به نام خدا

تمرین‌های سری ۳

4 Find a combination $x_1 \boldsymbol{w}_1 + x_2 \boldsymbol{w}_2 + x_3 \boldsymbol{w}_3$ that gives the zero vector:

$$\boldsymbol{w}_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \qquad \boldsymbol{w}_2 = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} \qquad \boldsymbol{w}_3 = \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix}.$$

Those vectors are (independent) (dependent). The three vectors lie in a ______. The matrix W with those columns is *not invertible*.

5 The rows of that matrix W produce three vectors (I write them as columns):

$$\boldsymbol{r}_1 = \begin{bmatrix} 1 \\ 4 \\ 7 \end{bmatrix} \qquad \boldsymbol{r}_2 = \begin{bmatrix} 2 \\ 5 \\ 8 \end{bmatrix} \qquad \boldsymbol{r}_3 = \begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix}.$$

Linear algebra says that these vectors must also lie in a plane. There must be many combinations with $y_1 \boldsymbol{r}_1 + y_2 \boldsymbol{r}_2 + y_3 \boldsymbol{r}_3 = \mathbf{0}$. Find two sets of y's.

6 Which values of c give dependent columns (combination equals zero)?

$$\begin{bmatrix} 1 & 3 & 5 \\ 1 & 2 & 4 \\ 1 & 1 & c \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & c \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \qquad \begin{bmatrix} c & c & c \\ 2 & 1 & 5 \\ 3 & 3 & 6 \end{bmatrix}$$