

- 3 Write the complete solution as \mathbf{x}_p plus any multiple of \mathbf{s} in the nullspace:

$$\begin{aligned}x + 3y + 3z &= 1 \\2x + 6y + 9z &= 5 \\-x - 3y + 3z &= 5.\end{aligned}$$

- 4 Find the complete solution (also called the *general solution*) to

$$\begin{bmatrix} 1 & 3 & 1 & 2 \\ 2 & 6 & 4 & 8 \\ 0 & 0 & 2 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ t \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}.$$

- 5 Under what condition on b_1, b_2, b_3 is this system solvable? Include \mathbf{b} as a fourth column in elimination. Find all solutions when that condition holds:

$$\begin{aligned}x + 2y - 2z &= b_1 \\2x + 5y - 4z &= b_2 \\4x + 9y - 8z &= b_3.\end{aligned}$$

- 6 What conditions on b_1, b_2, b_3, b_4 make each system solvable? Find \mathbf{x} in that case:

$$\begin{bmatrix} 1 & 2 \\ 2 & 4 \\ 2 & 5 \\ 3 & 9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} \quad \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 2 & 5 & 7 \\ 3 & 9 & 12 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}.$$

- 7 Show by elimination that (b_1, b_2, b_3) is in the column space if $b_3 - 2b_2 + 4b_1 = 0$.

$$A = \begin{bmatrix} 1 & 3 & 1 \\ 3 & 8 & 2 \\ 2 & 4 & 0 \end{bmatrix}.$$

What combination of the rows of A gives the zero row?

- 8 Which vectors (b_1, b_2, b_3) are in the column space of A ? Which combinations of the rows of A give zero?

$$(a) \quad A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 6 & 3 \\ 0 & 2 & 5 \end{bmatrix} \quad (b) \quad A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 2 & 4 & 8 \end{bmatrix}.$$