maximum operating frequency. Then the maximum power capacity of the guide can be shown to be

$$P_{\max} = \frac{0.11}{\eta_0}\left(\frac{cE_d}{f_{\max}}\right)^2 = 2.6 \times 10^{13}\left(\frac{E_d}{f_{\max}}\right)^2.$$

As an example, at 10 GHz the maximum peak power capacity of a rectangular waveguide operating in the TE_{10} mode is about 2300 kW, which is considerably higher than the power capacity of a coaxial cable at the same frequency.

Because arcing and voltage breakdown are high-speed transient effects, these voltage and power limits are peak values; average power capacity is lower. In addition, it is good engineering practice to provide a safety factor of at least two, so the maximum powers that can be safely transmitted should be limited to about half of the above values. If there are reflections on the line or guide, the power capacity is further reduced. In the worst case, a reflection coefficient magnitude of unity will double the maximum voltage on the line, so the power capacity will be reduced by a factor of four.

The power capacity of a line can be increased by pressurizing the line with air or an inert gas or by using a dielectric. The dielectric strength (E_d) of most dielectric materials is greater than that of air, but the power capacity may be further limited by the heating of the dielectric due to ohmic loss.

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PROBLEMS

3.1 Devise at least two variations of the basic coaxial transmission line geometry of Section 3.5, and discuss the advantages and disadvantages of your proposed lines in terms of size, loss, cost, higher order modes, dispersion, or other considerations. Repeat this exercise for the microstrip line geometry of Section 3.8.

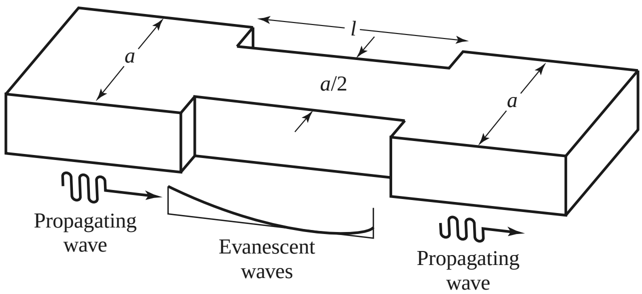
3.2 Derive equations (3.5a)–(3.5d) from equations (3.3) and (3.4).

3.3 Calculate the attenuation due to conductor loss for the TE_n mode of a parallel plate waveguide.

3.4 Consider a section of air-filled K-band waveguide. From the dimensions given in Appendix I, determine the cutoff frequencies of the first two propagating modes. From the recommended operating range given in Appendix I for this guide, determine the percentage reduction in bandwidth that this operating range represents, relative to the theoretical bandwidth for a single propagating mode.

3.5 A 10 cm length of a K-band copper waveguide is filled with a dielectric material with $\epsilon_r = 2.55$ and $\tan\delta = 0.0015$. If the operating frequency is 15 GHz, find the total loss through the guide and the phase delay from the input to the output of the guide.

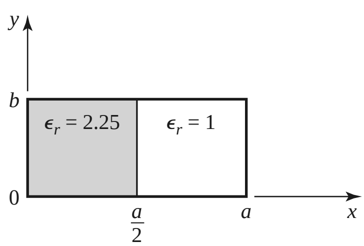
3.6 An attenuator can be made using a section of waveguide operating below cutoff, as shown in the accompanying figure. If $a = 2.286$ cm and the operating frequency is 12 GHz, determine the required length of the below-cutoff section of waveguide to achieve an attenuation of 100 dB between the input and output guides. Ignore the effect of reflections at the step discontinuities.



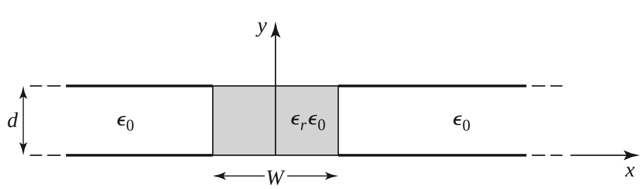
3.7 Find expressions for the electric surface current density on the walls of a rectangular waveguide for a TE_{10} mode. Why can a narrow slot be cut along the centerline of the broad wall of a rectangular waveguide without perturbing the operation of the guide? (Such a slot is often used in a slotted line for a probe to sample the standing wave field inside the guide.)

3.8 Derive the expression for the attenuation of the TM_{mn} mode of a rectangular waveguide due to imperfectly conducting walls.

3.9 For the partially loaded rectangular waveguide shown in the accompanying figure, solve (3.109) with $\beta = 0$ to find the cutoff frequency of the TE_{10} mode. Assume $a = 2.286$ cm, $t = a/2$, and $\epsilon_r = 2.25$.



3.10 Consider the partially filled parallel plate waveguide shown in the accompanying figure. Derive the solution (fields and cutoff frequency) for the lowest order TE mode of this structure. Assume the metal plates are infinitely wide. Can a TEM wave propagate on this structure?



3.11 Derive equations (3.110a)–(3.110d) for the transverse field components in terms of longitudinal fields, in cylindrical coordinates.

3.12 Derive the expression for the attenuation of the TM_{nm} mode in a circular waveguide with finite conductivity.

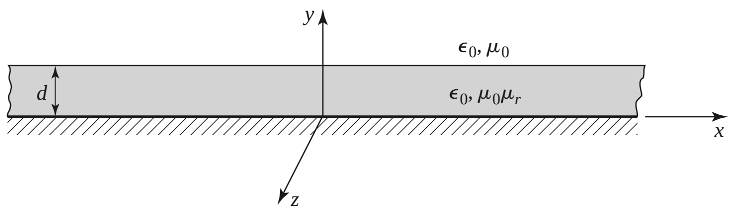
3.13 A circular copper waveguide has a radius of 0.4 cm and is filled with a dielectric material having $\epsilon_r = 1.5$ and $\tan\delta = 0.0002$. Identify the first four propagating modes and their cutoff frequencies. For the dominant mode, calculate the total attenuation at 20 GHz.

3.14 Derive the $\bar{E}$ and $\bar{H}$ fields of a coaxial line from the expression for the potential given in (3.153). Also find expressions for the voltage and current on the line and the characteristic impedance.

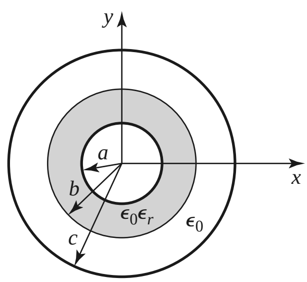
3.15 Derive a transcendental equation for the cutoff frequency of the TM modes of a coaxial waveguide. Using tables, obtain an approximate value of $k_c a$ for the TM_{01} mode if $b/a = 2$.

3.16 Derive an expression for the attenuation of a TE surface wave on a grounded dielectric substrate when the ground plane has finite conductivity.

3.17 Consider the grounded magnetic substrate shown in the accompanying figure. Derive a solution for the TM surface waves that can propagate on this structure.



3.18 Consider the partially filled coaxial line shown in the accompanying figure. Can a TEM wave propagate on this line? Derive the solution for the TM_{0m} (no azimuthal variation) modes of this geometry.



3.19 A copper stripline transmission line is to be designed for a 100 Ω characteristic impedance. The ground plane separation is 1.02 mm and the dielectric constant is 2.20, with $\tan\delta = 0.001$. At 5 GHz, find the guide wavelength on the line and the total attenuation.

3.20 A copper microstrip transmission line is to be designed for a 100 Ω characteristic impedance. The substrate is 0.51 mm thick, with $\epsilon_r = 2.20$ and $\tan\delta = 0.001$. At 5 GHz, find the guide wavelength on the line and the total attenuation. Compare these results with those for the similar stripline case of the preceding problem.

3.21 A 100 Ω microstrip line is printed on a substrate of thickness 0.0762 cm with a dielectric constant of 2.2. Ignoring losses and fringing fields, find the shortest length of this line that appears at its input as a capacitor of 5 pF at 2.5 GHz. Repeat for an inductance of 5 nH. Using a microwave CAD package with a physical model for the microstrip line, compute the actual input impedance seen when losses are included (assume copper conductors and $\tan\delta = 0.001$).

3.22 A microwave antenna feed network operating at 5 GHz requires a 50 Ω printed transmission line that is 16 λ long. Possible choices are (1) copper microstrip, with $d = 0.16$ cm, $\epsilon_r = 2.20$, and $\tan\delta = 0.001$, or (2) copper stripline, with $b = 0.32$ cm, $\epsilon_r = 2.20$, $t = 0.01$ mm, and $\tan\delta = 0.001$. Which line should be used if attenuation is to be minimized?

3.23 Consider the TE modes of an arbitrary uniform waveguiding structure in which the transverse fields are related to H_z as in (3.19). If H_z is of the form $H_z(x, y, z) = h_z(x, y)e^{-j\beta z}$, where $h_z(x, y)$ is a real function, compute the Poynting vector and show that real power flow occurs only in the z direction. Assume that β is real, corresponding to a propagating mode.

3.24 A piece of rectangular waveguide is air filled for $z < 0$ and dielectric filled for $z > 0$. Assume that both regions can support only the dominant TE_{10} mode and that a TE_{10} mode is incident on the interface from $z < 0$. Using a field analysis, write general expressions for the transverse field components of the incident, reflected, and transmitted waves in the two regions and enforce the boundary conditions at the dielectric interface to find the reflection and transmission coefficients. Compare these results to those obtained with an impedance approach, using Z_{TE} for each region.

3.25 Use the transverse resonance technique to derive a transcendental equation for the propagation constant of the TM modes of a rectangular waveguide that is air filled for $0 < x < d$ and dielectric filled for $d < x < a$.

3.26 Apply the transverse resonance technique to find the propagation constants for the TE surface waves that can be supported by the structure of Problem 3.17.

3.27 An X-band waveguide filled with Rexolite is operating at 9.0 GHz. Calculate the speed of light in this material and the phase and group velocities in the waveguide.

3.28 As discussed in the Point of Interest on the power-handling capacity of transmission lines, the maximum power capacity of a coaxial line is limited by voltage breakdown and is given by

$$P_{\max} = \frac{\pi a^2 E_d^2}{\eta_0} \ln \frac{b}{a},$$

where E_d is the field strength at breakdown. Find the value of b/a that maximizes the maximum power capacity and show that the corresponding characteristic impedance is about 30 Ω.

3.29 A microstrip circuit is fabricated on an alumina substrate having a dielectric constant of 9.9, a thickness of 2.0 mm, and a 50 Ω linewidth of 1.93 mm. Find the threshold frequencies of the four higher order modes discussed in Section 3.8, and recommend the maximum operating frequency for this microstrip circuit.