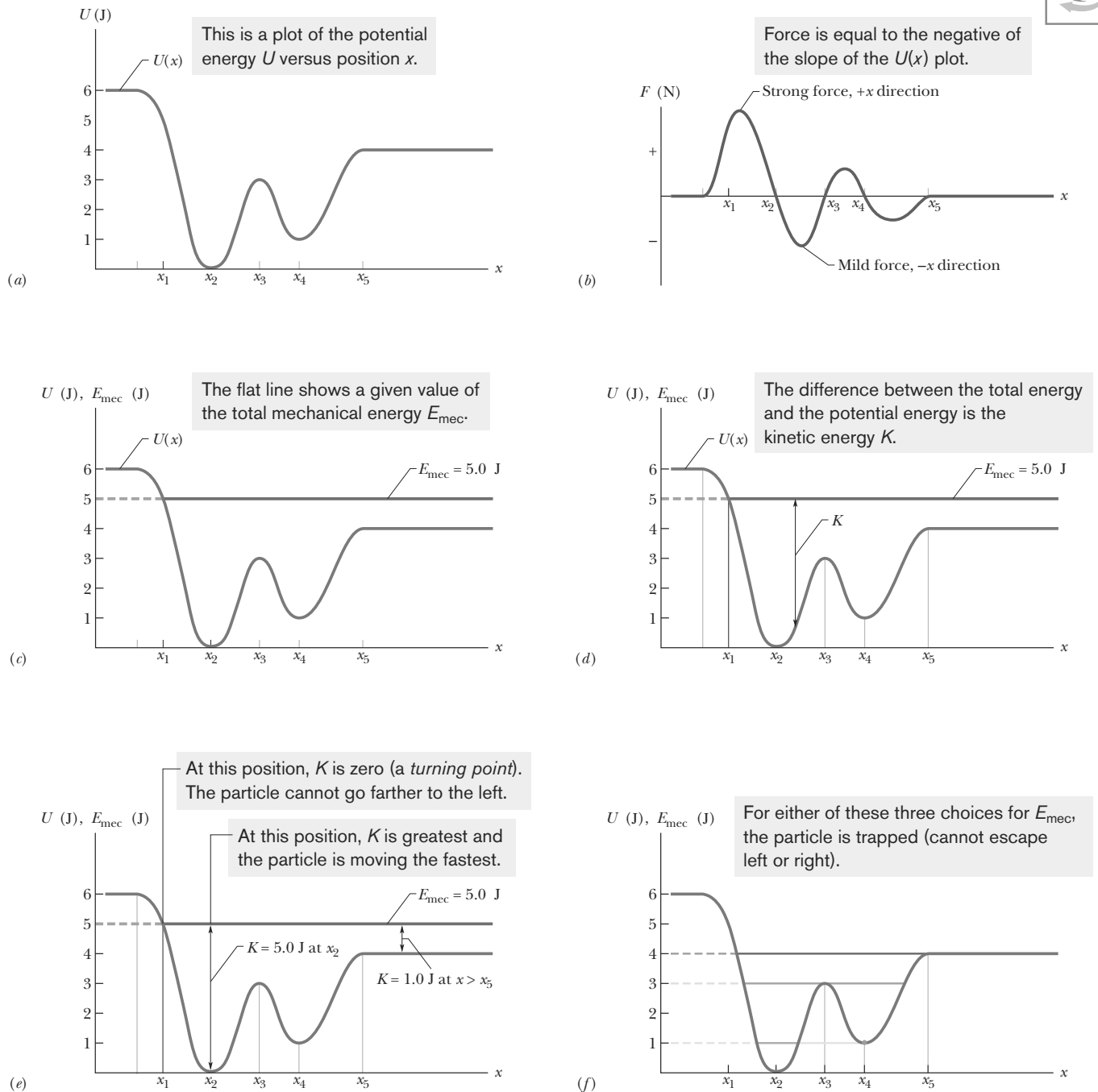


Figure 8-9 (a) A plot of $U(x)$, the potential energy function of a system containing a particle confined to move along an x axis. There is no friction, so mechanical energy is conserved. (b) A plot of the force $F(x)$ acting on the particle, derived from the potential energy plot by taking its slope at various points. (c)–(e) How to determine the kinetic energy. (f) The $U(x)$ plot of (a) with three possible values of E_{mec} shown. **In WileyPLUS, this figure is available as an animation with voiceover.**

Equilibrium Points

Figure 8-9f shows three different values for E_{mec} superposed on the plot of the potential energy function $U(x)$ of Fig. 8-9a. Let us see how they change the situation. If $E_{\text{mec}} = 4.0 \text{ J}$ (purple line), the turning point shifts from x_1 to a point between x_1 and x_2 . Also, at any point to the right of x_3 , the system's mechanical energy is equal to its potential energy; thus, the particle has no kinetic energy and (by Eq. 8-22) no force acts on it, and so it must be stationary. A particle at such a position is said to be in **neutral equilibrium**. (A marble placed on a horizontal tabletop is in that state.)

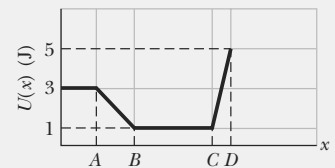
If $E_{\text{mec}} = 3.0 \text{ J}$ (pink line), there are two turning points: One is between x_1 and x_2 , and the other is between x_4 and x_5 . In addition, x_3 is a point at which $K = 0$. If the particle is located exactly there, the force on it is also zero, and the particle remains stationary. However, if it is displaced even slightly in either direction, a nonzero force pushes it farther in the same direction, and the particle continues to move. A particle at such a position is said to be in **unstable equilibrium**. (A marble balanced on top of a bowling ball is an example.)

Next consider the particle's behavior if $E_{\text{mec}} = 1.0 \text{ J}$ (green line). If we place it at x_4 , it is stuck there. It cannot move left or right on its own because to do so would require a negative kinetic energy. If we push it slightly left or right, a restoring force appears that moves it back to x_4 . A particle at such a position is said to be in **stable equilibrium**. (A marble placed at the bottom of a hemispherical bowl is an example.) If we place the particle in the cup-like *potential well* centered at x_2 , it is between two turning points. It can still move somewhat, but only partway to x_1 or x_3 .



Checkpoint 4

The figure gives the potential energy function $U(x)$ for a system in which a particle is in one-dimensional motion. (a) Rank regions AB , BC , and CD according to the magnitude of the force on the particle, greatest first. (b) What is the direction of the force when the particle is in region AB ?



Sample Problem 8.04 Reading a potential energy graph

A 2.00 kg particle moves along an x axis in one-dimensional motion while a conservative force along that axis acts on it. The potential energy $U(x)$ associated with the force is plotted in Fig. 8-10a. That is, if the particle were placed at any position between $x = 0$ and $x = 7.00 \text{ m}$, it would have the plotted value of U . At $x = 6.5 \text{ m}$, the particle has velocity $\vec{v}_0 = (-4.00 \text{ m/s})\hat{i}$.

(a) From Fig. 8-10a, determine the particle's speed at $x_1 = 4.5 \text{ m}$.

KEY IDEAS

(1) The particle's kinetic energy is given by Eq. 7-1 ($K = \frac{1}{2}mv^2$). (2) Because only a conservative force acts on the particle, the mechanical energy $E_{\text{mec}} (= K + U)$ is conserved as the particle moves. (3) Therefore, on a plot of $U(x)$ such as Fig. 8-10a, the kinetic energy is equal to the difference between E_{mec} and U .

Calculations: At $x = 6.5 \text{ m}$, the particle has kinetic energy

$$\begin{aligned} K_0 &= \frac{1}{2}mv_0^2 = \frac{1}{2}(2.00 \text{ kg})(4.00 \text{ m/s})^2 \\ &= 16.0 \text{ J}. \end{aligned}$$

Because the potential energy there is $U = 0$, the mechanical energy is

$$E_{\text{mec}} = K_0 + U_0 = 16.0 \text{ J} + 0 = 16.0 \text{ J}.$$

This value for E_{mec} is plotted as a horizontal line in Fig. 8-10a. From that figure we see that at $x = 4.5 \text{ m}$, the potential energy is $U_1 = 7.0 \text{ J}$. The kinetic energy K_1 is the difference between E_{mec} and U_1 :

$$K_1 = E_{\text{mec}} - U_1 = 16.0 \text{ J} - 7.0 \text{ J} = 9.0 \text{ J}.$$

Because $K_1 = \frac{1}{2}mv_1^2$, we find

$$v_1 = 3.0 \text{ m/s}. \quad (\text{Answer})$$

(b) Where is the particle's turning point located?

KEY IDEA

The turning point is where the force momentarily stops and then reverses the particle's motion. That is, it is where the particle momentarily has $v = 0$ and thus $K = 0$.

Calculations: Because K is the difference between E_{mec} and U , we want the point in Fig. 8-10a where the plot of U rises to meet the horizontal line of E_{mec} , as shown in Fig. 8-10b. Because the plot of U is a straight line in Fig. 8-10b, we can draw nested right triangles as shown and then write the proportionality of distances

$$\frac{16 - 7.0}{d} = \frac{20 - 7.0}{4.0 - 1.0},$$

which gives us $d = 2.08$ m. Thus, the turning point is at

$$x = 4.0 \text{ m} - d = 1.9 \text{ m}. \quad (\text{Answer})$$

(c) Evaluate the force acting on the particle when it is in the region $1.9 \text{ m} < x < 4.0 \text{ m}$.

KEY IDEA

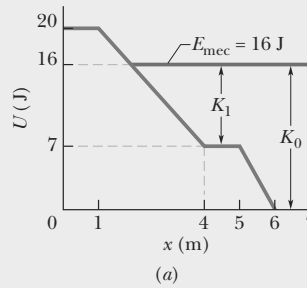
The force is given by Eq. 8-22 ($F(x) = -dU(x)/dx$): The force is equal to the negative of the slope on a graph of $U(x)$.

Calculations: For the graph of Fig. 8-10b, we see that for the range $1.0 \text{ m} < x < 4.0 \text{ m}$ the force is

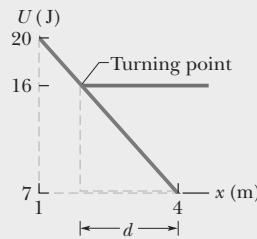
$$F = -\frac{20 \text{ J} - 7.0 \text{ J}}{1.0 \text{ m} - 4.0 \text{ m}} = 4.3 \text{ N}. \quad (\text{Answer})$$



Additional examples, video, and practice available at WileyPLUS



Kinetic energy is the difference between the total energy and the potential energy.



The kinetic energy is zero at the turning point (the particle speed is zero).

Figure 8-10 (a) A plot of potential energy U versus position x . (b) A section of the plot used to find where the particle turns around.

Thus, the force has magnitude 4.3 N and is in the positive direction of the x axis. This result is consistent with the fact that the initially leftward-moving particle is stopped by the force and then sent rightward.



8-4 WORK DONE ON A SYSTEM BY AN EXTERNAL FORCE

Learning Objectives

After reading this module, you should be able to . . .

8.13 When work is done on a system by an external force with no friction involved, determine the changes in kinetic energy and potential energy.

8.14 When work is done on a system by an external force with friction involved, relate that work to the changes in kinetic energy, potential energy, and thermal energy.

Key Ideas

- Work W is energy transferred to or from a system by means of an external force acting on the system.
- When more than one force acts on a system, their net work is the transferred energy.
- When friction is not involved, the work done on the system and the change ΔE_{mec} in the mechanical energy of the system are equal:

$$W = \Delta E_{\text{mec}} = \Delta K + \Delta U.$$

- When a kinetic frictional force acts within the system, then the thermal energy E_{th} of the system changes. (This energy is associated with the random motion of atoms and molecules in the system.) The work done on the system is then

$$W = \Delta E_{\text{mec}} + \Delta E_{\text{th}}.$$

- The change ΔE_{th} is related to the magnitude f_k of the frictional force and the magnitude d of the displacement caused by the external force by

$$\Delta E_{\text{th}} = f_k d.$$

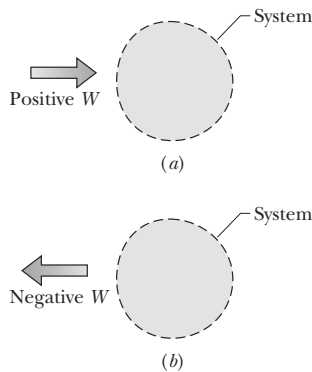


Figure 8-11 (a) Positive work W done on an arbitrary system means a transfer of energy to the system. (b) Negative work W means a transfer of energy from the system.

Work Done on a System by an External Force

In Chapter 7, we defined work as being energy transferred to or from an object by means of a force acting on the object. We can now extend that definition to an external force acting on a system of objects.



Work is energy transferred to or from a system by means of an external force acting on that system.

Figure 8-11a represents positive work (a transfer of energy *to* a system), and Fig. 8-11b represents negative work (a transfer of energy *from* a system). When more than one force acts on a system, their *net work* is the energy transferred to or from the system.

These transfers are like transfers of money to and from a bank account. If a system consists of a single particle or particle-like object, as in Chapter 7, the work done on the system by a force can change only the kinetic energy of the system. The energy statement for such transfers is the work–kinetic energy theorem of Eq. 7-10 ($\Delta K = W$); that is, a single particle has only one energy account, called kinetic energy. External forces can transfer energy into or out of that account. If a system is more complicated, however, an external force can change other forms of energy (such as potential energy); that is, a more complicated system can have multiple energy accounts.

Let us find energy statements for such systems by examining two basic situations, one that does not involve friction and one that does.

No Friction Involved

To compete in a bowling-ball-hurling contest, you first squat and cup your hands under the ball on the floor. Then you rapidly straighten up while also pulling your hands up sharply, launching the ball upward at about face level. During your upward motion, your applied force on the ball obviously does work; that is, it is an external force that transfers energy, but to what system?

To answer, we check to see which energies change. There is a change ΔK in the ball's kinetic energy and, because the ball and Earth become more separated, there is a change ΔU in the gravitational potential energy of the ball–Earth system. To include both changes, we need to consider the ball–Earth system. Then your force is an external force doing work on that system, and the work is

$$W = \Delta K + \Delta U, \quad (8-25)$$

$$\text{or} \quad W = \Delta E_{\text{mec}} \quad (\text{work done on system, no friction involved}), \quad (8-26)$$

where ΔE_{mec} is the change in the mechanical energy of the system. These two equations, which are represented in Fig. 8-12, are equivalent energy statements for work done on a system by an external force when friction is not involved.

Friction Involved

We next consider the example in Fig. 8-13a. A constant horizontal force \vec{F} pulls a block along an x axis and through a displacement of magnitude d , increasing the block's velocity from \vec{v}_0 to \vec{v} . During the motion, a constant kinetic frictional force \vec{f}_k from the floor acts on the block. Let us first choose the block as our system and apply Newton's second law to it. We can write that law for components along the x axis ($F_{\text{net},x} = ma_x$) as

$$F - f_k = ma. \quad (8-27)$$

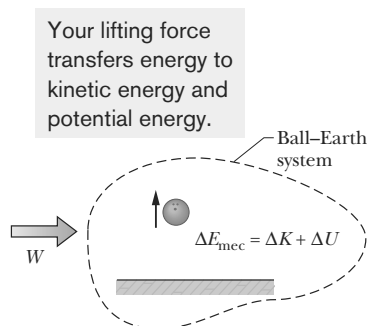


Figure 8-12 Positive work W is done on a system of a bowling ball and Earth, causing a change ΔE_{mec} in the mechanical energy of the system, a change ΔK in the ball's kinetic energy, and a change ΔU in the system's gravitational potential energy.

The applied force supplies energy. The frictional force transfers some of it to thermal energy.

So, the work done by the applied force goes into kinetic energy and also thermal energy.

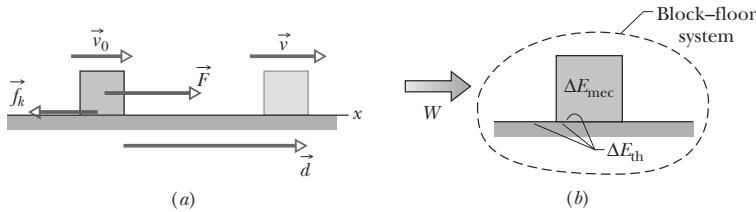


Figure 8-13 (a) A block is pulled across a floor by force \vec{F} while a kinetic frictional force \vec{f}_k opposes the motion. The block has velocity \vec{v}_0 at the start of a displacement \vec{d} and velocity \vec{v} at the end of the displacement. (b) Positive work W is done on the block–floor system by force \vec{F} , resulting in a change ΔE_{mec} in the block's mechanical energy and a change ΔE_{th} in the thermal energy of the block and floor.

Because the forces are constant, the acceleration \vec{a} is also constant. Thus, we can use Eq. 2-16 to write

$$v^2 = v_0^2 + 2ad.$$

Solving this equation for a , substituting the result into Eq. 8-27, and rearranging then give us

$$Fd = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 + f_k d \quad (8-28)$$

or, because $\frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 = \Delta K$ for the block,

$$Fd = \Delta K + f_k d. \quad (8-29)$$

In a more general situation (say, one in which the block is moving up a ramp), there can be a change in potential energy. To include such a possible change, we generalize Eq. 8-29 by writing

$$Fd = \Delta E_{\text{mec}} + f_k d. \quad (8-30)$$

By experiment we find that the block and the portion of the floor along which it slides become warmer as the block slides. As we shall discuss in Chapter 18, the temperature of an object is related to the object's thermal energy E_{th} (the energy associated with the random motion of the atoms and molecules in the object). Here, the thermal energy of the block and floor increases because (1) there is friction between them and (2) there is sliding. Recall that friction is due to the cold-welding between two surfaces. As the block slides over the floor, the sliding causes repeated tearing and re-forming of the welds between the block and the floor, which makes the block and floor warmer. Thus, the sliding increases their thermal energy E_{th} .

Through experiment, we find that the increase ΔE_{th} in thermal energy is equal to the product of the magnitudes f_k and d :

$$\Delta E_{\text{th}} = f_k d \quad (\text{increase in thermal energy by sliding}). \quad (8-31)$$

Thus, we can rewrite Eq. 8-30 as

$$Fd = \Delta E_{\text{mec}} + \Delta E_{\text{th}}. \quad (8-32)$$

Fd is the work W done by the external force \vec{F} (the energy transferred by the force), but on which system is the work done (where are the energy transfers made)? To answer, we check to see which energies change. The block's mechanical energy

changes, and the thermal energies of the block and floor also change. Therefore, the work done by force \vec{F} is done on the block–floor system. That work is

$$W = \Delta E_{\text{mec}} + \Delta E_{\text{th}} \quad (\text{work done on system, friction involved}). \quad (8-33)$$

This equation, which is represented in Fig. 8-13b, is the energy statement for the work done on a system by an external force when friction is involved.



Checkpoint 5

In three trials, a block is pushed by a horizontal applied force across a floor that is not frictionless, as in Fig. 8-13a. The magnitudes F of the applied force and the results of the pushing on the block's speed are given in the table. In all three trials, the block is pushed through the same distance d . Rank the three trials according to the change in the thermal energy of the block and floor that occurs in that distance d , greatest first.

Trial	F	Result on Block's Speed
a	5.0 N	decreases
b	7.0 N	remains constant
c	8.0 N	increases

Sample Problem 8.05 Work, friction, change in thermal energy, cabbage heads

A food shipper pushes a wood crate of cabbage heads (total mass $m = 14 \text{ kg}$) across a concrete floor with a constant horizontal force \vec{F} of magnitude 40 N. In a straight-line displacement of magnitude $d = 0.50 \text{ m}$, the speed of the crate decreases from $v_0 = 0.60 \text{ m/s}$ to $v = 0.20 \text{ m/s}$.

(a) How much work is done by force \vec{F} , and on what system does it do the work?

KEY IDEA

Because the applied force \vec{F} is constant, we can calculate the work it does by using Eq. 7-7 ($W = Fd \cos \phi$).

Calculation: Substituting given data, including the fact that force \vec{F} and displacement \vec{d} are in the same direction, we find

$$\begin{aligned} W &= Fd \cos \phi = (40 \text{ N})(0.50 \text{ m}) \cos 0^\circ \\ &= 20 \text{ J}. \end{aligned} \quad (\text{Answer})$$

Reasoning: To determine the system on which the work is done, let's check which energies change. Because the crate's speed changes, there is certainly a change ΔK in the crate's kinetic energy. Is there friction between the floor and the crate, and thus a change in thermal energy? Note that \vec{F} and the crate's velocity have the same direction. Thus, if there is no friction, then \vec{F} should be accelerating the crate to a greater speed. However, the crate is *slowing*, so there must

be friction and a change ΔE_{th} in thermal energy of the crate and the floor. Therefore, the system on which the work is done is the crate–floor system, because both energy changes occur in that system.

(b) What is the increase ΔE_{th} in the thermal energy of the crate and floor?

KEY IDEA

We can relate ΔE_{th} to the work W done by \vec{F} with the energy statement of Eq. 8-33 for a system that involves friction:

$$W = \Delta E_{\text{mec}} + \Delta E_{\text{th}}. \quad (8-34)$$

Calculations: We know the value of W from (a). The change ΔE_{mec} in the crate's mechanical energy is just the change in its kinetic energy because no potential energy changes occur, so we have

$$\Delta E_{\text{mec}} = \Delta K = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2.$$

Substituting this into Eq. 8-34 and solving for ΔE_{th} , we find

$$\begin{aligned} \Delta E_{\text{th}} &= W - \left(\frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 \right) = W - \frac{1}{2}m(v^2 - v_0^2) \\ &= 20 \text{ J} - \frac{1}{2}(14 \text{ kg})[(0.20 \text{ m/s})^2 - (0.60 \text{ m/s})^2] \\ &= 22.2 \text{ J} \approx 22 \text{ J}. \end{aligned} \quad (\text{Answer})$$

Without further experiments, we cannot say how much of this thermal energy ends up in the crate and how much in the floor. We simply know the total amount.



Additional examples, video, and practice available at WileyPLUS

8-5 CONSERVATION OF ENERGY

Learning Objectives

After reading this module, you should be able to . . .

- 8.15** For an isolated system (no net external force), apply the conservation of energy to relate the initial total energy (energies of all kinds) to the total energy at a later instant.
- 8.16** For a nonisolated system, relate the work done on the system by a net external force to the changes in the various types of energies within the system.

- 8.17** Apply the relationship between average power, the associated energy transfer, and the time interval in which that transfer is made.
- 8.18** Given an energy transfer as a function of time (either as an equation or a graph), determine the instantaneous power (the transfer at any given instant).

Key Ideas

- The total energy E of a system (the sum of its mechanical energy and its internal energies, including thermal energy) can change only by amounts of energy that are transferred to or from the system. This experimental fact is known as the law of conservation of energy.

- If work W is done on the system, then

$$W = \Delta E = \Delta E_{\text{mec}} + \Delta E_{\text{th}} + \Delta E_{\text{int}}.$$

If the system is isolated ($W = 0$), this gives

$$\Delta E_{\text{mec}} + \Delta E_{\text{th}} + \Delta E_{\text{int}} = 0$$

and $E_{\text{mec},2} = E_{\text{mec},1} - \Delta E_{\text{th}} - \Delta E_{\text{int}},$

where the subscripts 1 and 2 refer to two different instants.

- The power due to a force is the *rate* at which that force transfers energy. If an amount of energy ΔE is transferred by a force in an amount of time Δt , the average power of the force is

$$P_{\text{avg}} = \frac{\Delta E}{\Delta t}.$$

- The instantaneous power due to a force is

$$P = \frac{dE}{dt}.$$

On a graph of energy E versus time t , the power is the slope of the plot at any given time.

Conservation of Energy

We now have discussed several situations in which energy is transferred to or from objects and systems, much like money is transferred between accounts. In each situation we assume that the energy that was involved could always be accounted for; that is, energy could not magically appear or disappear. In more formal language, we assumed (correctly) that energy obeys a law called the **law of conservation of energy**, which is concerned with the **total energy** E of a system. That total is the sum of the system's mechanical energy, thermal energy, and any type of *internal energy* in addition to thermal energy. (We have not yet discussed other types of internal energy.) The law states that



The total energy E of a system can change only by amounts of energy that are transferred to or from the system.

The only type of energy transfer that we have considered is work W done on a system by an external force. Thus, for us at this point, this law states that

$$W = \Delta E = \Delta E_{\text{mec}} + \Delta E_{\text{th}} + \Delta E_{\text{int}}, \quad (8-35)$$

where ΔE_{mec} is any change in the mechanical energy of the system, ΔE_{th} is any change in the thermal energy of the system, and ΔE_{int} is any change in any other type of internal energy of the system. Included in ΔE_{mec} are changes ΔK in kinetic energy and changes ΔU in potential energy (elastic, gravitational, or any other type we might find).

This law of conservation of energy is *not* something we have derived from basic physics principles. Rather, it is a law based on countless experiments.



Tyler Stableford/The Image Bank/Getty Images

Figure 8-14 To descend, the rock climber must transfer energy from the gravitational potential energy of a system consisting of him, his gear, and Earth. He has wrapped the rope around metal rings so that the rope rubs against the rings. This allows most of the transferred energy to go to the thermal energy of the rope and rings rather than to his kinetic energy.

Scientists and engineers have never found an exception to it. Energy simply cannot magically appear or disappear.

Isolated System

If a system is isolated from its environment, there can be no energy transfers to or from it. For that case, the law of conservation of energy states:



The total energy E of an isolated system cannot change.

Many energy transfers may be going on *within* an isolated system—between, say, kinetic energy and a potential energy or between kinetic energy and thermal energy. However, the total of all the types of energy in the system cannot change. Here again, energy cannot magically appear or disappear.

We can use the rock climber in Fig. 8-14 as an example, approximating him, his gear, and Earth as an isolated system. As he rappels down the rock face, changing the configuration of the system, he needs to control the transfer of energy from the gravitational potential energy of the system. (That energy cannot just disappear.) Some of it is transferred to his kinetic energy. However, he obviously does not want very much transferred to that type or he will be moving too quickly, so he has wrapped the rope around metal rings to produce friction between the rope and the rings as he moves down. The sliding of the rings on the rope then transfers the gravitational potential energy of the system to thermal energy of the rings and rope in a way that he can control. The total energy of the climber–gear–Earth system (the total of its gravitational potential energy, kinetic energy, and thermal energy) does not change during his descent.

For an isolated system, the law of conservation of energy can be written in two ways. First, by setting $W = 0$ in Eq. 8-35, we get

$$\Delta E_{\text{mec}} + \Delta E_{\text{th}} + \Delta E_{\text{int}} = 0 \quad (\text{isolated system}). \quad (8-36)$$

We can also let $\Delta E_{\text{mec}} = E_{\text{mec},2} - E_{\text{mec},1}$, where the subscripts 1 and 2 refer to two different instants—say, before and after a certain process has occurred. Then Eq. 8-36 becomes

$$E_{\text{mec},2} = E_{\text{mec},1} - \Delta E_{\text{th}} - \Delta E_{\text{int}}. \quad (8-37)$$

Equation 8-37 tells us:



In an isolated system, we can relate the total energy at one instant to the total energy at another instant *without considering the energies at intermediate times*.

This fact can be a very powerful tool in solving problems about isolated systems when you need to relate energies of a system before and after a certain process occurs in the system.

In Module 8-2, we discussed a special situation for isolated systems—namely, the situation in which nonconservative forces (such as a kinetic frictional force) do not act within them. In that special situation, ΔE_{th} and ΔE_{int} are both zero, and so Eq. 8-37 reduces to Eq. 8-18. In other words, the mechanical energy of an isolated system is conserved when nonconservative forces do not act in it.

External Forces and Internal Energy Transfers

An external force can change the kinetic energy or potential energy of an object without doing work on the object—that is, without transferring energy to the object. Instead, the force is responsible for transfers of energy from one type to another inside the object.

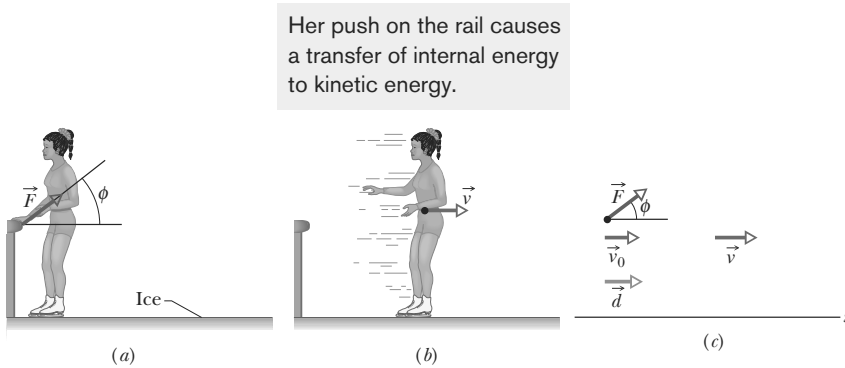


Figure 8-15 (a) As a skater pushes herself away from a railing, the force on her from the railing is \vec{F} . (b) After the skater leaves the railing, she has velocity \vec{v} . (c) External force \vec{F} acts on the skater, at angle ϕ with a horizontal x axis. When the skater goes through displacement \vec{d} , her velocity is changed from \vec{v}_0 ($= 0$) to \vec{v} by the horizontal component of \vec{F} .

Figure 8-15 shows an example. An initially stationary ice-skater pushes away from a railing and then slides over the ice (Figs. 8-15a and b). Her kinetic energy increases because of an external force \vec{F} on her from the rail. However, that force does not transfer energy from the rail to her. Thus, the force does no work on her. Rather, her kinetic energy increases as a result of internal transfers from the biochemical energy in her muscles.

Figure 8-16 shows another example. An engine increases the speed of a car with four-wheel drive (all four wheels are made to turn by the engine). During the acceleration, the engine causes the tires to push backward on the road surface. This push produces frictional forces \vec{f} that act on each tire in the forward direction. The net external force \vec{F} from the road, which is the sum of these frictional forces, accelerates the car, increasing its kinetic energy. However, \vec{F} does not transfer energy from the road to the car and so does no work on the car. Rather, the car's kinetic energy increases as a result of internal transfers from the energy stored in the fuel.

In situations like these two, we can sometimes relate the external force \vec{F} on an object to the change in the object's mechanical energy if we can simplify the situation. Consider the ice-skater example. During her push through distance d in Fig. 8-15c, we can simplify by assuming that the acceleration is constant, her speed changing from $v_0 = 0$ to v . (That is, we assume \vec{F} has constant magnitude F and angle ϕ .) After the push, we can simplify the skater as being a particle and neglect the fact that the exertions of her muscles have increased the thermal energy in her muscles and changed other physiological features. Then we can apply Eq. 7-5 ($\frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 = F_x d$) to write

$$K - K_0 = (F \cos \phi)d,$$

or

$$\Delta K = Fd \cos \phi. \quad (8-38)$$

If the situation also involves a change in the elevation of an object, we can include the change ΔU in gravitational potential energy by writing

$$\Delta U + \Delta K = Fd \cos \phi. \quad (8-39)$$

The force on the right side of this equation does no work on the object but is still responsible for the changes in energy shown on the left side.

Power

Now that you have seen how energy can be transferred from one type to another, we can expand the definition of power given in Module 7-6. There power is

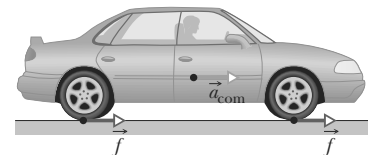


Figure 8-16 A vehicle accelerates to the right using four-wheel drive. The road exerts four frictional forces (two of them shown) on the bottom surfaces of the tires. Taken together, these four forces make up the net external force \vec{F} acting on the car.

defined as the rate at which work is done by a force. In a more general sense, power P is the rate at which energy is transferred by a force from one type to another. If an amount of energy ΔE is transferred in an amount of time Δt , the **average power** due to the force is

$$P_{\text{avg}} = \frac{\Delta E}{\Delta t}. \quad (8-40)$$

Similarly, the **instantaneous power** due to the force is

$$P = \frac{dE}{dt}. \quad (8-41)$$



Sample Problem 8.06 Lots of energies at an amusement park water slide

Figure 8-17 shows a water-slide ride in which a glider is shot by a spring along a water-drenched (frictionless) track that takes the glider from a horizontal section down to ground level. As the glider then moves along ground-level track, it is gradually brought to rest by friction. The total mass of the glider and its rider is $m = 200$ kg, the initial compression of the spring is $d = 5.00$ m, the spring constant is $k = 3.20 \times 10^3$ N/m, the initial height is $h = 35.0$ m, and the coefficient of kinetic friction along the ground-level track is $\mu_k = 0.800$. Through what distance L does the glider slide along the ground-level track until it stops?

KEY IDEAS

Before we touch a calculator and start plugging numbers into equations, we need to examine all the forces and then determine what our system should be. Only then can we decide what equation to write. Do we have an isolated system (our equation would be for the conservation of energy) or a system on which an external force does work (our equation would relate that work to the system's change in energy)?

Forces: The normal force on the glider from the track does no work on the glider because the direction of this force is always perpendicular to the direction of the glider's displacement. The gravitational force does work on the glider, and because the force is conservative we can associate a potential energy with it. As the spring pushes

on the glider to get it moving, a spring force does work on it, transferring energy from the elastic potential energy of the compressed spring to kinetic energy of the glider. The spring force also pushes against a rigid wall. Because there is friction between the glider and the ground-level track, the sliding of the glider along that track section increases their thermal energies.

System: Let's take the system to contain all the interacting bodies: glider, track, spring, Earth, and wall. Then, because all the force interactions are *within* the system, the system is *isolated* and thus its total energy cannot change. So, the equation we should use is not that of some external force doing work on the system. Rather, it is a conservation of energy. We write this in the form of Eq. 8-37:

$$E_{\text{mec},2} = E_{\text{mec},1} - \Delta E_{\text{th}}. \quad (8-42)$$

This is like a money equation: The final money is equal to the initial money *minus* the amount stolen away by a thief. Here, the final mechanical energy is equal to the initial mechanical energy *minus* the amount stolen away by friction. None has magically appeared or disappeared.

Calculations: Now that we have an equation, let's find distance L . Let subscript 1 correspond to the initial state of the glider (when it is still on the compressed spring) and subscript 2 correspond to the final state of the glider (when it has come to rest on the ground-level track). For both states, the mechanical energy of the system is the sum of any potential energy and any kinetic energy.

We have two types of potential energy: the elastic potential energy ($U_e = \frac{1}{2}kx^2$) associated with the compressed spring and the gravitational potential energy ($U_g = mgy$) associated with the glider's elevation. For the latter, let's take ground level as the reference level. That means that the glider is initially at height $y = h$ and finally at height $y = 0$.

In the initial state, with the glider stationary and elevated and the spring compressed, the energy is

$$\begin{aligned} E_{\text{mec},1} &= K_1 + U_{e1} + U_{g1} \\ &= 0 + \frac{1}{2}kd^2 + mgh. \end{aligned} \quad (8-43)$$

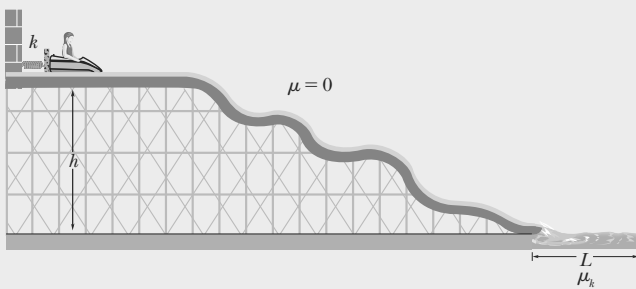


Figure 8-17 A spring-loaded amusement park water slide.

In the final state, with the spring now in its relaxed state and the glider again stationary but no longer elevated, the final mechanical energy of the system is

$$E_{\text{mec},2} = K_2 + U_{e2} + U_{g2} \\ = 0 + 0 + 0. \quad (8-44)$$

Let's next go after the change ΔE_{th} of the thermal energy of the glider and ground-level track. From Eq. 8-31, we can substitute for ΔE_{th} with $f_k L$ (the product of the frictional force magnitude and the distance of rubbing). From Eq. 6-2, we know that $f_k = \mu_k F_N$, where F_N is the normal force. Because the glider moves horizontally through the region with friction, the magnitude of F_N is equal to mg (the upward force matches the downward force). So, the friction's theft from the mechanical energy amounts to

$$\Delta E_{\text{th}} = \mu_k mgL. \quad (8-45)$$

(By the way, without further experiments, we *cannot* say how much of this thermal energy ends up in the glider and how much in the track. We simply know the total amount.)

Substituting Eqs. 8-43 through 8-45 into Eq. 8-42, we find

$$0 = \frac{1}{2}kd^2 + mgh - \mu_k mgL, \quad (8-46)$$

and

$$L = \frac{kd^2}{2\mu_k mg} + \frac{h}{\mu_k} \\ = \frac{(3.20 \times 10^3 \text{ N/m})(5.00 \text{ m})^2}{2(0.800)(200 \text{ kg})(9.8 \text{ m/s}^2)} + \frac{35 \text{ m}}{0.800} \\ = 69.3 \text{ m}. \quad (\text{Answer})$$

Finally, note how algebraically simple our solution is. By carefully defining a system and realizing that we have an isolated system, we get to use the law of the conservation of energy. That means we can relate the initial and final states of the system with no consideration of the intermediate states. In particular, we did not need to consider the glider as it slides over the uneven track. If we had, instead, applied Newton's second law to the motion, we would have had to know the details of the track and would have faced a far more difficult calculation.

 Additional examples, video, and practice available at WileyPLUS



Review & Summary

Conservative Forces A force is a **conservative force** if the net work it does on a particle moving around any closed path, from an initial point and then back to that point, is zero. Equivalently, a force is conservative if the net work it does on a particle moving between two points does not depend on the path taken by the particle. The gravitational force and the spring force are conservative forces; the kinetic frictional force is a **nonconservative force**.

Potential Energy A **potential energy** is energy that is associated with the configuration of a system in which a conservative force acts. When the conservative force does work W on a particle within the system, the change ΔU in the potential energy of the system is

$$\Delta U = -W. \quad (8-1)$$

If the particle moves from point x_i to point x_f , the change in the potential energy of the system is

$$\Delta U = - \int_{x_i}^{x_f} F(x) dx. \quad (8-6)$$

Gravitational Potential Energy The potential energy associated with a system consisting of Earth and a nearby particle is **gravitational potential energy**. If the particle moves from height y_i to height y_f , the change in the gravitational potential energy of the particle–Earth system is

$$\Delta U = mg(y_f - y_i) = mg \Delta y. \quad (8-7)$$

If the **reference point** of the particle is set as $y_i = 0$ and the corresponding gravitational potential energy of the system is set as $U_i = 0$, then the gravitational potential energy U when the parti-

cle is at any height y is

$$U(y) = mgy. \quad (8-9)$$

Elastic Potential Energy **Elastic potential energy** is the energy associated with the state of compression or extension of an elastic object. For a spring that exerts a spring force $F = -kx$ when its free end has displacement x , the elastic potential energy is

$$U(x) = \frac{1}{2}kx^2. \quad (8-11)$$

The **reference configuration** has the spring at its relaxed length, at which $x = 0$ and $U = 0$.

Mechanical Energy The **mechanical energy** E_{mec} of a system is the sum of its kinetic energy K and potential energy U :

$$E_{\text{mec}} = K + U. \quad (8-12)$$

An **isolated system** is one in which no **external force** causes energy changes. If only conservative forces do work within an isolated system, then the mechanical energy E_{mec} of the system cannot change. This **principle of conservation of mechanical energy** is written as

$$K_2 + U_2 = K_1 + U_1, \quad (8-17)$$

in which the subscripts refer to different instants during an energy transfer process. This conservation principle can also be written as

$$\Delta E_{\text{mec}} = \Delta K + \Delta U = 0. \quad (8-18)$$

Potential Energy Curves If we know the potential energy function $U(x)$ for a system in which a one-dimensional force $F(x)$

acts on a particle, we can find the force as

$$F(x) = -\frac{dU(x)}{dx}. \quad (8-22)$$

If $U(x)$ is given on a graph, then at any value of x , the force $F(x)$ is the negative of the slope of the curve there and the kinetic energy of the particle is given by

$$K(x) = E_{\text{mec}} - U(x), \quad (8-24)$$

where E_{mec} is the mechanical energy of the system. A **turning point** is a point x at which the particle reverses its motion (there, $K = 0$). The particle is in **equilibrium** at points where the slope of the $U(x)$ curve is zero (there, $F(x) = 0$).

Work Done on a System by an External Force Work W is energy transferred to or from a system by means of an external force acting on the system. When more than one force acts on a system, their *net work* is the transferred energy. When friction is not involved, the work done on the system and the change ΔE_{mec} in the mechanical energy of the system are equal:

$$W = \Delta E_{\text{mec}} = \Delta K + \Delta U. \quad (8-26, 8-25)$$

When a kinetic frictional force acts within the system, then the thermal energy E_{th} of the system changes. (This energy is associated with the random motion of atoms and molecules in the system.) The work done on the system is then

$$W = \Delta E_{\text{mec}} + \Delta E_{\text{th}}. \quad (8-33)$$

The change ΔE_{th} is related to the magnitude f_k of the frictional force and the magnitude d of the displacement caused by the external force by

$$\Delta E_{\text{th}} = f_k d. \quad (8-31)$$

Conservation of Energy The **total energy** E of a system (the sum of its mechanical energy and its internal energies, including thermal energy) can change only by amounts of energy that are transferred to or from the system. This experimental fact is known as the **law of conservation of energy**. If work W is done on the system, then

$$W = \Delta E = \Delta E_{\text{mec}} + \Delta E_{\text{th}} + \Delta E_{\text{int}}. \quad (8-35)$$

If the system is isolated ($W = 0$), this gives

$$\Delta E_{\text{mec}} + \Delta E_{\text{th}} + \Delta E_{\text{int}} = 0 \quad (8-36)$$

and

$$E_{\text{mec},2} = E_{\text{mec},1} - \Delta E_{\text{th}} - \Delta E_{\text{int}}, \quad (8-37)$$

where the subscripts 1 and 2 refer to two different instants.

Power The **power** due to a force is the *rate* at which that force transfers energy. If an amount of energy ΔE is transferred by a force in an amount of time Δt , the **average power** of the force is

$$P_{\text{avg}} = \frac{\Delta E}{\Delta t}. \quad (8-40)$$

The **instantaneous power** due to a force is

$$P = \frac{dE}{dt}. \quad (8-41)$$

Questions

1 In Fig. 8-18, a horizontally moving block can take three frictionless routes, differing only in elevation, to reach the dashed finish line. Rank the routes according to (a) the speed of the block at the finish line and (b) the travel time of the block to the finish line, greatest first.

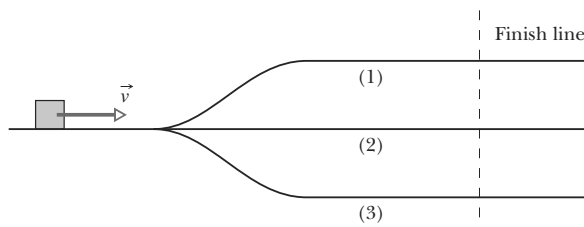


Figure 8-18 Question 1.

2 Figure 8-19 gives the potential energy function of a particle. (a) Rank regions AB, BC, CD, and DE according to the magni-

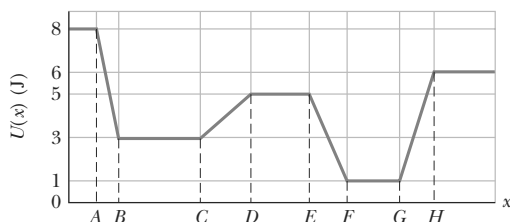


Figure 8-19 Question 2.

tude of the force on the particle, greatest first. What value must the mechanical energy E_{mec} of the particle not exceed if the particle is to be (b) trapped in the potential well at the left, (c) trapped in the potential well at the right, and (d) able to move between the two potential wells but not to the right of point H? For the situation of (d), in which of regions BC, DE, and FG will the particle have (e) the greatest kinetic energy and (f) the least speed?

3 Figure 8-20 shows one direct path and four indirect paths from point i to point f . Along the direct path and three of the indirect paths, only a conservative force F_c acts on a certain object. Along the fourth indirect path, both F_c and a nonconservative force F_{nc} act on the object. The change ΔE_{mec} in the object's mechanical energy (in joules) in going from i to f is indicated along each straight-line segment of the indirect paths. What is ΔE_{mec} (a) from i to f along the direct path and (b) due to F_{nc} along the one path where it acts?

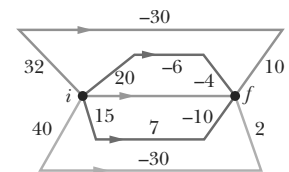


Figure 8-20 Question 3.

4 In Fig. 8-21, a small, initially stationary block is released on a frictionless ramp at a height of 3.0 m. Hill heights along the ramp are as shown in the figure. The hills have identical circular tops, and the block does not fly off any hill. (a) Which hill is the first the block cannot cross? (b) What does the block do after failing to cross that hill? Of the hills that the block can cross, on which hill-

top is (c) the centripetal acceleration of the block greatest and (d) the normal force on the block least?

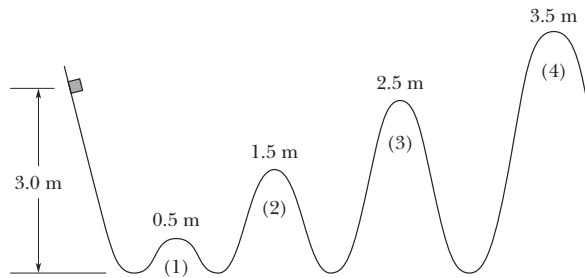


Figure 8-21 Question 4.

5 In Fig. 8-22, a block slides from A to C along a frictionless ramp, and then it passes through horizontal region CD , where a frictional force acts on it. Is the block's kinetic energy increasing, decreasing, or constant in (a) region AB , (b) region BC , and (c) region CD ? (d) Is the block's mechanical energy increasing, decreasing, or constant in those regions?

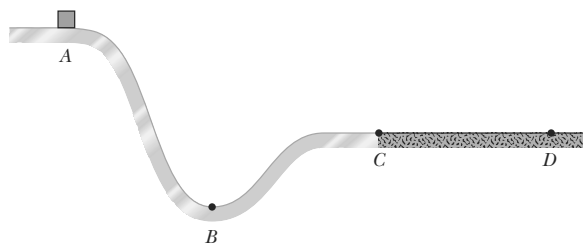


Figure 8-22 Question 5.

6 In Fig. 8-23a, you pull upward on a rope that is attached to a cylinder on a vertical rod. Because the cylinder fits tightly on the rod, the cylinder slides along the rod with considerable friction. Your force does work $W = +100 \text{ J}$ on the cylinder-rod-Earth system (Fig. 8-23b). An "energy statement" for the system is shown in Fig. 8-23c: the kinetic energy K increases by 50 J , and the gravitational potential energy U_g increases by 20 J . The only other change in energy within the system is for the thermal energy E_{th} . What is the change ΔE_{th} ?

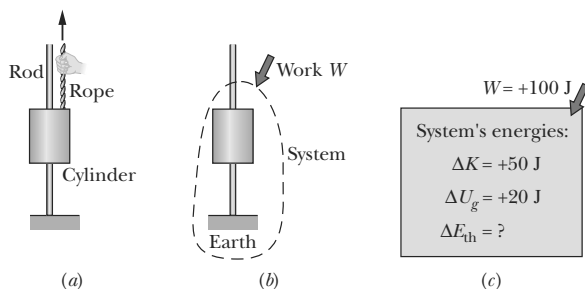


Figure 8-23 Question 6.

7 The arrangement shown in Fig. 8-24 is similar to that in Question 6. Here you pull downward on the rope that is attached to the cylinder, which fits tightly on the rod. Also, as the cylinder

descends, it pulls on a block via a second rope, and the block slides over a lab table. Again consider the cylinder-rod-Earth system, similar to that shown in Fig. 8-23b. Your work on the system is 200 J . The system does work of 60 J on the block. Within the system, the kinetic energy increases by 130 J and the gravitational potential energy decreases by 20 J . (a) Draw an "energy statement" for the system, as in Fig. 8-23c. (b) What is the change in the thermal energy within the system?

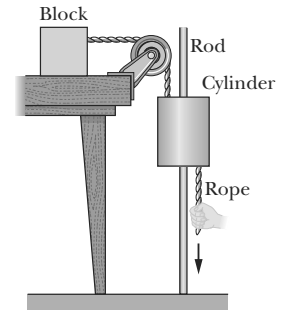


Figure 8-24 Question 7.

8 In Fig. 8-25, a block slides along a track that descends through distance h . The track is frictionless except for the lower section. There the block slides to a stop in a certain distance D because of friction. (a) If we decrease h , will the block now slide to a stop in a distance that is greater than, less than, or equal to D ? (b) If, instead, we increase the mass of the block, will the stopping distance now be greater than, less than, or equal to D ?



Figure 8-25 Question 8.

9 Figure 8-26 shows three situations involving a plane that is not frictionless and a block sliding along the plane. The block begins with the same speed in all three situations and slides until the kinetic frictional force has stopped it. Rank the situations according to the increase in thermal energy due to the sliding, greatest first.

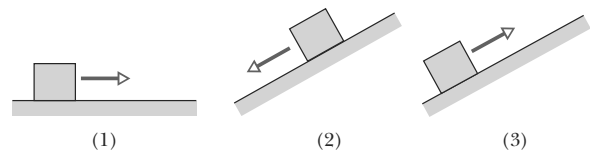


Figure 8-26 Question 9.

10 Figure 8-27 shows three plums that are launched from the same level with the same speed. One moves straight upward, one is launched at a small angle to the vertical, and one is launched along a frictionless incline. Rank the plums according to their speed when they reach the level of the dashed line, greatest first.

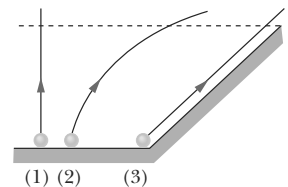


Figure 8-27 Question 10.

11 When a particle moves from f to i and from j to i along the paths shown in Fig. 8-28, and in the indicated directions, a conservative force \vec{F} does the indicated amounts of work on it. How much work is done on the particle by \vec{F} when the particle moves directly from f to j ?

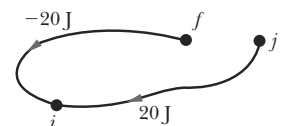


Figure 8-28 Question 11.

Problems



Tutoring problem available (at instructor's discretion) in WileyPLUS and WebAssign



Worked-out solution available in Student Solutions Manual

WWW Worked-out solution is at



Number of dots indicates level of problem difficulty



Interactive solution is at

<http://www.wiley.com/college/halliday>Additional information available in *The Flying Circus of Physics* and at flyingcircusofphysics.com

Module 8-1 Potential Energy

•1 **SSM** What is the spring constant of a spring that stores 25 J of elastic potential energy when compressed by 7.5 cm?

•2 In Fig. 8-29, a single frictionless roller-coaster car of mass $m = 825$ kg tops the first hill with speed $v_0 = 17.0$ m/s at height $h = 42.0$ m. How much work does the gravitational force do on the car from that point to (a) point A, (b) point B, and (c) point C? If the gravitational potential energy of the car–Earth system is taken to be zero at C, what is its value when the car is at (d) B and (e) A? (f) If mass m were doubled, would the change in the gravitational potential energy of the system between points A and B increase, decrease, or remain the same?

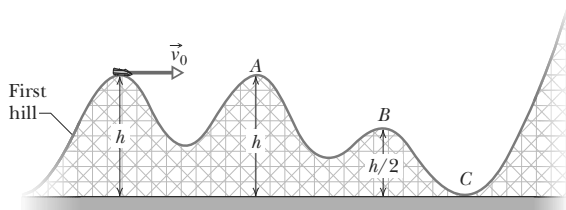


Figure 8-29 Problems 2 and 9.

•3 You drop a 2.00 kg book to a friend who stands on the ground at distance $D = 10.0$ m below. If your friend's outstretched hands are at distance $d = 1.50$ m above the ground (Fig. 8-30), (a) how much work W_g does the gravitational force do on the book as it drops to her hands? (b) What is the change ΔU in the gravitational potential energy of the book–Earth system during the drop? If the gravitational potential energy U of that system is taken to be zero at ground level, what is U (c) when the book is released and (d) when it reaches her hands? Now take U to be 100 J at ground level and again find (e) W_g , (f) ΔU , (g) U at the release point, and (h) U at her hands.

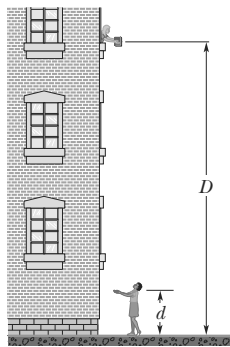


Figure 8-30 Problems 3 and 10.

•4 Figure 8-31 shows a ball with mass $m = 0.341$ kg attached to the end of a thin rod with length $L = 0.452$ m and negligible mass. The other end of the rod is pivoted so that the ball can move in a vertical circle. The rod is held horizontally as shown and then given enough of a downward push to cause the ball to swing down and around and just reach the vertically up position, with zero speed there. How much work is done on the ball by the gravitational force from the initial point

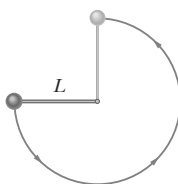


Figure 8-31 Problems 4 and 14.

to (a) the lowest point, (b) the highest point, and (c) the point on the right level with the initial point? If the gravitational potential energy of the ball–Earth system is taken to be zero at the initial point, what is it when the ball reaches (d) the lowest point, (e) the highest point, and (f) the point on the right level with the initial point? (g) Suppose the rod were pushed harder so that the ball passed through the highest point with a nonzero speed. Would ΔU_g from the lowest point to the highest point then be greater than, less than, or the same as it was when the ball stopped at the highest point?

•5 **SSM** In Fig. 8-32, a 2.00 g ice flake is released from the edge of a hemispherical bowl whose radius r is 22.0 cm. The flake–bowl contact is frictionless. (a) How much work is done on the flake by the gravitational force during the flake's descent to the bottom of the bowl? (b) What is the change in the potential energy of the flake–Earth system during that descent? (c) If that potential energy is taken to be zero at the bottom of the bowl, what is its value when the flake is released? (d) If, instead, the potential energy is taken to be zero at the release point, what is its value when the flake reaches the bottom of the bowl? (e) If the mass of the flake were doubled, would the magnitudes of the answers to (a) through (d) increase, decrease, or remain the same?

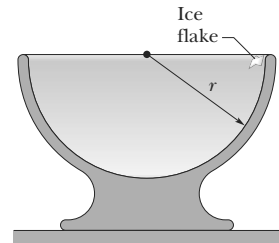


Figure 8-32 Problems 5 and 11.

••6 In Fig. 8-33, a small block of mass $m = 0.032$ kg can slide along the frictionless loop-the-loop, with loop radius $R = 12$ cm. The block is released from rest at point P, at height $h = 5.0R$ above the bottom of the loop. How much work does the gravitational force do on the block as the block travels from point P to (a) point Q and (b) the top of the loop? If the gravitational potential energy of the block–Earth system is taken to be zero at the bottom of the loop, what is that potential energy when the block is (c) at point P, (d) at point Q, and (e) at the top of the loop? (f) If, instead of merely being released, the block is given some initial speed downward along the track, do the answers to (a) through (e) increase, decrease, or remain the same?

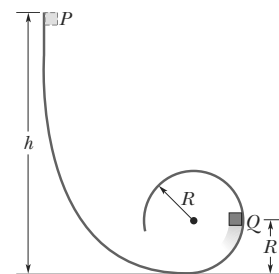


Figure 8-33 Problems 6 and 17.

••7 Figure 8-34 shows a thin rod, of length $L = 2.00$ m and negligible mass, that can pivot about one end to rotate in a vertical circle. A ball of mass $m = 5.00$ kg is attached to the other end. The rod is pulled aside to angle $\theta_0 = 30.0^\circ$ and released with initial velocity $\vec{v}_0 = 0$. As the ball descends to its lowest point, (a) how much work does the gravitational force do on it and (b) what is the change in the gravitational potential energy of

the ball–Earth system? (c) If the gravitational potential energy is taken to be zero at the lowest point, what is its value just as the ball is released? (d) Do the magnitudes of the answers to (a) through (c) increase, decrease, or remain the same if angle θ_0 is increased?

- 8 A 1.50 kg snowball is fired from a cliff 12.5 m high. The snowball's initial velocity is 14.0 m/s, directed 41.0° above the horizontal. (a) How much work is done on the snowball by the gravitational force during its flight to the flat ground below the cliff? (b) What is the change in the gravitational potential energy of the snowball–Earth system during the flight? (c) If that gravitational potential energy is taken to be zero at the height of the cliff, what is its value when the snowball reaches the ground?

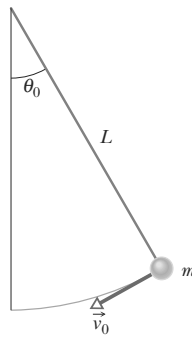


Figure 8-34
Problems 7, 18,
and 21.

Module 8-2 Conservation of Mechanical Energy

- 9 **GO** In Problem 2, what is the speed of the car at (a) point A, (b) point B, and (c) point C? (d) How high will the car go on the last hill, which is too high for it to cross? (e) If we substitute a second car with twice the mass, what then are the answers to (a) through (d)?

- 10 (a) In Problem 3, what is the speed of the book when it reaches the hands? (b) If we substituted a second book with twice the mass, what would its speed be? (c) If, instead, the book were thrown down, would the answer to (a) increase, decrease, or remain the same?

- 11 **SSM WWW** (a) In Problem 5, what is the speed of the flake when it reaches the bottom of the bowl? (b) If we substituted a second flake with twice the mass, what would its speed be? (c) If, instead, we gave the flake an initial downward speed along the bowl, would the answer to (a) increase, decrease, or remain the same?

- 12 (a) In Problem 8, using energy techniques rather than the techniques of Chapter 4, find the speed of the snowball as it reaches the ground below the cliff. What is that speed (b) if the launch angle is changed to 41.0° below the horizontal and (c) if the mass is changed to 2.50 kg?

- 13 **SSM** A 5.0 g marble is fired vertically upward using a spring gun. The spring must be compressed 8.0 cm if the marble is to just reach a target 20 m above the marble's position on the compressed spring. (a) What is the change ΔU_g in the gravitational potential energy of the marble–Earth system during the 20 m ascent? (b) What is the change ΔU_s in the elastic potential energy of the spring during its launch of the marble? (c) What is the spring constant of the spring?

- 14 (a) In Problem 4, what initial speed must be given the ball so that it reaches the vertically upward position with zero speed? What then is its speed at (b) the lowest point and (c) the point on the right at which the ball is level with the initial point? (d) If the ball's mass were doubled, would the answers to (a) through (c) increase, decrease, or remain the same?

- 15 **SSM** In Fig. 8-35, a runaway truck with failed brakes is moving downgrade at 130 km/h just before the driver steers the truck up a frictionless emergency escape ramp with an inclination of $\theta = 15^\circ$. The truck's mass is 1.2×10^4 kg. (a) What minimum length

L must the ramp have if the truck is to stop (momentarily) along it? (Assume the truck is a particle, and justify that assumption.) Does the minimum length L increase, decrease, or remain the same if (b) the truck's mass is decreased and (c) its speed is decreased?

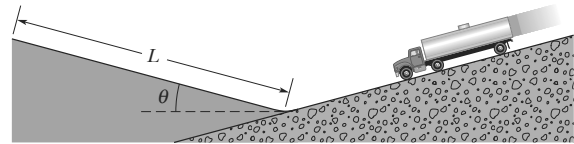


Figure 8-35 Problem 15.

- 16 A 700 g block is released from rest at height h_0 above a vertical spring with spring constant $k = 400$ N/m and negligible mass. The block sticks to the spring and momentarily stops after compressing the spring 19.0 cm. How much work is done (a) by the block on the spring and (b) by the spring on the block? (c) What is the value of h_0 ? (d) If the block were released from height $2.00h_0$ above the spring, what would be the maximum compression of the spring?

- 17 In Problem 6, what are the magnitudes of (a) the horizontal component and (b) the vertical component of the *net* force acting on the block at point Q? (c) At what height h should the block be released from rest so that it is on the verge of losing contact with the track at the top of the loop? (*On the verge of losing contact* means that the normal force on the block from the track has just then become zero.) (d) Graph the magnitude of the normal force on the block at the top of the loop versus initial height h , for the range $h = 0$ to $h = 6R$.

- 18 (a) In Problem 7, what is the speed of the ball at the lowest point? (b) Does the speed increase, decrease, or remain the same if the mass is increased?

- 19 **GO** Figure 8-36 shows an 8.00 kg stone at rest on a spring. The spring is compressed 10.0 cm by the stone. (a) What is the spring constant? (b) The stone is pushed down an additional 30.0 cm and released. What is the elastic potential energy of the compressed spring just before that release? (c) What is the change in the gravitational potential energy of the stone–Earth system when the stone moves from the release point to its maximum height? (d) What is that maximum height, measured from the release point?

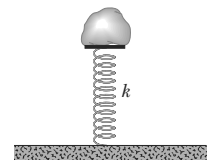



Figure 8-36
Problem 19.

- 20 **GO** A pendulum consists of a 2.0 kg stone swinging on a 4.0 m string of negligible mass. The stone has a speed of 8.0 m/s when it passes its lowest point. (a) What is the speed when the string is at 60° to the vertical? (b) What is the greatest angle with the vertical that the string will reach during the stone's motion? (c) If the potential energy of the pendulum–Earth system is taken to be zero at the stone's lowest point, what is the total mechanical energy of the system?

- 21 Figure 8-34 shows a pendulum of length $L = 1.25$ m. Its bob (which effectively has all the mass) has speed v_0 when the cord makes an angle $\theta_0 = 40.0^\circ$ with the vertical. (a) What is the speed of the bob when it is in its lowest position if $v_0 = 8.00$ m/s? What is the least value that v_0 can have if the pendulum is to swing down and then up (b) to a horizontal position, and (c) to a vertical position with the cord remaining straight? (d) Do the answers to (b) and (c) increase, decrease, or remain the same if θ_0 is increased by a few degrees?

- 22  A 60 kg skier starts from rest at height $H = 20$ m above the end of a ski-jump ramp (Fig. 8-37) and leaves the ramp at angle $\theta = 28^\circ$. Neglect the effects of air resistance and assume the ramp is frictionless. (a) What is the maximum height h of his jump above the end of the ramp? (b) If he increased his weight by putting on a backpack, would h then be greater, less, or the same?

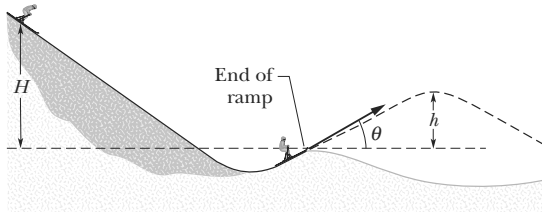



Figure 8-37 Problem 22.

- 23  The string in Fig. 8-38 is $L = 120$ cm long, has a ball attached to one end, and is fixed at its other end. The distance d from the fixed end to a fixed peg at point P is 75.0 cm. When the initially stationary ball is released with the string horizontal as shown, it will swing along the dashed arc. What is its speed when it reaches (a) its lowest point and (b) its highest point after the string catches on the peg?

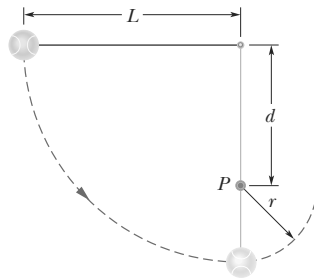


Figure 8-38 Problems 23 and 70.

- 24 A block of mass $m = 2.0$ kg is dropped from height $h = 40$ cm onto a spring of spring constant $k = 1960$ N/m (Fig. 8-39). Find the maximum distance the spring is compressed.

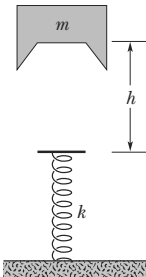


Figure 8-39 Problem 24.

- 25 At $t = 0$ a 1.0 kg ball is thrown from a tall tower with $\vec{v} = (18 \text{ m/s})\hat{i} + (24 \text{ m/s})\hat{j}$. What is ΔU of the ball–Earth system between $t = 0$ and $t = 6.0$ s (still free fall)?

- 26 A conservative force $\vec{F} = (6.0x - 12)\hat{i}$ N, where x is in meters, acts on a particle moving along an x axis. The potential energy U associated with this force is assigned a value of 27 J at $x = 0$. (a) Write an expression for U as a function of x , with U in joules and x in meters. (b) What is the maximum positive potential energy? At what (c) negative value and (d) positive value of x is the potential energy equal to zero?

- 27 Tarzan, who weighs 688 N, swings from a cliff at the end of a vine 18 m long (Fig. 8-40). From the top of the cliff to the bottom of the swing, he descends by 3.2 m. The vine will break if the force on it exceeds 950 N. (a) Does the vine break? (b) If no, what is the greatest force on it during the swing? If yes, at what angle with the vertical does it break?

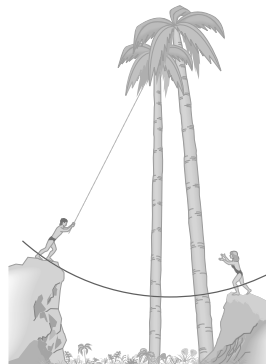
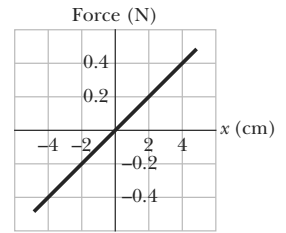
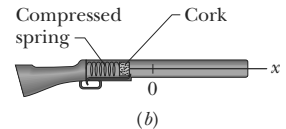


Figure 8-40 Problem 27.

- 28 Figure 8-41a applies to the spring in a cork gun (Fig. 8-41b); it shows the spring force as a function of the stretch or compression of the spring. The spring is compressed by 5.5 cm and used to propel a 3.8 g cork from the gun. (a) What is the speed of the cork if it is released as the spring passes through its relaxed position? (b) Suppose, instead, that the cork sticks to the spring and stretches it 1.5 cm before separation occurs. What now is the speed of the cork at the time of release?





(a)



(b)

Figure 8-41 Problem 28.

- 29   In Fig. 8-42, a block of mass $m = 12$ kg is released from rest on a frictionless incline of angle $\theta = 30^\circ$. Below the block is a spring that can be compressed 2.0 cm by a force of 270 N. The block momentarily stops when it compresses the spring by 5.5 cm. (a) How far does the block move down the incline from its rest position to this stopping point? (b) What is the speed of the block just as it touches the spring?

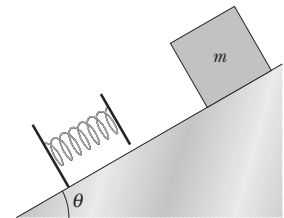



Figure 8-42 Problems 29 and 35.

- 30  A 2.0 kg breadbox on a frictionless incline of angle $\theta = 40^\circ$ is connected, by a cord that runs over a pulley, to a light spring of spring constant $k = 120$ N/m, as shown in Fig. 8-43. The box is released from rest when the spring is unstretched. Assume that the pulley is massless and frictionless. (a) What is the speed of the box when it has moved 10 cm down the incline? (b) How far down the incline from its point of release does the box slide before momentarily stopping, and what are the (c) magnitude and (d) direction (up or down the incline) of the box's acceleration at the instant the box momentarily stops?

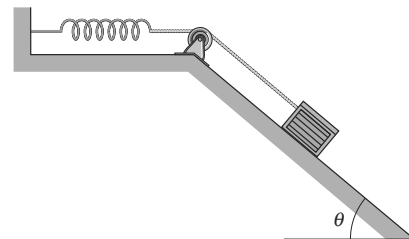



Figure 8-43 Problem 30.

- 31  A block with mass $m = 2.00$ kg is placed against a spring on a frictionless incline with angle $\theta = 30.0^\circ$ (Fig. 8-44). (The block is not attached to the spring.) The spring, with spring constant $k = 19.6$ N/cm, is compressed 20.0 cm and then released. (a) What is the elastic potential energy of the compressed spring? (b) What is the change in the gravitational potential energy of the block–Earth system as the block moves from the release point to its highest point on the incline? (c) How far along the incline is the highest point from the release point?

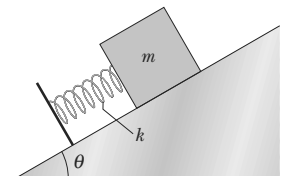


Figure 8-44 Problem 31.

••32 In Fig. 8-45, a chain is held on a frictionless table with one-fourth of its length hanging over the edge. If the chain has length $L = 28$ cm and mass $m = 0.012$ kg, how much work is required to pull the hanging part back onto the table?

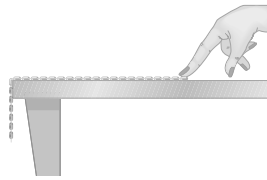


Figure 8-45 Problem 32.

••33 GO In Fig. 8-46, a spring with $k = 170$ N/m is at the top of a frictionless incline of angle $\theta = 37.0^\circ$. The lower end of the incline is distance $D = 1.00$ m from the end of the spring, which is at its relaxed length. A 2.00 kg canister is pushed against the spring until the spring is compressed 0.200 m and released from rest. (a) What is the speed of the canister at the instant the spring returns to its relaxed length (which is when the canister loses contact with the spring)? (b) What is the speed of the canister when it reaches the lower end of the incline?

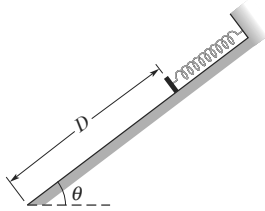


Figure 8-46 Problem 33.

••34 GO A boy is initially seated on the top of a hemispherical ice mound of radius $R = 13.8$ m. He begins to slide down the ice, with a negligible initial speed (Fig. 8-47). Approximate the ice as being frictionless. At what height does the boy lose contact with the ice?

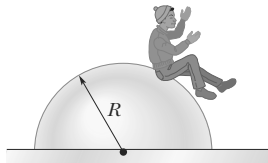


Figure 8-47 Problem 34.

••35 GO In Fig. 8-42, a block of mass $m = 3.20$ kg slides from rest a distance d down a frictionless incline at angle $\theta = 30.0^\circ$ where it runs into a spring of spring constant 431 N/m. When the block momentarily stops, it has compressed the spring by 21.0 cm. What are (a) distance d and (b) the distance between the point of the first block-spring contact and the point where the block's speed is greatest?

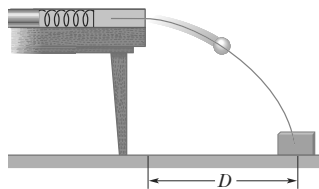


Figure 8-48 Problem 36.

••36 GO Two children are playing a game in which they try to hit a small box on the floor with a marble fired from a spring-loaded gun that is mounted on a table. The target box is horizontal distance $D = 2.20$ m from the edge of the table;

see Fig. 8-48. Bobby compresses the spring 1.10 cm, but the center of the marble falls 27.0 cm short of the center of the box. How far should Rhoda compress the spring to score a direct hit? Assume that neither the spring nor the ball encounters friction in the gun.

••37 A uniform cord of length 25 cm and mass 15 g is initially stuck to a ceiling. Later, it hangs vertically from the ceiling with only one end still stuck. What is the change in the gravitational potential energy of the cord with this change in orientation? (Hint: Consider a differential slice of the cord and then use integral calculus.)

Module 8-3 Reading a Potential Energy Curve

••38 Figure 8-49 shows a plot of potential energy U versus position x of a 0.200 kg particle that can travel only along an x axis under the influence of a conservative force. The graph has these

values: $U_A = 9.00$ J, $U_C = 20.00$ J, and $U_D = 24.00$ J. The particle is released at the point where U forms a “potential hill” of “height” $U_B = 12.00$ J, with kinetic energy 4.00 J. What is the speed of the particle at (a) $x = 3.5$ m and (b) $x = 6.5$ m? What is the position of the turning point on (c) the right side and (d) the left side?

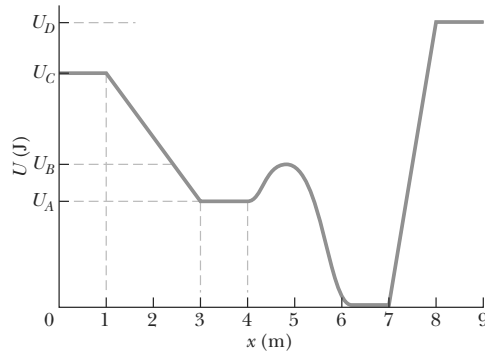


Figure 8-49 Problem 38.

••39 GO Figure 8-50 shows a plot of potential energy U versus position x of a 0.90 kg particle that can travel only along an x axis. (Nonconservative forces are not involved.) Three values are $U_A = 15.0$ J, $U_B = 35.0$ J, and $U_C = 45.0$ J. The particle is released at $x = 4.5$ m with an initial speed of 7.0 m/s, headed in the negative x direction.

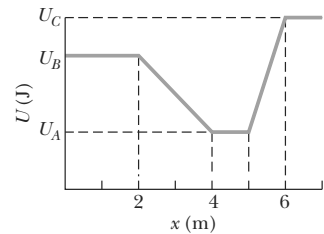


Figure 8-50 Problem 39.

(a) If the particle can reach $x = 1.0$ m, what is its speed there, and if it cannot, what is its turning point? What are the (b) magnitude and (c) direction of the force on the particle as it begins to move to the left of $x = 4.0$ m? Suppose, instead, the particle is headed in the positive x direction when it is released at $x = 4.5$ m at speed 7.0 m/s. (d) If the particle can reach $x = 7.0$ m, what is its speed there, and if it cannot, what is its turning point? What are the (e) magnitude and (f) direction of the force on the particle as it begins to move to the right of $x = 5.0$ m?

••40 The potential energy of a diatomic molecule (a two-atom system like H_2 or O_2) is given by

$$U = \frac{A}{r^{12}} - \frac{B}{r^6},$$

where r is the separation of the two atoms of the molecule and A and B are positive constants. This potential energy is associated with the force that binds the two atoms together. (a) Find the *equilibrium separation*—that is, the distance between the atoms at which the force on each atom is zero. Is the force repulsive (the atoms are pushed apart) or attractive (they are pulled together) if their separation is (b) smaller and (c) larger than the equilibrium separation?

••41 A single conservative force $F(x)$ acts on a 1.0 kg particle that moves along an x axis. The potential energy $U(x)$ associated with $F(x)$ is given by

$$U(x) = -4x e^{-x/4} \text{ J},$$

where x is in meters. At $x = 5.0$ m the particle has a kinetic energy of 2.0 J. (a) What is the mechanical energy of the system? (b) Make

a plot of $U(x)$ as a function of x for $0 \leq x \leq 10$ m, and on the same graph draw the line that represents the mechanical energy of the system. Use part (b) to determine (c) the least value of x the particle can reach and (d) the greatest value of x the particle can reach. Use part (b) to determine (e) the maximum kinetic energy of the particle and (f) the value of x at which it occurs. (g) Determine an expression in newtons and meters for $F(x)$ as a function of x . (h) For what (finite) value of x does $F(x) = 0$?

Module 8-4 Work Done on a System by an External Force

•42 A worker pushed a 27 kg block 9.2 m along a level floor at constant speed with a force directed 32° below the horizontal. If the coefficient of kinetic friction between block and floor was 0.20, what were (a) the work done by the worker's force and (b) the increase in thermal energy of the block–floor system?

•43 A collie drags its bed box across a floor by applying a horizontal force of 8.0 N. The kinetic frictional force acting on the box has magnitude 5.0 N. As the box is dragged through 0.70 m along the way, what are (a) the work done by the collie's applied force and (b) the increase in thermal energy of the bed and floor?

•44 A horizontal force of magnitude 35.0 N pushes a block of mass 4.00 kg across a floor where the coefficient of kinetic friction is 0.600. (a) How much work is done by that applied force on the block–floor system when the block slides through a displacement of 3.00 m across the floor? (b) During that displacement, the thermal energy of the block increases by 40.0 J. What is the increase in thermal energy of the floor? (c) What is the increase in the kinetic energy of the block?

•45 **SSM** A rope is used to pull a 3.57 kg block at constant speed 4.06 m along a horizontal floor. The force on the block from the rope is 7.68 N and directed 15.0° above the horizontal. What are (a) the work done by the rope's force, (b) the increase in thermal energy of the block–floor system, and (c) the coefficient of kinetic friction between the block and floor?

Module 8-5 Conservation of Energy

•46 An outfielder throws a baseball with an initial speed of 81.8 mi/h. Just before an infielder catches the ball at the same level, the ball's speed is 110 ft/s. In foot-pounds, by how much is the mechanical energy of the ball–Earth system reduced because of air drag? (The weight of a baseball is 9.0 oz.)

•47 A 75 g Frisbee is thrown from a point 1.1 m above the ground with a speed of 12 m/s. When it has reached a height of 2.1 m, its speed is 10.5 m/s. What was the reduction in E_{mec} of the Frisbee–Earth system because of air drag?

•48 In Fig. 8-51, a block slides down an incline. As it moves from point A to point B, which are 5.0 m apart, force \vec{F} acts on the block, with magnitude 2.0 N and directed down the incline. The magnitude of the frictional force acting on the block is 10 N. If the kinetic energy of the block increases by 35 J between A and B, how much work is done on the block by the gravitational force as the block moves from A to B?

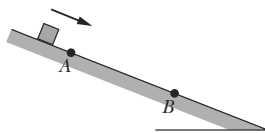


Figure 8-51 Problems 48 and 71.

•49 **SSM ILW** A 25 kg bear slides, from rest, 12 m down a lodgepole pine tree, moving with a speed of 5.6 m/s just before hitting the ground. (a) What change occurs in the gravitational

potential energy of the bear–Earth system during the slide? (b) What is the kinetic energy of the bear just before hitting the ground? (c) What is the average frictional force that acts on the sliding bear?

•50 **ILW** A 60 kg skier leaves the end of a ski-jump ramp with a velocity of 24 m/s directed 25° above the horizontal. Suppose that as a result of air drag the skier returns to the ground with a speed of 22 m/s, landing 14 m vertically below the end of the ramp. From the launch to the return to the ground, by how much is the mechanical energy of the skier–Earth system reduced because of air drag?

•51 During a rockslide, a 520 kg rock slides from rest down a hillside that is 500 m long and 300 m high. The coefficient of kinetic friction between the rock and the hill surface is 0.25. (a) If the gravitational potential energy U of the rock–Earth system is zero at the bottom of the hill, what is the value of U just before the slide? (b) How much energy is transferred to thermal energy during the slide? (c) What is the kinetic energy of the rock as it reaches the bottom of the hill? (d) What is its speed then?

•52 A large fake cookie sliding on a horizontal surface is attached to one end of a horizontal spring with spring constant $k = 400$ N/m; the other end of the spring is fixed in place. The cookie has a kinetic energy of 20.0 J as it passes through the spring's equilibrium position. As the cookie slides, a frictional force of magnitude 10.0 N acts on it. (a) How far will the cookie slide from the equilibrium position before coming momentarily to rest? (b) What will be the kinetic energy of the cookie as it slides back through the equilibrium position?

•53 **GO** In Fig. 8-52, a 3.5 kg block is accelerated from rest by a compressed spring of spring constant 640 N/m. The block leaves the spring at the spring's relaxed length and then travels over a horizontal floor with a coefficient of kinetic friction $\mu_k = 0.25$. The frictional force stops the block in distance $D = 7.8$ m. What are (a) the increase in the thermal energy of the block–floor system, (b) the maximum kinetic energy of the block, and (c) the original compression distance of the spring?

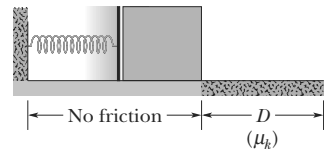


Figure 8-52 Problem 53.

•54 A child whose weight is 267 N slides down a 6.1 m playground slide that makes an angle of 20° with the horizontal. The coefficient of kinetic friction between slide and child is 0.10. (a) How much energy is transferred to thermal energy? (b) If she starts at the top with a speed of 0.457 m/s, what is her speed at the bottom?

•55 **ILW** In Fig. 8-53, a block of mass $m = 2.5$ kg slides head on into a spring of spring constant $k = 320$ N/m. When the block stops, it has compressed the spring by 7.5 cm. The coefficient of kinetic friction between block and floor is 0.25. While the block is in contact with the spring and being brought to rest, what are (a) the work done by the spring force and (b) the increase in thermal energy of the block–floor system? (c) What is the block's speed just as it reaches the spring?

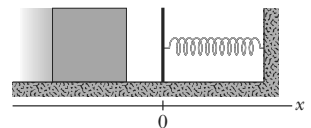


Figure 8-53 Problem 55.

•56 You push a 2.0 kg block against a horizontal spring, compressing the spring by 15 cm. Then you release the block, and the

spring sends it sliding across a tabletop. It stops 75 cm from where you released it. The spring constant is 200 N/m. What is the block–table coefficient of kinetic friction?

••57 **GO** In Fig. 8-54, a block slides along a track from one level to a higher level after passing through an intermediate valley. The track is frictionless until the block reaches the higher level. There a frictional force stops the block in a distance d . The block's initial speed v_0 is 6.0 m/s, the height difference h is 1.1 m, and μ_k is 0.60. Find d .

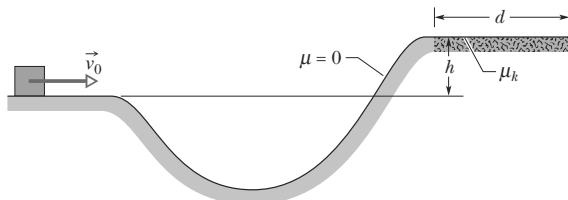


Figure 8-54 Problem 57.

••58 A cookie jar is moving up a 40° incline. At a point 55 cm from the bottom of the incline (measured along the incline), the jar has a speed of 1.4 m/s. The coefficient of kinetic friction between jar and incline is 0.15. (a) How much farther up the incline will the jar move? (b) How fast will it be going when it has slid back to the bottom of the incline? (c) Do the answers to (a) and (b) increase, decrease, or remain the same if we decrease the coefficient of kinetic friction (but do not change the given speed or location)?

••59 A stone with a weight of 5.29 N is launched vertically from ground level with an initial speed of 20.0 m/s, and the air drag on it is 0.265 N throughout the flight. What are (a) the maximum height reached by the stone and (b) its speed just before it hits the ground?

••60 A 4.0 kg bundle starts up a 30° incline with 128 J of kinetic energy. How far will it slide up the incline if the coefficient of kinetic friction between bundle and incline is 0.30?

••61 **GO** When a click beetle is upside down on its back, it jumps upward by suddenly arching its back, transferring energy stored in a muscle to mechanical energy. This launching mechanism produces an audible click, giving the beetle its name. Videotape of a certain click-beetle jump shows that a beetle of mass $m = 4.0 \times 10^{-6}$ kg moved directly upward by 0.77 mm during the launch and then to a maximum height of $h = 0.30$ m. During the launch, what are the average magnitudes of (a) the external force on the beetle's back from the floor and (b) the acceleration of the beetle in terms of g ?

••62 **GO** In Fig. 8-55, a block slides along a path that is without friction until the block reaches the section of length $L = 0.75$ m, which begins at height $h = 2.0$ m on a ramp of angle $\theta = 30^\circ$. In that section, the coefficient of kinetic friction is 0.40. The block passes through point A with a speed of 8.0 m/s. If the block can reach point B (where the friction ends), what is its speed there, and if it cannot, what is its greatest height above A?

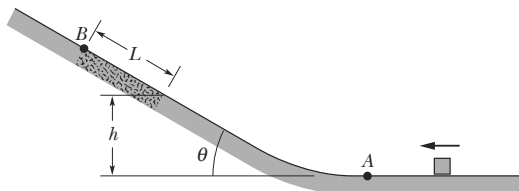


Figure 8-55 Problem 62.

••63 The cable of the 1800 kg elevator cab in Fig. 8-56 snaps when the cab is at rest at the first floor, where the cab bottom is a distance $d = 3.7$ m above a spring of spring constant $k = 0.15$ MN/m. A safety device clamps the cab against guide rails so that a constant frictional force of 4.4 kN opposes the cab's motion. (a) Find the speed of the cab just before it hits the spring. (b) Find the maximum distance x that the spring is compressed (the frictional force still acts during this compression). (c) Find the distance that the cab will bounce back up the shaft. (d) Using conservation of energy, find the approximate total distance that the cab will move before coming to rest. (Assume that the frictional force on the cab is negligible when the cab is stationary.)

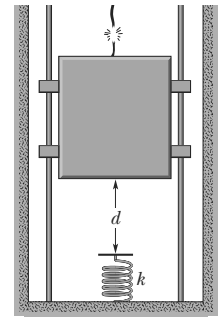


Figure 8-56 Problem 63.

••64 **GO** In Fig. 8-57, a block is released from rest at height $d = 40$ cm and slides down a frictionless ramp and onto a first plateau, which has length d and where the coefficient of kinetic friction is 0.50. If the block is still moving, it then slides down a second frictionless ramp through height $d/2$ and onto a lower plateau, which has length $d/2$ and where the coefficient of kinetic friction is again 0.50. If the block is still moving, it then slides up a frictionless ramp until it (momentarily) stops. Where does the block stop? If its final stop is on a plateau, state which one and give the distance L from the left edge of that plateau. If the block reaches the ramp, give the height H above the lower plateau where it momentarily stops.

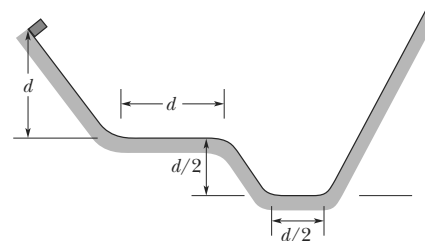


Figure 8-57 Problem 64.

••65 **GO** A particle can slide along a track with elevated ends and a flat central part, as shown in Fig. 8-58. The flat part has length $L = 40$ cm. The curved portions of the track are frictionless, but for the flat part the coefficient of kinetic friction is $\mu_k = 0.20$. The particle is released from rest at point A, which is at height $h = L/2$. How far from the left edge of the flat part does the particle finally stop?

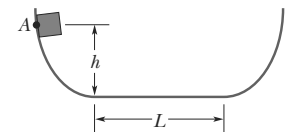


Figure 8-58 Problem 65.

Additional Problems

66 A 3.2 kg sloth hangs 3.0 m above the ground. (a) What is the gravitational potential energy of the sloth–Earth system if we take the reference point $y = 0$ to be at the ground? If the sloth drops to the ground and air drag on it is assumed to be negligible, what are the (b) kinetic energy and (c) speed of the sloth just before it reaches the ground?

67 SSM A spring ($k = 200 \text{ N/m}$) is fixed at the top of a frictionless plane inclined at angle $\theta = 40^\circ$ (Fig. 8-59). A 1.0 kg block is projected up the plane, from an initial position that is distance $d = 0.60 \text{ m}$ from the end of the relaxed spring, with an initial kinetic energy of 16 J . (a) What is the kinetic energy of the block at the instant it has compressed the spring 0.20 m ? (b) With what kinetic energy must the block be projected up the plane if it is to stop momentarily when it has compressed the spring by 0.40 m ?

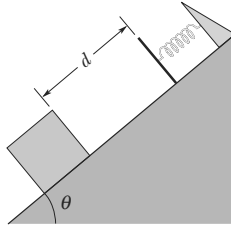


Figure 8-59 Problem 67.

68 From the edge of a cliff, a 0.55 kg projectile is launched with an initial kinetic energy of 1550 J . The projectile's maximum upward displacement from the launch point is $+140 \text{ m}$. What are the (a) horizontal and (b) vertical components of its launch velocity? (c) At the instant the vertical component of its velocity is 65 m/s , what is its vertical displacement from the launch point?

69 SSM In Fig. 8-60, the pulley has negligible mass, and both it and the inclined plane are frictionless. Block A has a mass of 1.0 kg , block B has a mass of 2.0 kg , and angle θ is 30° . If the blocks are released from rest with the connecting cord taut, what is their total kinetic energy when block B has fallen 25 cm ?

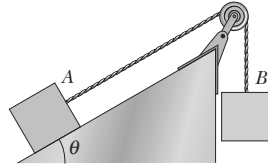


Figure 8-60 Problem 69.

70 GO In Fig. 8-38, the string is $L = 120 \text{ cm}$ long, has a ball attached to one end, and is fixed at its other end. A fixed peg is at point P. Released from rest, the ball swings down until the string catches on the peg; then the ball swings up, around the peg. If the ball is to swing completely around the peg, what value must distance d exceed? (Hint: The ball must still be moving at the top of its swing. Do you see why?)

71 SSM In Fig. 8-51, a block is sent sliding down a frictionless ramp. Its speeds at points A and B are 2.00 m/s and 2.60 m/s , respectively. Next, it is again sent sliding down the ramp, but this time its speed at point A is 4.00 m/s . What then is its speed at point B?

72 Two snowy peaks are at heights $H = 850 \text{ m}$ and $h = 750 \text{ m}$ above the valley between them. A ski run extends between the peaks, with a total length of 3.2 km and an average slope of $\theta = 30^\circ$ (Fig. 8-61). (a) A skier starts from rest at the top of the higher peak. At what speed will he arrive at the top of the lower peak if he coasts without using ski poles? Ignore friction. (b) Approximately what coefficient of kinetic friction

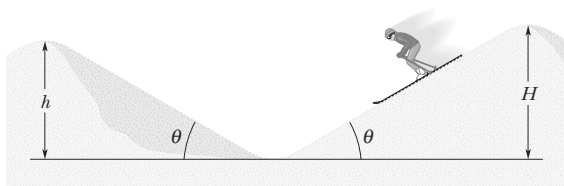


Figure 8-61 Problem 72.

between snow and skis would make him stop just at the top of the lower peak?

73 SSM The temperature of a plastic cube is monitored while the cube is pushed 3.0 m across a floor at constant speed by a horizontal force of 15 N . The thermal energy of the cube increases by 20 J . What is the increase in the thermal energy of the floor along which the cube slides?

74 A skier weighing 600 N goes over a frictionless circular hill of radius $R = 20 \text{ m}$ (Fig. 8-62). Assume that the effects of air resistance on the skier are negligible. As she comes up the hill, her speed is 8.0 m/s at point B, at angle $\theta = 20^\circ$. (a) What is her speed at the hilltop (point A) if she coasts without using her poles? (b) What minimum speed can she have at B and still coast to the hilltop? (c) Do the answers to these two questions increase, decrease, or remain the same if the skier weighs 700 N instead of 600 N ?

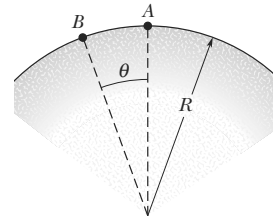


Figure 8-62 Problem 74.

75 SSM To form a pendulum, a 0.092 kg ball is attached to one end of a rod of length 0.62 m and negligible mass, and the other end of the rod is mounted on a pivot. The rod is rotated until it is straight up, and then it is released from rest so that it swings down around the pivot. When the ball reaches its lowest point, what are (a) its speed and (b) the tension in the rod? Next, the rod is rotated until it is horizontal, and then it is again released from rest. (c) At what angle from the vertical does the tension in the rod equal the weight of the ball? (d) If the mass of the ball is increased, does the answer to (c) increase, decrease, or remain the same?

76 We move a particle along an x axis, first outward from $x = 1.0 \text{ m}$ to $x = 4.0 \text{ m}$ and then back to $x = 1.0 \text{ m}$, while an external force acts on it. That force is directed along the x axis, and its x component can have different values for the outward trip and for the return trip. Here are the values (in newtons) for four situations, where x is in meters:

Outward	Inward
(a) $+3.0$	-3.0
(b) $+5.0$	$+5.0$
(c) $+2.0x$	$-2.0x$
(d) $+3.0x^2$	$+3.0x^2$

Find the net work done on the particle by the external force for the round trip for each of the four situations. (e) For which, if any, is the external force conservative?

77 SSM A conservative force $F(x)$ acts on a 2.0 kg particle that moves along an x axis. The potential energy $U(x)$ associated with $F(x)$ is graphed in Fig. 8-63. When the particle is at $x = 2.0 \text{ m}$, its

velocity is -1.5 m/s. What are the (a) magnitude and (b) direction of $F(x)$ at this position? Between what positions on the (c) left and (d) right does the particle move? (e) What is the particle's speed at $x = 7.0$ m?

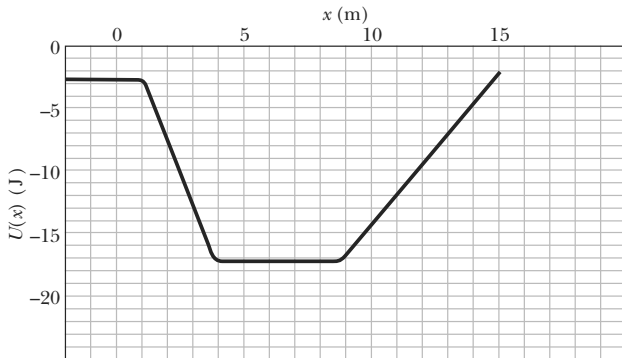


Figure 8-63 Problem 77.

78 At a certain factory, 300 kg crates are dropped vertically from a packing machine onto a conveyor belt moving at 1.20 m/s (Fig. 8-64). (A motor maintains the belt's constant speed.) The coefficient of kinetic friction between the belt and each crate is 0.400. After a short time, slipping between the belt and the crate ceases, and the crate then moves along with the belt. For the period of time during which the crate is being brought to rest relative to the belt, calculate, for a coordinate system at rest in the factory, (a) the kinetic energy supplied to the crate, (b) the magnitude of the kinetic frictional force acting on the crate, and (c) the energy supplied by the motor. (d) Explain why answers (a) and (c) differ.

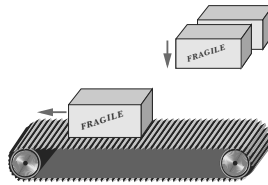


Figure 8-64 Problem 78.

79 SSM A 1500 kg car begins sliding down a 5.0° inclined road with a speed of 30 km/h. The engine is turned off, and the only forces acting on the car are a net frictional force from the road and the gravitational force. After the car has traveled 50 m along the road, its speed is 40 km/h. (a) How much is the mechanical energy of the car reduced because of the net frictional force? (b) What is the magnitude of that net frictional force?

80 GO In Fig. 8-65, a 1400 kg block of granite is pulled up an incline at a constant speed of 1.34 m/s by a cable and winch. The indicated distances are $d_1 = 40$ m and $d_2 = 30$ m. The coefficient of kinetic friction between the block and the incline is 0.40. What is the power due to the force applied to the block by the cable?

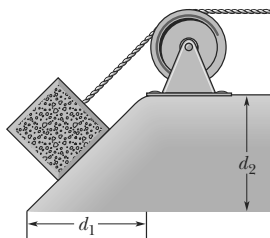


Figure 8-65 Problem 80.

81 A particle can move along only an x axis, where conservative forces act on it (Fig. 8-66 and the following table). The particle is released at $x = 5.00$ m with a kinetic energy of $K = 14.0$ J and a potential energy of $U = 0$. If its motion is in the negative direction of the x axis, what are its (a) K and (b) U at $x = 2.00$ m and its (c) K and (d) U at $x = 0$? If its motion is in the positive direction of the x axis, what are its (e) K and (f) U at $x = 11.0$ m, its (g) K and (h) U at $x = 12.0$ m, and its (i) K and (j) U at $x = 13.0$ m? (k) Plot $U(x)$ versus x for the range $x = 0$ to $x = 13.0$ m.

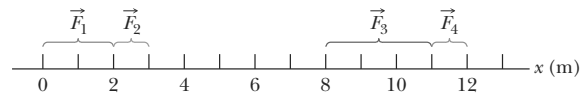


Figure 8-66 Problems 81 and 82.

Next, the particle is released from rest at $x = 0$. What are (l) its kinetic energy at $x = 5.0$ m and (m) the maximum positive position x_{max} it reaches? (n) What does the particle do after it reaches x_{max} ?


Range	Force
0 to 2.00 m	$\vec{F}_1 = +(3.00 \text{ N})\hat{i}$
2.00 m to 3.00 m	$\vec{F}_2 = +(5.00 \text{ N})\hat{i}$
3.00 m to 8.00 m	$F = 0$
8.00 m to 11.0 m	$\vec{F}_3 = -(4.00 \text{ N})\hat{i}$
11.0 m to 12.0 m	$\vec{F}_4 = -(1.00 \text{ N})\hat{i}$
12.0 m to 15.0 m	$F = 0$

82 For the arrangement of forces in Problem 81, a 2.00 kg particle is released at $x = 5.00$ m with an initial velocity of 3.45 m/s in the negative direction of the x axis. (a) If the particle can reach $x = 0$ m, what is its speed there, and if it cannot, what is its turning point? Suppose, instead, the particle is headed in the positive x direction when it is released at $x = 5.00$ m at speed 3.45 m/s. (b) If the particle can reach $x = 13.0$ m, what is its speed there, and if it cannot, what is its turning point?

83 SSM A 15 kg block is accelerated at 2.0 m/s^2 along a horizontal frictionless surface, with the speed increasing from 10 m/s to 30 m/s. What are (a) the change in the block's mechanical energy and (b) the average rate at which energy is transferred to the block? What is the instantaneous rate of that transfer when the block's speed is (c) 10 m/s and (d) 30 m/s?

84 A certain spring is found *not* to conform to Hooke's law. The force (in newtons) it exerts when stretched a distance x (in meters) is found to have magnitude $52.8x + 38.4x^2$ in the direction opposing the stretch. (a) Compute the work required to stretch the spring from $x = 0.500$ m to $x = 1.00$ m. (b) With one end of the spring fixed, a particle of mass 2.17 kg is attached to the other end of the spring when it is stretched by an amount $x = 1.00$ m. If the particle is then released from rest, what is its speed at the instant the stretch in the spring is $x = 0.500$ m? (c) Is the force exerted by the spring conservative or nonconservative? Explain.

85 SSM Each second, 1200 m^3 of water passes over a waterfall 100 m high. Three-fourths of the kinetic energy gained by the water in falling is transferred to electrical energy by a hydroelectric generator. At what rate does the generator produce electrical energy? (The mass of 1 m^3 of water is 1000 kg.)

86  In Fig. 8-67, a small block is sent through point *A* with a speed of 7.0 m/s. Its path is without friction until it reaches the section of length $L = 12$ m, where the coefficient of kinetic friction is 0.70. The indicated heights are $h_1 = 6.0$ m and $h_2 = 2.0$ m. What are the speeds of the block at (a) point *B* and (b) point *C*? (c) Does the block reach point *D*? If so, what is its speed there; if not, how far through the section of friction does it travel?

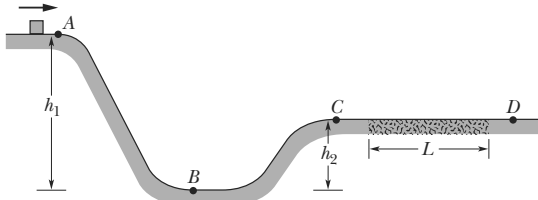



Figure 8-67 Problem 86.

87  A massless rigid rod of length L has a ball of mass m attached to one end (Fig. 8-68). The other end is pivoted in such a way that the ball will move in a vertical circle. First, assume that there is no friction at the pivot. The system is launched downward from the horizontal position *A* with initial speed v_0 . The ball just barely reaches point *D* and then stops. (a) Derive an expression for v_0 in terms of L , m , and g . (b) What is the tension in the rod when the ball passes through *B*? (c) A little grit is placed on the pivot to increase the friction there. Then the ball just barely reaches *C* when launched from *A* with the same speed as before. What is the decrease in the mechanical energy during this motion? (d) What is the decrease in the mechanical energy by the time the ball finally comes to rest at *B* after several oscillations?

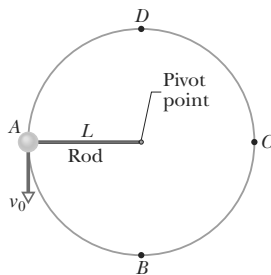



Figure 8-68 Problem 87.

88 A 1.50 kg water balloon is shot straight up with an initial speed of 3.00 m/s. (a) What is the kinetic energy of the balloon just as it is launched? (b) How much work does the gravitational force do on the balloon during the balloon's full ascent? (c) What is the change in the gravitational potential energy of the balloon–Earth system during the full ascent? (d) If the gravitational potential energy is taken to be zero at the launch point, what is its value when the balloon reaches its maximum height? (e) If, instead, the gravitational potential energy is taken to be zero at the maximum height, what is its value at the launch point? (f) What is the maximum height?

89 A 2.50 kg beverage can is thrown directly downward from a height of 4.00 m, with an initial speed of 3.00 m/s. The air drag on the can is negligible. What is the kinetic energy of the can (a) as it reaches the ground at the end of its fall and (b) when it is halfway to the ground? What are (c) the kinetic energy of the can and (d) the gravitational potential energy of the can–Earth system 0.200 s before the can reaches the ground? For the latter, take the reference point $y = 0$ to be at the ground.

90 A constant horizontal force moves a 50 kg trunk 6.0 m up a 30° incline at constant speed. The coefficient of kinetic friction is 0.20. What are (a) the work done by the applied force and (b) the increase in the thermal energy of the trunk and incline?

91  Two blocks, of masses $M = 2.0$ kg and $2M$, are connected to a spring of spring constant $k = 200$ N/m that has one end fixed, as shown in Fig. 8-69. The horizontal surface and the pulley are frictionless, and the pulley has negligible mass. The blocks are released from rest with the spring relaxed. (a) What is the combined kinetic energy of the two blocks when the hanging block has fallen 0.090 m? (b) What is the kinetic energy of the hanging block when it has fallen that 0.090 m? (c) What maximum distance does the hanging block fall before momentarily stopping?

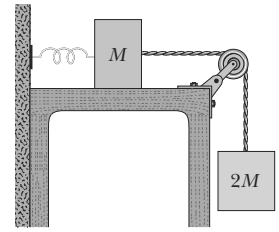


Figure 8-69 Problem 91.

92 A volcanic ash flow is moving across horizontal ground when it encounters a 10° upslope. The front of the flow then travels 920 m up the slope before stopping. Assume that the gases entrapped in the flow lift the flow and thus make the frictional force from the ground negligible; assume also that the mechanical energy of the front of the flow is conserved. What was the initial speed of the front of the flow?

93 A playground slide is in the form of an arc of a circle that has a radius of 12 m. The maximum height of the slide is $h = 4.0$ m, and the ground is tangent to the circle (Fig. 8-70). A 25 kg child starts from rest at the top of the slide and has a speed of 6.2 m/s at the bottom. (a) What is the length of the slide? (b) What average frictional force acts on the child over this distance? If, instead of the ground, a vertical line through the *top of the slide* is tangent to the circle, what are (c) the length of the slide and (d) the average frictional force on the child?

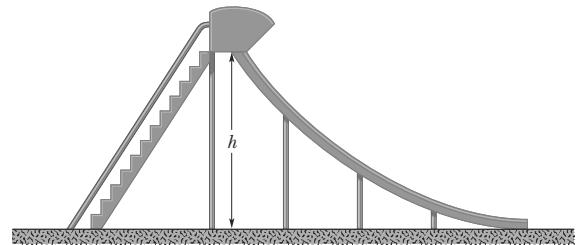


Figure 8-70 Problem 93.

94 The luxury liner *Queen Elizabeth 2* has a diesel-electric power plant with a maximum power of 92 MW at a cruising speed of 32.5 knots. What forward force is exerted on the ship at this speed? (1 knot = 1.852 km/h.)

95 A factory worker accidentally releases a 180 kg crate that was being held at rest at the top of a ramp that is 3.7 m long and inclined at 39° to the horizontal. The coefficient of kinetic friction between the crate and the ramp, and between the crate and the horizontal factory floor, is 0.28. (a) How fast is the crate moving as it reaches the bottom of the ramp? (b) How far will it subsequently slide across the floor? (Assume that the crate's kinetic energy does not change as it moves from the ramp onto the floor.) (c) Do the answers to (a) and (b) increase, decrease, or remain the same if we halve the mass of the crate?

96 If a 70 kg baseball player steals home by sliding into the plate with an initial speed of 10 m/s just as he hits the ground, (a) what

is the decrease in the player's kinetic energy and (b) what is the increase in the thermal energy of his body and the ground along which he slides?

97 A 0.50 kg banana is thrown directly upward with an initial speed of 4.00 m/s and reaches a maximum height of 0.80 m. What change does air drag cause in the mechanical energy of the banana–Earth system during the ascent?

98 A metal tool is sharpened by being held against the rim of a wheel on a grinding machine by a force of 180 N. The frictional forces between the rim and the tool grind off small pieces of the tool. The wheel has a radius of 20.0 cm and rotates at 2.50 rev/s. The coefficient of kinetic friction between the wheel and the tool is 0.320. At what rate is energy being transferred from the motor driving the wheel to the thermal energy of the wheel and tool and to the kinetic energy of the material thrown from the tool?

99 A swimmer moves through the water at an average speed of 0.22 m/s. The average drag force is 110 N. What average power is required of the swimmer?

100 An automobile with passengers has weight 16 400 N and is moving at 113 km/h when the driver brakes, sliding to a stop. The frictional force on the wheels from the road has a magnitude of 8230 N. Find the stopping distance.

101 A 0.63 kg ball thrown directly upward with an initial speed of 14 m/s reaches a maximum height of 8.1 m. What is the change in the mechanical energy of the ball–Earth system during the ascent of the ball to that maximum height?

102 The summit of Mount Everest is 8850 m above sea level. (a) How much energy would a 90 kg climber expend against the gravitational force on him in climbing to the summit from sea level? (b) How many candy bars, at 1.25 MJ per bar, would supply an energy equivalent to this? Your answer should suggest that work done against the gravitational force is a very small part of the energy expended in climbing a mountain.

103 A sprinter who weighs 670 N runs the first 7.0 m of a race in 1.6 s, starting from rest and accelerating uniformly. What are the sprinter's (a) speed and (b) kinetic energy at the end of the 1.6 s? (c) What average power does the sprinter generate during the 1.6 s interval?

104 A 20 kg object is acted on by a conservative force given by $F = -3.0x - 5.0x^2$, with F in newtons and x in meters. Take the potential energy associated with the force to be zero when the object is at $x = 0$. (a) What is the potential energy of the system associated with the force when the object is at $x = 2.0$ m? (b) If the object has a velocity of 4.0 m/s in the negative direction of the x axis when it is at $x = 5.0$ m, what is its speed when it passes through the origin? (c) What are the answers to (a) and (b) if the potential energy of the system is taken to be -8.0 J when the object is at $x = 0$?

105 A machine pulls a 40 kg trunk 2.0 m up a 40° ramp at constant velocity, with the machine's force on the trunk directed parallel to the ramp. The coefficient of kinetic friction between the trunk and the ramp is 0.40. What are (a) the work done on the trunk by the machine's force and (b) the increase in thermal energy of the trunk and the ramp?

106 The spring in the muzzle of a child's spring gun has a spring constant of 700 N/m. To shoot a ball from the gun, first the spring is compressed and then the ball is placed on it. The gun's trigger then

releases the spring, which pushes the ball through the muzzle. The ball leaves the spring just as it leaves the outer end of the muzzle. When the gun is inclined upward by 30° to the horizontal, a 57 g ball is shot to a maximum height of 1.83 m above the gun's muzzle. Assume air drag on the ball is negligible. (a) At what speed does the spring launch the ball? (b) Assuming that friction on the ball within the gun can be neglected, find the spring's initial compression distance.

107 The only force acting on a particle is conservative force \vec{F} . If the particle is at point A , the potential energy of the system associated with \vec{F} and the particle is 40 J. If the particle moves from point A to point B , the work done on the particle by \vec{F} is +25 J. What is the potential energy of the system with the particle at B ?

108 In 1981, Daniel Goodwin climbed 443 m up the *exterior* of the Sears Building in Chicago using suction cups and metal clips. (a) Approximate his mass and then compute how much energy he had to transfer from biomechanical (internal) energy to the gravitational potential energy of the Earth–Goodwin system to lift himself to that height. (b) How much energy would he have had to transfer if he had, instead, taken the stairs inside the building (to the same height)?

109 A 60.0 kg circus performer slides 4.00 m down a pole to the circus floor, starting from rest. What is the kinetic energy of the performer as she reaches the floor if the frictional force on her from the pole (a) is negligible (she will be hurt) and (b) has a magnitude of 500 N?

110 A 5.0 kg block is projected at 5.0 m/s up a plane that is inclined at 30° with the horizontal. How far up along the plane does the block go (a) if the plane is frictionless and (b) if the coefficient of kinetic friction between the block and the plane is 0.40? (c) In the latter case, what is the increase in thermal energy of block and plane during the block's ascent? (d) If the block then slides back down against the frictional force, what is the block's speed when it reaches the original projection point?

111 A 9.40 kg projectile is fired vertically upward. Air drag decreases the mechanical energy of the projectile–Earth system by 68.0 kJ during the projectile's ascent. How much higher would the projectile have gone were air drag negligible?

112 A 70.0 kg man jumping from a window lands in an elevated fire rescue net 11.0 m below the window. He momentarily stops when he has stretched the net by 1.50 m. Assuming that mechanical energy is conserved during this process and that the net functions like an ideal spring, find the elastic potential energy of the net when it is stretched by 1.50 m.

113 A 30 g bullet moving a horizontal velocity of 500 m/s comes to a stop 12 cm within a solid wall. (a) What is the change in the bullet's mechanical energy? (b) What is the magnitude of the average force from the wall stopping it?

114 A 1500 kg car starts from rest on a horizontal road and gains a speed of 72 km/h in 30 s. (a) What is its kinetic energy at the end of the 30 s? (b) What is the average power required of the car during the 30 s interval? (c) What is the instantaneous power at the end of the 30 s interval, assuming that the acceleration is constant?

115 A 1.50 kg snowball is shot upward at an angle of 34.0° to the horizontal with an initial speed of 20.0 m/s. (a) What is its initial kinetic energy? (b) By how much does the gravitational potential

energy of the snowball–Earth system change as the snowball moves from the launch point to the point of maximum height? (c) What is that maximum height?

116 A 68 kg sky diver falls at a constant terminal speed of 59 m/s. (a) At what rate is the gravitational potential energy of the Earth–sky diver system being reduced? (b) At what rate is the system’s mechanical energy being reduced?

117 A 20 kg block on a horizontal surface is attached to a horizontal spring of spring constant $k = 4.0$ kN/m. The block is pulled to the right so that the spring is stretched 10 cm beyond its relaxed length, and the block is then released from rest. The frictional force between the sliding block and the surface has a magnitude of 80 N. (a) What is the kinetic energy of the block when it has moved 2.0 cm from its point of release? (b) What is the kinetic energy of the block when it first slides back through the point at which the spring is relaxed? (c) What is the maximum kinetic energy attained by the block as it slides from its point of release to the point at which the spring is relaxed?

118 Resistance to the motion of an automobile consists of road friction, which is almost independent of speed, and air drag, which is proportional to speed-squared. For a certain car with a weight of 12 000 N, the total resistant force F is given by $F = 300 + 1.8v^2$, with F in newtons and v in meters per second. Calculate the power (in horsepower) required to accelerate the car at 0.92 m/s² when the speed is 80 km/h.

119 SSM A 50 g ball is thrown from a window with an initial velocity of 8.0 m/s at an angle of 30° above the horizontal. Using energy methods, determine (a) the kinetic energy of the ball at the top of its flight and (b) its speed when it is 3.0 m below the window. Does the answer to (b) depend on either (c) the mass of the ball or (d) the initial angle?

120 A spring with a spring constant of 3200 N/m is initially stretched until the elastic potential energy of the spring is 1.44 J. ($U = 0$ for the relaxed spring.) What is ΔU if the initial stretch is changed to (a) a stretch of 2.0 cm, (b) a compression of 2.0 cm, and (c) a compression of 4.0 cm?

121 A locomotive with a power capability of 1.5 MW can accelerate a train from a speed of 10 m/s to 25 m/s in 6.0 min. (a) Calculate the mass of the train. Find (b) the speed of the train and (c) the force accelerating the train as functions of time (in seconds) during the 6.0 min interval. (d) Find the distance moved by the train during the interval.

122 SSM A 0.42 kg shuffleboard disk is initially at rest when a player uses a cue to increase its speed to 4.2 m/s at constant acceleration. The acceleration takes place over a 2.0 m distance, at the end of which the cue loses contact with the disk. Then the disk slides an additional 12 m before stopping. Assume that the shuffleboard court is level and that the force of friction on the disk is constant. What is the increase in the thermal energy of the disk–court system (a) for that additional 12 m and (b) for the entire 14 m distance? (c) How much work is done on the disk by the cue?

123 A river descends 15 m through rapids. The speed of the water is 3.2 m/s upon entering the rapids and 13 m/s upon leaving. What percentage of the gravitational potential energy of the water–Earth system is transferred to kinetic energy during the descent? (*Hint:* Consider the descent of, say, 10 kg of water.)

124 The magnitude of the gravitational force between a particle of mass m_1 and one of mass m_2 is given by

$$F(x) = G \frac{m_1 m_2}{x^2},$$

where G is a constant and x is the distance between the particles. (a) What is the corresponding potential energy function $U(x)$? Assume that $U(x) \rightarrow 0$ as $x \rightarrow \infty$ and that x is positive. (b) How much work is required to increase the separation of the particles from $x = x_1$ to $x = x_1 + d$?

125 Approximately 5.5×10^6 kg of water falls 50 m over Niagara Falls each second. (a) What is the decrease in the gravitational potential energy of the water–Earth system each second? (b) If all this energy could be converted to electrical energy (it cannot be), at what rate would electrical energy be supplied? (The mass of 1 m³ of water is 1000 kg.) (c) If the electrical energy were sold at 1 cent/kW·h, what would be the yearly income?

126 To make a pendulum, a 300 g ball is attached to one end of a string that has a length of 1.4 m and negligible mass. (The other end of the string is fixed.) The ball is pulled to one side until the string makes an angle of 30.0° with the vertical; then (with the string taut) the ball is released from rest. Find (a) the speed of the ball when the string makes an angle of 20.0° with the vertical and (b) the maximum speed of the ball. (c) What is the angle between the string and the vertical when the speed of the ball is one-third its maximum value?

127 In a circus act, a 60 kg clown is shot from a cannon with an initial velocity of 16 m/s at some unknown angle above the horizontal. A short time later the clown lands in a net that is 3.9 m vertically above the clown’s initial position. Disregard air drag. What is the kinetic energy of the clown as he lands in the net?

128 A 70 kg firefighter slides, from rest, 4.3 m down a vertical pole. (a) If the firefighter holds onto the pole lightly, so that the frictional force of the pole on her is negligible, what is her speed just before reaching the ground floor? (b) If the firefighter grasps the pole more firmly as she slides, so that the average frictional force of the pole on her is 500 N upward, what is her speed just before reaching the ground floor?

129 The surface of the continental United States has an area of about 8×10^6 km² and an average elevation of about 500 m (above sea level). The average yearly rainfall is 75 cm. The fraction of this rainwater that returns to the atmosphere by evaporation is $\frac{2}{3}$; the rest eventually flows into the ocean. If the decrease in gravitational potential energy of the water–Earth system associated with that flow could be fully converted to electrical energy, what would be the average power? (The mass of 1 m³ of water is 1000 kg.)

130 A spring with spring constant $k = 200$ N/m is suspended vertically with its upper end fixed to the ceiling and its lower end at position $y = 0$. A block of weight 20 N is attached to the lower end, held still for a moment, and then released. What are (a) the kinetic energy K , (b) the change (from the initial value) in the gravitational potential energy ΔU_g , and (c) the change in the elastic potential energy ΔU_e of the spring–block system when the block is at $y = -5.0$ cm? What are (d) K , (e) ΔU_g , and (f) ΔU_e when $y = -10$ cm, (g) K , (h) ΔU_g , and (i) ΔU_e when $y = -15$ cm, and (j) K , (k) ΔU_g , and (l) ΔU_e when $y = -20$ cm?

131 Fasten one end of a vertical spring to a ceiling, attach a cabbage to the other end, and then slowly lower the cabbage until the upward force on it from the spring balances the gravitational force on it. Show that the loss of gravitational potential energy of the cabbage–Earth system equals twice the gain in the spring’s potential energy.

132 The maximum force you can exert on an object with one of your back teeth is about 750 N. Suppose that as you gradually bite on a clump of licorice, the licorice resists compression by one of your teeth by acting like a spring for which $k = 2.5 \times 10^5$ N/m. Find (a) the distance the licorice is compressed by your tooth and (b) the work the tooth does on the licorice during the compression. (c) Plot the magnitude of your force versus the compression distance. (d) If there is a potential energy associated with this compression, plot it versus compression distance.

In the 1990s the pelvis of a particular *Triceratops* dinosaur was found to have deep bite marks. The shape of the marks suggested that they were made by a *Tyrannosaurus rex* dinosaur. To test the idea, researchers made a replica of a *T. rex* tooth from bronze and aluminum and then used a hydraulic press to gradually drive the replica into cow bone to the depth seen in the *Triceratops* bone. A graph of the force required versus depth of penetration is given in Fig. 8-71 for one trial; the required force increased with depth because, as the nearly conical tooth penetrated the bone, more of the tooth came in contact with the bone. (e) How much work was done by the hydraulic press—and thus presumably by the *T. rex*—in such a penetration? (f) Is there a potential energy associated with this penetration? (The large biting force and energy expenditure

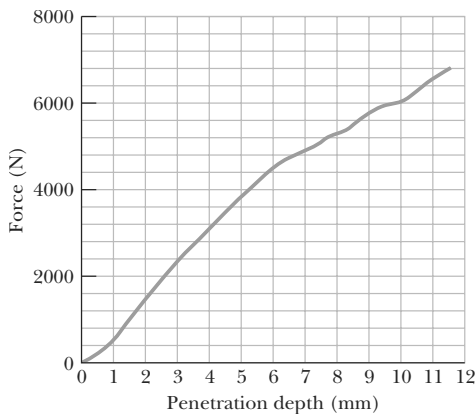


Figure 8-71 Problem 132.

attributed to the *T. rex* by this research suggest that the animal was a predator and not a scavenger.)

133 Conservative force $F(x)$ acts on a particle that moves along an x axis. Figure 8-72 shows how the potential energy $U(x)$ associated with force $F(x)$ varies with the position of the particle. (a) Plot $F(x)$ for the range $0 < x < 6$ m. (b) The mechanical energy E of the system is 4.0 J. Plot the kinetic energy $K(x)$ of the particle directly on Fig. 8-72.

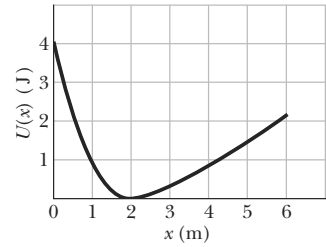


Figure 8-72 Problem 133.

134 Figure 8-73a shows a molecule consisting of two atoms of masses m and M (with $m \ll M$) and separation r . Figure 8-73b shows the potential energy $U(r)$ of the molecule as a function of r . Describe the motion of the atoms (a) if the total mechanical energy E of the two-atom system is greater than zero (as is E_1), and (b) if E is less than zero (as is E_2). For $E_1 = 1 \times 10^{-19}$ J and $r = 0.3$ nm, find (c) the potential energy of the system, (d) the total kinetic energy of the atoms, and (e) the force (magnitude and direction) acting on each atom. For what values of r is the force (f) repulsive, (g) attractive, and (h) zero?

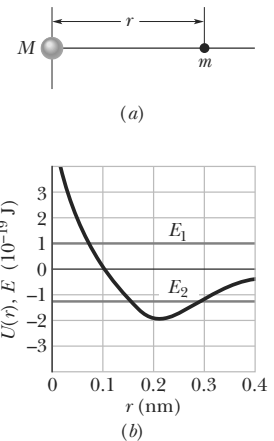


Figure 8-73 Problem 134.

135 Repeat Problem 83, but now with the block accelerated up a frictionless plane inclined at 5.0° to the horizontal.

136 A spring with spring constant $k = 620$ N/m is placed in a vertical orientation with its lower end supported by a horizontal surface. The upper end is depressed 25 cm, and a block with a weight of 50 N is placed (unattached) on the depressed spring. The system is then released from rest. Assume that the gravitational potential energy U_g of the block is zero at the release point ($y = 0$) and calculate the kinetic energy K of the block for y equal to (a) 0, (b) 0.050 m, (c) 0.10 m, (d) 0.15 m, and (e) 0.20 m. Also, (f) how far above its point of release does the block rise?