where $V_0 = abd$ is the volume of the unperturbed cavity. Then (6.107) gives

$$\frac{\omega - \omega_0}{\omega_0} = \frac{-2\ell\pi r_0^2}{abd} = \frac{-2\Delta V}{V_0},$$

which indicates a lowering of the resonant frequency. ■

REFERENCES

[1] R. E. Collin, *Foundations for Microwave Engineering*, 2nd edition, Wiley–IEEE Press, Hoboken, N.J., 2001.

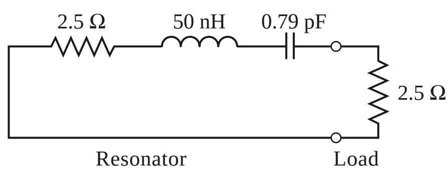
[2] S. B. Cohn, "Microwave Bandpass Filters Containing High-Q Dielectric Resonators," *IEEE Transactions on Microwave Theory and Techniques*, vol. MTT-16, pp. 218–227, April 1968.

[3] M. W. Pospieszalski, "Cylindrical Dielectric Resonators and Their Applications in TEM Line Microwave Circuits," *IEEE Transactions on Microwave Theory and Techniques*, vol. MTT-27, pp. 233–238, March 1979.

[4] R. E. Collin, *Field Theory of Guided Waves*, McGraw-Hill, New York, 1960.

PROBLEMS

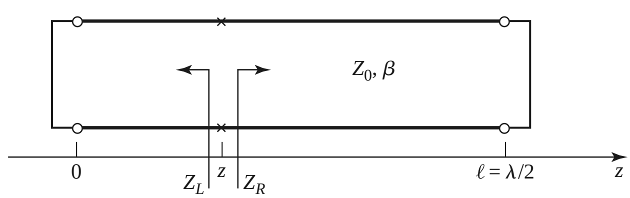
6.1 A series *RLC* resonator with an external load is shown below. Find the resonant frequency, the unloaded Q, and the loaded Q.



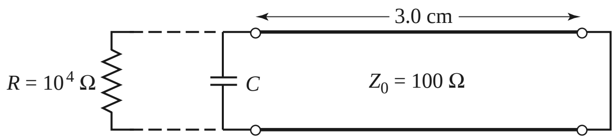
6.2 Derive an expression for the unloaded Q of a transmission line resonator consisting of a short-circuited transmission line 1λ long.

6.3 A transmission line resonator is fabricated from a $\lambda/4$ length of open-circuited line. Find the unloaded Q of this resonator if the complex propagation constant of the line is $\alpha + j\beta$.

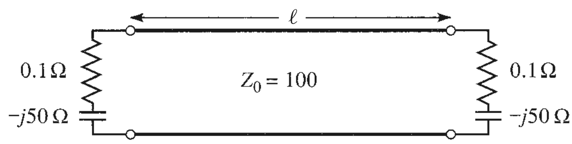
6.4 Consider the resonator shown below, consisting of a $\lambda/2$ length of lossless transmission line shorted at both ends. At an arbitrary point, z, on the line, compute the impedances Z_L and Z_R seen looking to the left and to the right, respectively, and show that $Z_L = Z_R^*$. (This condition holds true for any lossless transmission line resonator and is the basis for the transverse resonance technique discussed in Section 3.9.)



6.5 A resonator is constructed from a 3.0 cm length of 100 Ω air-filled coaxial line, shorted at one end and terminated with a capacitor at the other end, as shown below. (a) Determine the capacitor value to achieve the lowest order resonance at 6.0 GHz. (b) Now assume that loss is introduced by placing a 10,000 Ω resistor in parallel with the capacitor. Calculate the unloaded Q.

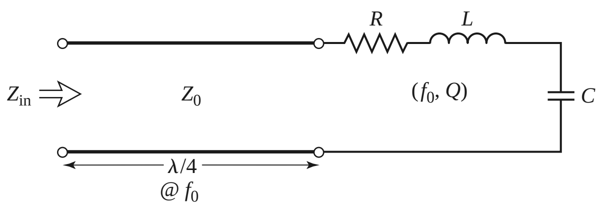


6.6 A transmission line resonator is made from a length ℓ of lossless transmission line of characteristic impedance $Z_0 = 100\ \Omega$. If the line is terminated at both ends as shown below, find ℓ/λ for the first resonance, and the unloaded Q of this resonator.



6.7 Write the expressions for the $\bar{E}$ and $\bar{H}$ fields for a short-circuited $\lambda/2$ coaxial line resonator, and show that the time-average stored electric and magnetic energies are equal.

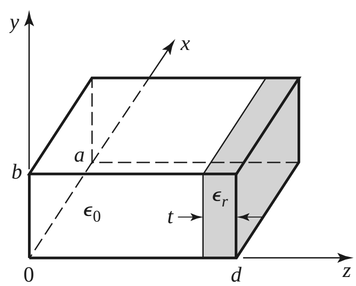
6.8 A series *RLC* resonant circuit is connected to a length of transmission line that is $\lambda/4$ long at its resonant frequency, as shown below. Show that, in the vicinity of resonance, the input impedance behaves like that of a parallel *RLC* circuit.



6.9 A rectangular cavity resonator is constructed from a 2.0 cm length of aluminum X-band waveguide. The cavity is air filled. Find the resonant frequency and unloaded Q of the TE_{101} and TE_{102} resonant modes.

6.10 Derive the unloaded Q for the TM_{111} mode of a rectangular cavity, assuming lossy conducting walls and lossless dielectric.

6.11 Consider the rectangular cavity resonator partially filled with dielectric as shown below. Derive a transcendental equation for the resonant frequency of the dominant mode by writing the fields in the air- and dielectric-filled regions in terms of TE_{10} waveguide modes, and enforcing boundary conditions at $z = 0$, $d - t$, and d.



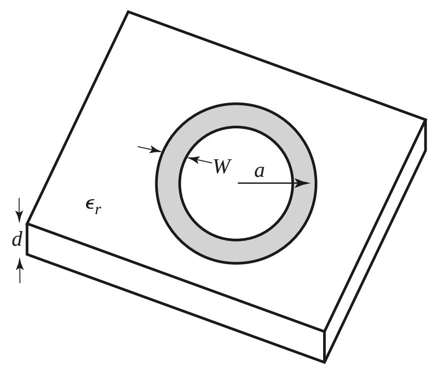
6.12 Determine the resonant frequencies of a rectangular cavity by carrying out a full separation-of-variables solution to the wave equation for E_z (for TM modes) and H_z (for TE modes), subject to the appropriate boundary conditions of the cavity. [Assume a solution of the form $X(x)Y(y)Z(z)$.]

6.13 Find the unloaded Q for the TM_{nm0} resonant mode of a circular cavity. Consider both conductor and dielectric losses.

6.14 Design a circular cavity resonator to operate in the TE_{111} mode with maximum unloaded Q at a frequency of 6 GHz. The cavity is gold plated and filled with a dielectric material having $\epsilon_r = 1.5$ and $\tan\delta = 0.0005$. Find the cavity dimensions and the resulting unloaded Q.

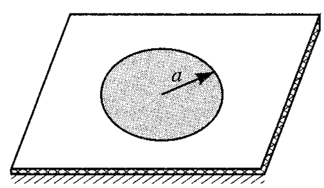
6.15 An air-filled rectangular cavity resonator has its first three resonant modes at the frequencies 5.2, 6.5, and 7.2 GHz. Find the dimensions of the cavity.

6.16 Consider the microstrip ring resonator shown below. If the effective dielectric constant of the microstrip line is ϵ_e, find an equation for the frequency of the first resonance. Suggest some methods of coupling to this resonator.



6.17 A circular microstrip disk resonator is shown below. Solve the wave equation for TM_{nm0} modes for this structure, using the magnetic wall approximation that $H_\varphi = 0$ at $\rho = a$. If fringing fields are neglected, show that the resonant frequency of the dominant mode is given by

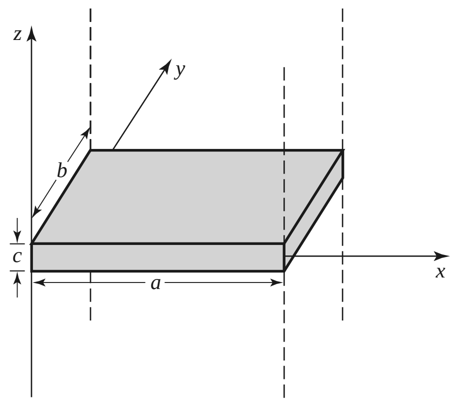
$$f_{110} = \frac{1.841c}{2\pi a\sqrt{\epsilon_r}}$$



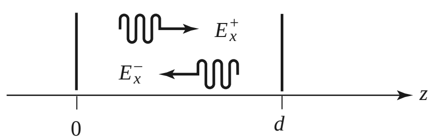
6.18 Compute the resonant frequency of a cylindrical dielectric resonator with $\epsilon_r = 36.2$, $2a = 7.99$ mm, and $L = 2.14$ mm.

6.19 Extend the analysis of Section 6.5 to derive a transcendental equation for the resonant frequency of the next resonant mode of the cylindrical dielectric resonator. (H_z odd in z.)

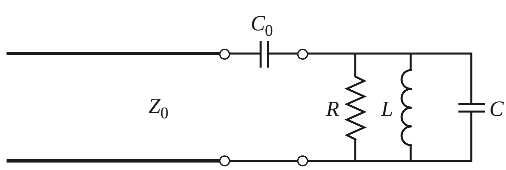
6.20 Consider the rectangular dielectric resonator shown below. Assume a magnetic wall boundary condition around the edges of the cavity, and allow evanescent fields in the $\pm z$ directions away from the dielectric, similar to the analysis of Section 6.5. Derive a transcendental equation for the resonant frequency.



6.21 A high-Q resonator useful at millimeter wave frequencies is the Fabry-Perot resonator, which consists of two parallel metal plates (see figure below). A plane wave traveling at normal incidence between the two plates will exhibit resonance when the plate separation is equal to a multiple of $\lambda/2$. (a) Derive an expression for the resonant frequency of a Fabry-Perot resonator having a plate separation d and mode number ℓ. (b) If the plates have conductivity σ, derive an expression for the unloaded Q of the resonator. (c) Use these results to find the resonant frequency and unloaded Q of a Fabry-Perot resonator having $d = 4.0$ cm, with copper plates, and with a mode number $\ell = 25$.



6.22 A parallel RLC circuit, with $R = 1000\ \Omega$, $L = 1.26$ nH, $C = 0.804$ pF, is coupled with a series capacitor, C_0, to a 50-Ω transmission line, as shown below. Determine C_0 for critical coupling to the line. What is the resonant frequency?

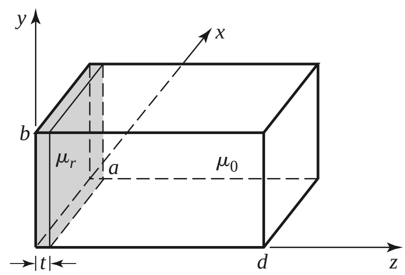


6.23 An aperture-coupled rectangular waveguide cavity has a resonant frequency of 9.0 GHz and an unloaded Q of 11,000. If the waveguide dimensions are $a = 2.5$ cm and $b = 1.25$ cm, find the normalized aperture reactance required for critical coupling.

6.24 A microwave resonator is connected as a one-port circuit, and its return loss is measured versus frequency. At resonance the return loss is 14 dB, while at 2.9985 GHz and at 3.0015 GHz the return loss is 11 dB (the half-power points). Determine the unloaded Q of the resonator. Do this for both series and parallel resonators.

6.25 A microwave resonator is measured in a two-port configuration like that shown in Figure 6.21. The minimum insertion loss is measured as 1.94 dB at 3.0000 GHz. The insertion loss is 4.95 dB at 2.9925 GHz and at 3.0075 GHz. What is the unloaded Q of the resonator?

6.26 A thin slab of magnetic material is inserted next to the $z = 0$ wall of the rectangular cavity shown below. If the cavity is operating in the TE_{101} mode, derive a perturbational expression for the change in resonant frequency caused by the magnetic material.



6.27 Derive an expression for the change in resonant frequency for the screw-tuned rectangular cavity of Example 6.8 if the screw is located at $x = a/2,\ z = 0$, where H_x is maximum and E_y is minimum.