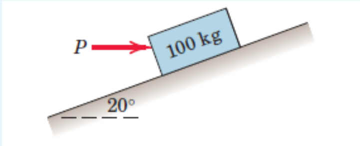
بسمه تعالی

مبحث اصطکاک

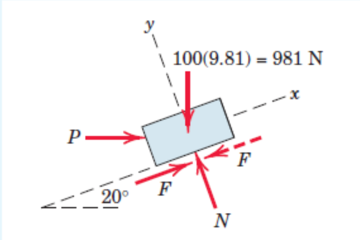
مثالهای تکمیلی

SAMPLE PROBLEM 6/3

Determine the magnitude and direction of the friction force acting on the 100-kg block shown if, first, $P = 500$ N and, second, $P = 100$ N. The coefficient of static friction is 0.20, and the coefficient of kinetic friction is 0.17. The forces are applied with the block initially at rest.



Solution. There is no way of telling from the statement of the problem whether the block will remain in equilibrium or whether it will begin to slip following the application of P. It is therefore necessary that we make an assumption, so we will take the friction force to be up the plane, as shown by the solid arrow. From the free-body diagram a balance of forces in both x- and y-directions gives



$[\Sigma F_x = 0]$ $\qquad P \cos 20° + F - 981 \sin 20° = 0$

$[\Sigma F_y = 0]$ $\qquad N - P \sin 20° - 981 \cos 20° = 0$

Case I. $P = 500$ N

Substitution into the first of the two equations gives

$$F = -134.3 \text{ N}$$

The negative sign tells us that *if* the block is in equilibrium, the friction force acting on it is in the direction opposite to that assumed and therefore is down the plane, as represented by the dashed arrow. We cannot reach a conclusion on the magnitude of F, however, until we verify that the surfaces are capable of supporting 134.3 N of friction force. This may be done by substituting $P = 500$ N into the second equation, which gives

$$N = 1093 \text{ N}$$

The maximum static friction force which the surfaces can support is then

$[F_{max} = \mu_s N]$ $\qquad F_{max} = 0.20(1093) = 219$ N

Since this force is greater than that required for equilibrium, we conclude that the assumption of equilibrium was correct. The answer is, then,

$$F = 134.3 \text{ N down the plane} \qquad \textit{Ans.}$$

Case II. $P = 100$ N

Substitution into the two equilibrium equations gives

$$F = 242 \text{ N} \qquad N = 956 \text{ N}$$

But the maximum possible static friction force is

$[F_{max} = \mu_s N]$ $\qquad F_{max} = 0.20(956) = 191.2$ N

① It follows that 242 N of friction cannot be supported. Therefore, equilibrium cannot exist, and we obtain the correct value of the friction force by using the kinetic coefficient of friction accompanying the motion down the plane. Hence, the answer is

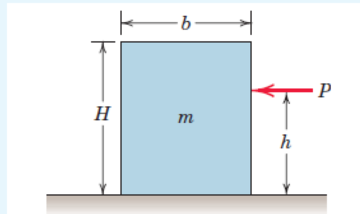
$[F_k = \mu_k N]$ $\qquad F = 0.17(956) = 162.5$ N up the plane $\qquad$ *Ans.*

Helpful Hint

① We should note that even though ΣF_x is no longer equal to zero, equilibrium does exist in the y-direction, so that $\Sigma F_y = 0$. Therefore, the normal force N is 956 N whether or not the block is in equilibrium.

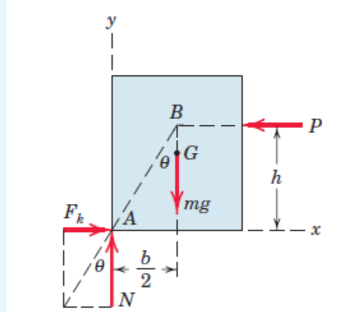
SAMPLE PROBLEM 6/4

The homogeneous rectangular block of mass m, width b, and height H is placed on the horizontal surface and subjected to a horizontal force P which moves the block along the surface with a constant velocity. The coefficient of kinetic friction between the block and the surface is μ_k. Determine (*a*) the greatest value which h may have so that the block will slide without tipping over and (*b*) the location of a point C on the bottom face of the block through which the resultant of the friction and normal forces acts if $h = H/2$.



Solution. (*a*) With the block on the verge of tipping, we see that the entire reaction between the plane and the block will necessarily be at A. The free-body diagram of the block shows this condition. Since slipping occurs, the friction force is the limiting value $\mu_k N$, and the angle θ becomes $\theta = \tan^{-1} \mu_k$. The resultant of F_k and N passes through a point B through which P must also pass, since
1 three coplanar forces in equilibrium are concurrent. Hence, from the geometry of the block

$$\tan\theta = \mu_k = \frac{b/2}{h} \qquad h = \frac{b}{2\mu_k} \qquad \textit{Ans.}$$



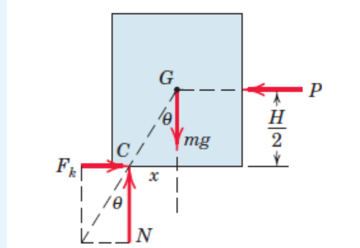
If h were greater than this value, moment equilibrium about A would not be satisfied, and the block would tip over.

Alternatively, we may find h by combining the equilibrium requirements for the x- and y-directions with the moment-equilibrium equation about A. Thus,

$[\Sigma F_y = 0]$ $\qquad N - mg = 0 \qquad N = mg$

$[\Sigma F_x = 0]$ $\qquad F_k - P = 0 \qquad P = F_k = \mu_k N = \mu_k mg$

$[\Sigma M_A = 0]$ $\qquad Ph - mg\dfrac{b}{2} = 0 \qquad h = \dfrac{mgb}{2P} = \dfrac{mgb}{2\mu_k mg} = \dfrac{b}{2\mu_k}$ $\qquad$ *Ans.*



(*b*) With $h = H/2$ we see from the free-body diagram for case (*b*) that the resultant of F_k and N passes through a point C which is a distance x to the left of the vertical centerline through G. The angle θ is still $\theta = \phi = \tan^{-1} \mu_k$ as long as the block is slipping. Thus, from the geometry of the figure we have

2 $$\frac{x}{H/2} = \tan\theta = \mu_k \qquad \text{so} \qquad x = \mu_k H/2 \qquad \textit{Ans.}$$

If we were to replace μ_k by the static coefficient μ_s, then our solutions would describe the conditions under which the block is (*a*) on the verge of tipping and (*b*) on the verge of slipping, both from a rest position.

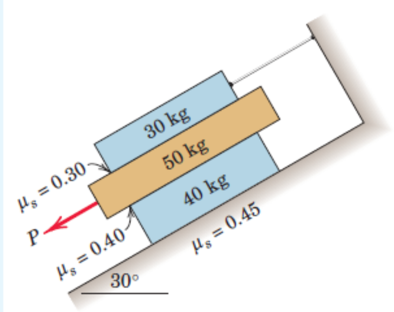
Helpful Hints

1 Recall that the equilibrium equations apply to a body moving with a constant velocity (zero acceleration) just as well as to a body at rest.

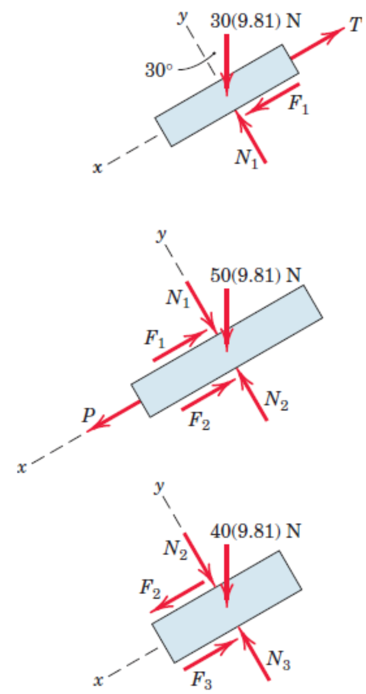
2 Alternatively, we could equate the moments about G to zero, which would give us $F(H/2) - Nx = 0$. Thus, with $F_k = \mu_k N$ we get $x = \mu_x H/2$.

SAMPLE PROBLEM 6/5

The three flat blocks are positioned on the 30° incline as shown, and a force P parallel to the incline is applied to the middle block. The upper block is prevented from moving by a wire which attaches it to the fixed support. The coefficient of static friction for each of the three pairs of mating surfaces is shown. Determine the maximum value which P may have before any slipping takes place.



Solution. The free-body diagram of each block is drawn. The friction forces are assigned in the directions to oppose the relative motion which would occur if ① no friction were present. There are two possible conditions for impending motion. Either the 50-kg block slips and the 40-kg block remains in place, or the 50- and 40-kg blocks move together with slipping occurring between the 40-kg block and the incline.



The normal forces, which are in the y-direction, may be determined without reference to the friction forces, which are all in the x-direction. Thus,

$[\Sigma F_y = 0]$ (30-kg) $N_1 - 30(9.81)\cos 30° = 0$ $N_1 = 255$ N

(50-kg) $N_2 - 50(9.81)\cos 30° - 255 = 0$ $N_2 = 680$ N

(40-kg) $N_3 - 40(9.81)\cos 30° - 680 = 0$ $N_3 = 1019$ N

We will assume arbitrarily that only the 50-kg block slips, so that the 40-kg block remains in place. Thus, for impending slippage at both surfaces of the 50-kg block, we have

$[F_{max} = \mu_s N]$ $F_1 = 0.30(255) = 76.5$ N $F_2 = 0.40(680) = 272$ N

The assumed equilibrium of forces at impending motion for the 50-kg block gives

$[\Sigma F_x = 0]$ $P - 76.5 - 272 + 50(9.81)\sin 30° = 0$ $P = 103.1$ N

We now check on the validity of our initial assumption. For the 40-kg block with $F_2 = 272$ N the friction force F_3 would be given by

$[\Sigma F_x = 0]$ $272 + 40(9.81)\sin 30° - F_3 = 0$ $F_3 = 468$ N

But the maximum possible value of F_3 is $F_3 = \mu_s N_3 = 0.45(1019) = 459$ N. Thus, 468 N cannot be supported and our initial assumption was wrong. We conclude, therefore, that slipping occurs first between the 40-kg block and the incline. With the corrected value $F_3 = 459$ N, equilibrium of the 40-kg block for its impending motion requires

② $[\Sigma F_x = 0]$ $F_2 + 40(9.81)\sin 30° - 459 = 0$ $F_2 = 263$ N

Equilibrium of the 50-kg block gives, finally,

$[\Sigma F_x = 0]$ $P + 50(9.81)\sin 30° - 263 - 76.5 = 0$

$P = 93.8$ N *Ans.*

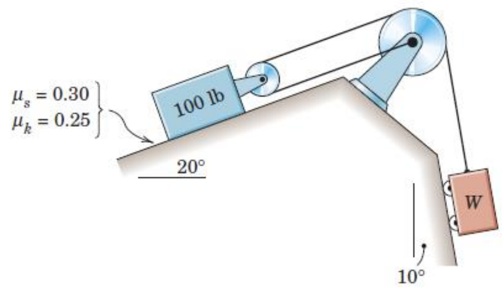
Thus, with $P = 93.8$ N, motion impends for the 50-kg and 40-kg blocks as a unit.

Helpful Hints

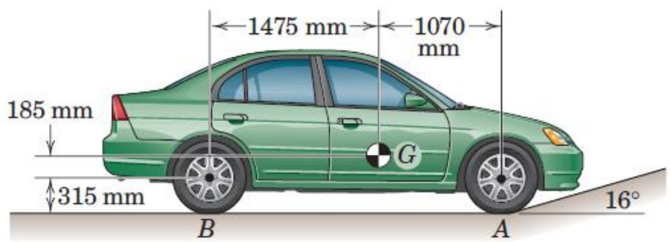
① In the absence of friction the middle block, under the influence of P, would have a greater movement than the 40-kg block, and the friction force F_2 will be in the direction to oppose this motion as shown.

② We see now that F_2 is less than $\mu_s N_2 = 272$ N.

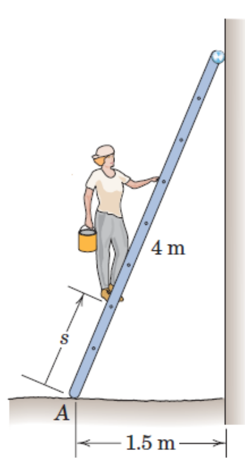
تمرین ۱- محدوده وزن W را به نحوی تعیین کنید که بلوک 100 lb در حال تعادل باشد. اصطکاک در قرقره‌ها ناچیز است.



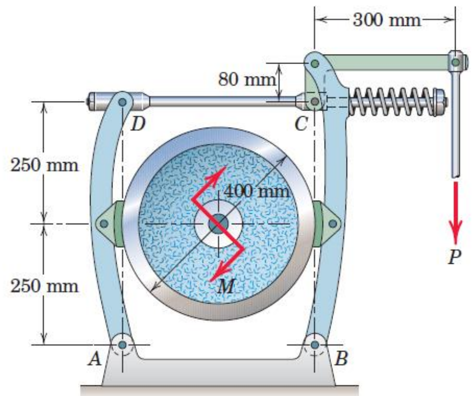
تمرین ۲- یک خودروی 1600 kg در آستانه بالا رفتن از یک شیب ۱۶ درجه قرار دارد. اگر قدرت از سمت چرخ عقب به خودرو منتقل شود (دیفرانسیل عقب باشد) حداقل ضریب اصطکاک استاتیکی لازم در نقطه B را بدست آورید.



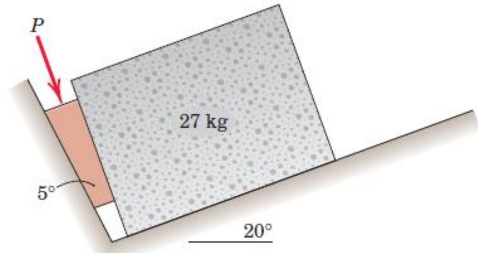
تمرین ۳- فاصله s را به نحوی تعیین کنید که نقاش 90 kg بتواند بدون سر خوردن انتهای نردبان (نقطه A) از نردبان بالا برود. وزن نردبان 15 kg و ضریب اصطکاک استاتیکی بین زمین و نردبان 0.25 است. بالای نردبان در نقطه B غلتک کوچکی قرار دارد و مرکز جرم شخص بالای پاهای او میباشد.



تمرین ۴- چرخ طیار نشان داده شده در شکل توسط نوعی ترمز دو کفشکی مهار شده است. جهت آزادسازی ترمز، نیروی P به میله کنترل اعمال میشود. در حال ترمز، P=0 و فنر به اندازه 30 mm فشرده شده است. اگر چرخ طیار تحت گشتاور 100 N.m باشد، ضریب سفتی مناسب برای فنر برای اینکه مانع چرخش چرخ طیار شود را تعیین کنید. ضریب اصطکاک بین کفشکها و چرخ طیار 0.2 است. از ابعاد کفشکها صرفنظر کنید.



تمرین ۵- ضریب اصطکاک برای هر دو سطح گوه 0.4 و بین بلوک بتنی 27 kg و سطح شیبدار 0.7 است. کمترین نیروی P لازم برای آنکه بلوک شروع به بالا رفتن از سطح بکند را تعیین کنید. از وزن گوه صرفنظر شود.



موفق باشید.