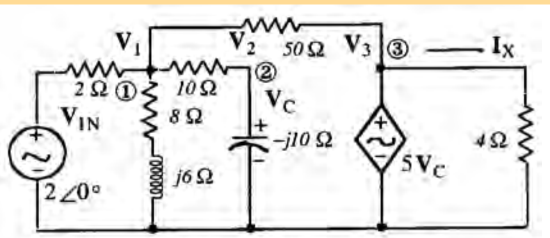
Circuit Analysis I
with MATLAB® Applications

Steven T. Karris



$$\left(\frac{35}{50} - j\frac{3}{50}\right)V_1 - \frac{1}{5}V_C = 1\angle 0^\circ$$

$$-\frac{1}{10}V_1 + \left(\frac{1}{10} + j\frac{1}{10}\right)V_C = 0$$

```
G=[35/50  –j*3/50;  –1/5  1/10+j*1/10];  I=[1  0]';  V=G\I;
Ix=5*V(2,1)/4;                                   % Multiply Vc by 5 and divide by 4 to get current Ix
magIx=abs(Ix); theta=angle(Ix)*180/pi;           % Convert current Ix to polar form
fprintf(' \n'); disp(' Ix = ' ); disp(Ix);...
fprintf('magIx = %4.2f A \t', magIx); fprintf('theta = %4.2f deg \t', theta);...
fprintf(' \n'); fprintf(' \n');

Ix = 2.1176-1.7546i  magIx = 2.75 A  theta = -39.64 deg
```

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Circuit Analysis I with MATLAB® Applications

Students and working professionals will find *Circuit Analysis I with MATLAB® Applications* to be a concise and easy-to-learn text. It provides complete, clear, and detailed explanations of the principal electrical engineering concepts, and these are illustrated with numerous practical examples.

This text includes the following chapters and appendices:

• Basic Concepts and Definitions • Analysis of Simple Circuits • Nodal and Mesh Equations - Circuit Theorems • Introduction to Operational Amplifiers • Inductance and Capacitance • Sinusoidal Circuit Analysis • Phasor Circuit Analysis • Average and RMS Values, Complex Power, and Instruments • Natural Response • Forced and Total Response in RL and RC Circuits • Introduction to MATLAB • Review of Complex Numbers • Matrices and Determinants

Each chapter contains numerous practical applications supplemented with detailed instructions for using MATLAB to obtain quick and accurate answers.

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Chapter 1

Basic Concepts and Definitions

This chapter begins with the basic definitions in electric circuit analysis. It introduces the concepts and conventions used in introductory circuit analysis, the unit and quantities used in circuit analysis, and includes several practical examples to illustrate these concepts.

1.1 The Coulomb

Two identically charged (both positive or both negative) particles possess a charge of one *coulomb* when being separated by one meter in a vacuum, repel each other with a force of $10^{-7}c^2$ newton where $c = velocity\ of\ light \approx 3 \times 10^8\ m/s$. The definition of coulomb is illustrated in Figure 1.1.

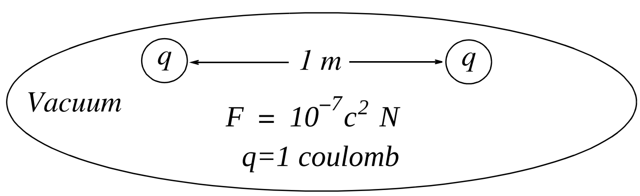


Figure 1.1. Definition of the coulomb

The coulomb, abbreviated as C, is the fundamental unit of charge. In terms of this unit, the charge of an electron is $1.6 \times 10^{-19}\ C$ and one negative coulomb is equal to 6.24×10^{18} electrons. Charge, positive or negative, is denoted by the letter q or Q.

1.2 Electric Current and Ampere

Electric *current* i at a specified point and flowing in a specified direction is defined as the instantaneous rate at which net positive charge is moving past this point in that specified direction, that is,

$$\boxed{i = \frac{dq}{dt} = \lim_{\Delta t \to 0} \frac{\Delta q}{\Delta t}} \tag{1.1}$$

The unit of current is the *ampere* abbreviated as A and corresponds to charge q moving at the *rate* of one coulomb per second. In other words,

$$\boxed{1\ ampere = \frac{1\ coulomb}{1\ second}} \tag{1.2}$$

1.4 Voltage (Potential Difference)

The voltage (potential difference) across a two-terminal device is defined as the work required to move a positive charge of one coulomb from one terminal of the device to the other terminal.

The unit of voltage is the *volt* (abbreviated as V or v) and it is defined as

$$\boxed{1\ volt = \frac{1\ joule}{1\ coulomb}} \tag{1.4}$$

Convention: We denote the voltage v by a plus (+) minus (–) pair. For example, in Figure 1.5, we say that terminal A is $10\ V$ positive with respect to terminal B or there is a potential difference of $10\ V$ between points A and B. We can also say that there is a *voltage drop* of $10\ V$ in going from point A to point B. Alternately, we can say that there is a *voltage rise* of $10\ V$ in going from B to A.

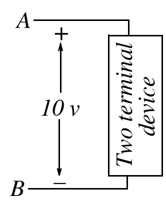


Figure 1.5. Illustration of voltage polarity for a two-terminal device

Caution: The (+) and (–) pair may or may not indicate the actual voltage drop or voltage rise. As in the case with the current, in some circuits the actual polarity cannot be determined by inspection. In such a case, again we assume a voltage reference polarity for the voltage; if this reference polarity turns out to be negative, this means that the potential at the (+) sign terminal is at a lower potential than the potential at the (–) sign terminal.

In the case of time-varying voltages which change (+) and (–) polarity from time-to-time, it is convenient to think the *instantaneous voltage*, that is, the voltage reference polarity at some particular instance. As before, we assume a voltage reference polarity by placing (+) and (–) polarity signs at the terminals of the device, and if a negative value of the voltage is obtained, we conclude that the actual polarity is opposite to that of the assumed reference polarity. We must remember that reversing the reference polarity reverses the algebraic sign of the voltage as shown in Figure 1.6.

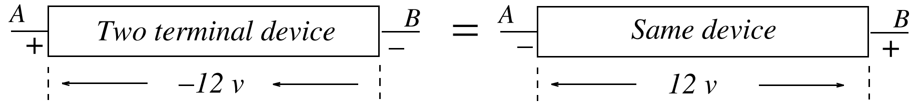


Figure 1.6. Alternate ways of denoting voltage polarity in a two-terminal device

Example 1.3

The $i-v$ (current-voltage) relation of a non-linear electrical device is given by

$$i(t) = 0.1(e^{0.2\sin 3t} - 1) \tag{10.5}$$

a. Use MATLAB®* to sketch this function for the interval $0 \le t \le 10\ s$

b. Use the MATLAB **quad** function to find the charge at $t = 5\ s$ given that $q(0) = 0$

Solution:

a. We use the following code to sketch $i(t)$.

```
t=0: 0.1: 10;
it=0.1.*(exp(0.2.*sin(3.*t))–1);
plot(t,it), grid, xlabel('time in sec.'), ylabel('current in amp.')
```

The plot for $i(t)$ is shown in Figure 1.7.

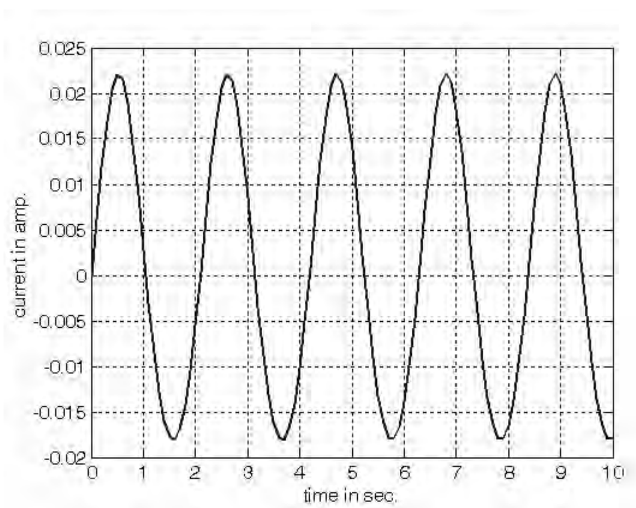


Figure 1.7. Plot of $i(t)$ for Example 1.3

b. The charge $q(t)$ is the integral of the current $i(t)$, that is,

$$q(t) = \int_{t_0}^{t_1} i(t)dt = 0.1\int_{0}^{t_1} (e^{0.2\sin 3t} - 1)dt \tag{1.6}$$

* *MATLAB and SIMULINK are registered marks of The MathWorks, Inc., 3 Apple Hill Drive, Natick, MA, 01760, www.mathworks.com. An introduction to MATLAB is given in Appendix A.*

1.5 Power and Energy

Power p is the rate at which *energy* (or *work*) W is expended. That is,

$$\boxed{Power = p = \frac{dW}{dt}} \tag{1.7}$$

Absorbed power is proportional both to the current and the voltage needed to transfer one coulomb through the device. The unit of power is the $watt$. Then,

$$Power = p = volts \times amperes = vi = \frac{joul}{coul} \times \frac{coul}{\sec} = \frac{joul}{\sec} = watts \tag{1.8}$$

and

$$\boxed{1\ watt = 1\ volt \times 1\ ampere} \tag{1.9}$$

Passive Sign Convention: Consider the two-terminal device shown in Figure 1.8.

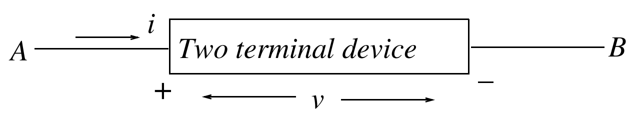
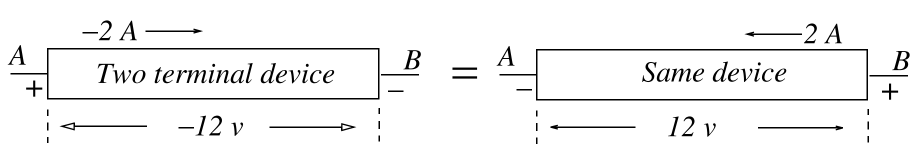


Figure 1.8. Illustration of the passive sign convention

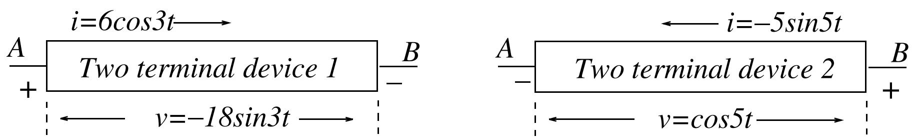
In Figure 1.8, terminal A is v volts positive with respect to terminal B and current *i* enters the device through the positive terminal A. In this case, we satisfy the *passive sign convention* and $power = p = vi$ is said to be *absorbed* by the device.

The passive sign convention states that if the arrow representing the current *i* and the (+) (−) pair are placed at the device terminals in such a way that the current enters the device terminal marked with the (+) sign, and if both the arrow and the sign pair are labeled with the appropriate algebraic quantities, the power absorbed or delivered to the device can be expressed as $p = vi$. If the numerical value of this product is positive, we say that the device is *absorbing power* which is equivalent to saying that power is delivered to the device. If, on the other hand, the numerical value of the product $p = vi$ is negative, we say that the device delivers power to some other device. The passive sign convention is illustrated with the examples in Figures 1.9 and 1.10.



Power = p = (−12)(−2) = 24 w *Power = p = (12)(2) = 24 w*

Figure 1.9. Examples where power is absorbed by a two-terminal device



$p = (-18\sin 3t)(6\cos 3t) = -54\sin 6t\ w$ $\quad$ $p = (\cos 5t)(-5\sin 5t) = -2.5\sin 10t\ w$

Figure 1.10. Examples where power is delivered to a two-terminal device

In Figure 1.9, power is absorbed by the device, whereas in Figure 1.10, power is delivered to the device.

Example 1.4

It is assumed a 12-volt automotive battery is completely discharged and at some reference time $t = 0$, is connected to a battery charger to trickle charge it for the next 8 hours. It is also assumed that the charging rate is

$$i(t) = \begin{cases} 8e^{-t/3600}\ A & 0 \le t \le 8\ hr \\ 0 & otherwise \end{cases}$$

For this 8-hour interval compute:

a. the total charge delivered to the battery

b. the *maximum* power (in watts) absorbed by the battery

c. the total energy (in joules) supplied

d. the *average* power (in watts) absorbed by the battery

Solution:

The current entering the positive terminal of the battery is the decaying exponential shown in Figure 1.11 where the time has been converted to seconds.

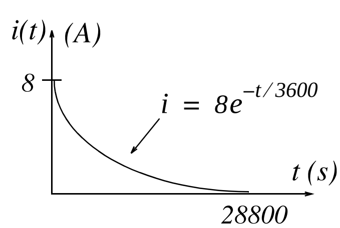
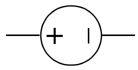


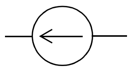
Figure 1.11. Decaying exponential for Example 1.4

Then,

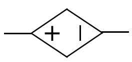
Independent and Dependent Sources



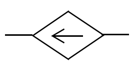
Ideal Independent Voltage Source – Maintains same voltage regardless of the amount of current that flows through it. Its value is either constant (DC) or sinusoidal (AC).



Ideal Independent Current Source – Maintains same current regardless of the voltage that appears across its terminals. Its value is either constant (DC) or sinusoidal (AC).



Dependent Voltage Source – Its value depends on another voltage or current elsewhere in the circuit. Here, k_1 is a constant and k_2 is a resistance as defined in linear devices below. When denoted as k_1v it is referred to as *voltage controlled voltage source*, and when denoted as k_2i it is referred to *as current controlled voltage source*.



Dependent Current Source – Its value depends on another current or voltage elsewhere in the circuit. Here, k_3 is a constant and k_4 is a conductance as defined in linear devices below. When denoted as k_3i it is referred to as *current controlled current source* and when denoted as k_4v it is referred to as *voltage controlled current source*.

Linear Devices

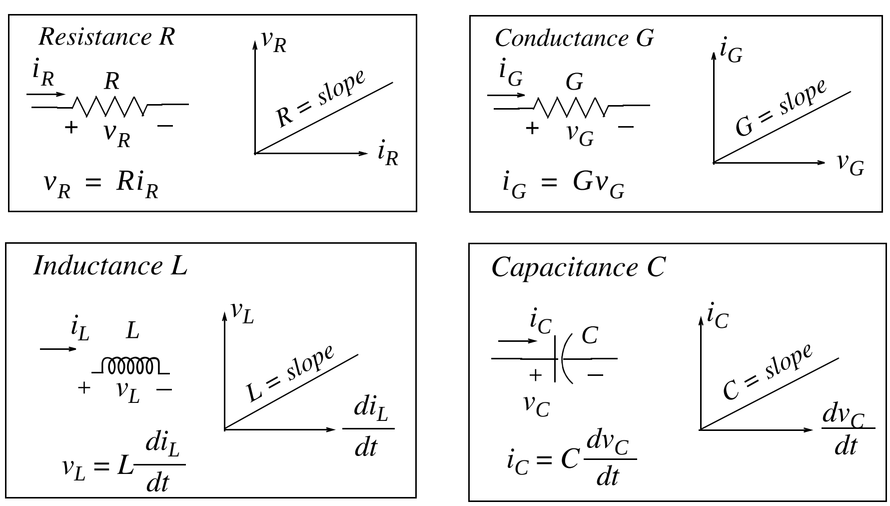


Figure 1.12. Voltage and current sources and linear devices

wood and it is turned on for 8 hours. It is known that 1 BTU is equivalent to 778.3 ft-lb of energy, and 1 joule is equivalent to 0.7376 ft-lb.

Compute:

a. the energy consumption during this 8-hour interval

b. the cost for this energy consumption if the rate is $0.15 per kw-hr

c. the amount of wood in lbs burned during this time interval.

Solution:

a. Energy consumption for 8 hours is

$$Energy\ W = P_{ave}t = 500\ w\ \times 8\ hrs\ \times \frac{3600\ s}{1\ hr} = 14.4\ Mjoules$$

b. Since $1\ kilowatt-hour = 3.6 \times 10^6\ joules$,

$$Cost = \frac{\$0.15}{kw-hr} \times \frac{1\ kw-hr}{3.6 \times 10^6\ joules} \times 14.4 \times 10^6 = \$0.60$$

c. Wood burned in 8 hours,

$$14.4 \times 10^6\ joules \times 0.7376\ \frac{ft-lb}{joule} \times \frac{1\ BTU}{778.3\ ft-lb} \times \frac{1\ lb}{12000\ BTU} = 1.137\ lb$$

1.12 Summary

- Two identically charged (both positive or both negative) particles possess a charge of one coulomb when being separated by one meter in a vacuum, repel each other with a force of $10^{-7}c^2$ newton where $c = velocity\ of\ light \approx 3 \times 10^8\ m/s$. Thus, the force with which two electrically charged bodies attract or repel one another depends on the product of the charges (in coulombs) in both objects, and also on the distance between the objects. If the polarities are the same (negative/negative or positive/positive), the so-called coulumb force is repulsive; if the polarities are opposite (negative/positive or positive/negative), the force is attractive. For any two charged bodies, the coulomb force decreases in proportion to the square of the distance between their charge centers.

- Electric current is defined as the instantaneous rate at which net positive charge is moving past this point in that specified direction, that is,

$$i = \frac{dq}{dt} = \lim_{\Delta t \to 0} \frac{\Delta q}{\Delta t}$$

1.13 Exercises

Multiple choice

1. The unit of charge is the

   A. ampere

   B. volt

   C. watt

   D. coulomb

   E. none of the above

2. The unit of current is the

   A. ampere

   B. coulomb

   C. watt

   D. joule

   E. none of the above

3. The unit of electric power is the

   A. ampere

   B. coulomb

   C. watt

   D. joule

   E. none of the above

4. The unit of energy is the

   A. ampere

   B. volt

   C. watt

   D. joule

   E. none of the above

5. Power is

   A. the integral of energy

E. none of the above

10. The value of a dependent current source can be denoted as

A. kV where k is a conductance value

B. kI where k is a resistance value

C. kV where k is an inductance value

D. kI where k is a capacitance value

E. none of the above

Problems

1. A two terminal device consumes energy as shown by the waveform of Figure 1.18 below, and the current through this device is $i(t) = 2\cos 4000\pi t\ A$. Find the voltage across this device at t = 0.5, 1.5, 4.75 and 6.5 ms. Answers: *2.5 V*, *0 V*, *2.5 V*, *–2.5 V*

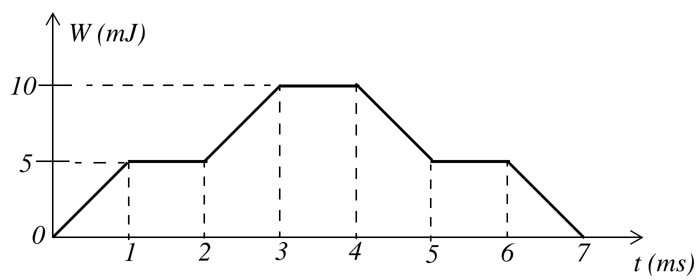


Figure 1.18. Waveform for Problem 1

2. A household light bulb is rated 75 watts at 120 volts. Compute the number of electrons per second that flow through this bulb when it is connected to a 120 volt source.

   Answer: $3.9 \times 10^{18}\ electrons/s$

3. An airplane, whose total mass is *50,000* metric tons, reaches a height of 32,808 feet in 20 minutes after takeoff.

   a. Compute the potential energy that the airplane has gained at this height. Answer: *1, 736 MJ*

   b. If this energy could be converted to electric energy with a conversion loss of 10%, how much would this energy be worth at $0.15 per kilowatt-hour? Answer: *$65.10*

   c. If this energy were converted into electric energy during the period of 20 minutes, what average number of kilowatts would be generated? Answer: *1, 450 Kw*

1.14 Answers to Exercises

Dear Reader:

The remaining pages on this chapter contain answers to the multiple-choice questions and solutions to the exercises.

You must, for your benefit, make an honest effort to answer the multiple-choice questions and solve the problems without first looking at the solutions that follow. It is recommended that first you go through and answer those you feel that you know. For the multiple-choice questions and problems that you are uncertain, review this chapter and try again. If your answers to the problems do not agree with those provided, look over your procedures for inconsistencies and computational errors. Refer to the solutions as a last resort and rework those problems at a later date.

You should follow this practice with the multiple-choice and problems on all chapters of this book.

Chapter 2

Analysis of Simple Circuits

This chapter defines constant and instantaneous values, Ohm's law, and Kirchhoff's Current and Voltage laws. Series and parallel circuits are also defined and nodal, mesh, and loop analyses are introduced. Combinations of voltage and current sources and resistance combinations are discussed, and the voltage and current division formulas are derived.

2.1 Conventions

We will use lower case letters such as v, i, and p to denote *instantaneous values* of voltage, current, and power respectively, and we will use subscripts to denote specific voltages, currents, resistances, etc. For example, v_S and i_S will be used to denote voltage and current sources respectively. Notations like v_{R1} and i_{R2} will be used to denote the voltage across resistance R_1 and the current through resistance R_2 respectively. Other notations like v_A or v_1 will represent the voltage (potential difference) between point A or point 1 *with respect* to some arbitrarily chosen reference point taken as "zero" volts or "ground".

The designations v_{AB} or v_{12} will be used to denote the voltage between point A or point 1 *with respect* to point B or 2 respectively. We will denote voltages as $v(t)$ and $i(t)$ whenever we wish to emphasize that these quantities are time dependent. Thus, sinusoidal (AC) voltages and currents will be denoted as $v(t)$ and $i(t)$ respectively. Phasor quantities, to be inroduced in Chapter 6, will be represented with bold capital letters, $\mathbf{V}$ for phasor voltage and $\mathbf{I}$ for phasor current.

2.2 Ohm's Law

We recall from Chapter 1 that *resistance* R is a *constant* that relates the voltage and the current as:

$$\boxed{v_R = Ri_R} \tag{2.1}$$

This relation is known as *Ohm's law.*

The unit of resistance is the *Ohm* and its symbol is the Greek capital letter Ω. One ohm is the resistance of a conductor such that a constant current of one ampere through it produces a voltage of one volt between its ends. Thus,

$$\boxed{1\ \Omega = \frac{1\ V}{1\ A}} \tag{2.2}$$

then, the current arrow will be pointing to the right direction as shown in Figure 2.14.

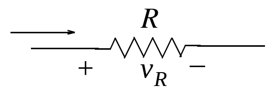


Figure 2.14. Direction of conventional current flow in device with established voltage polarity

Alternately, if current flows in an assumed specific direction through a device thus producing a voltage, we will assign a (+) sign at the terminal of the device at which the current enters. For example, if we are given this designation a device in which the current direction has been established as shown in Figure 2.15,

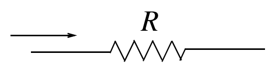


Figure 2.15. Device with established conventional current direction

then we assign (+) and (−) as shown in Figure 2.16.

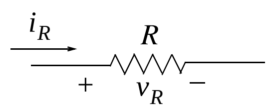


Figure 2.16. Voltage polarity in a device with established conventional current flow

Note: Active devices, such as voltage and current sources, have their voltage polarity and current direction respectively, established as part of their notation. The current through and the voltage across these devices can easily be determined *if these devices deliver power to the rest of the circuit.* Thus with the voltage polarity as given in the circuit of Figure 2.17 (a), we assign a clockwise direction to the current as shown in Figure 2.17 (b). This is consistent with the passive sign convention since we have assumed that the voltage source delivers power to the rest of the circuit.

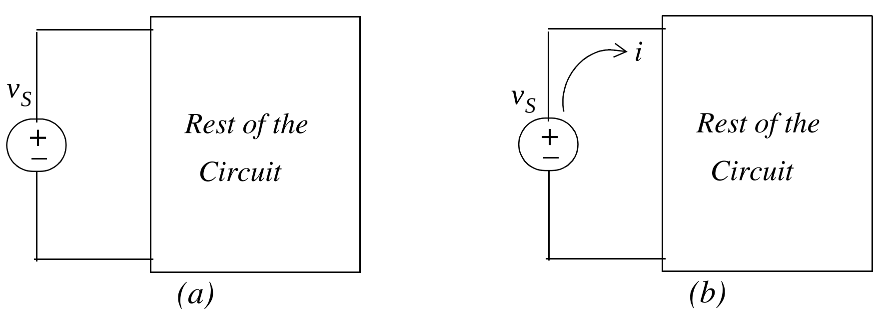


Figure 2.17. Direction of conventional current flow produced by voltage sources

Likewise, in the circuit of Figure 2.18 (a) below, the direction of the current source is clockwise, and assuming that this source delivers power to the rest of the circuit, we assign the voltage polarity shown in Figure 2.18 (b) to be consistent with the passive sign convention.

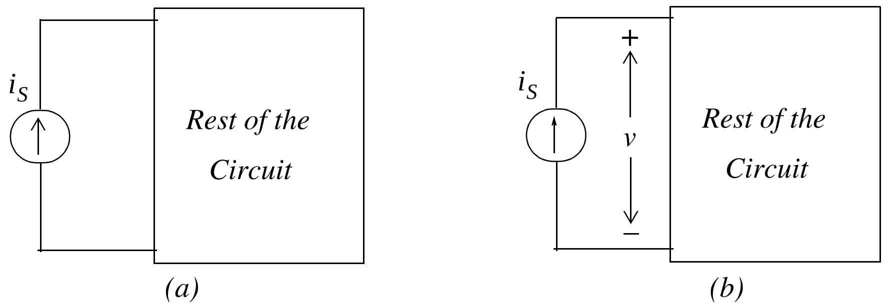


Figure 2.18. Voltage polarity across current sources

The following facts were discussed in the previous chapter but they are repeated here for emphasis.

There are two conditions required to setup and maintain the flow of an electric current:

1. *There must be some voltage (potential difference) to provide the energy (work) which will force electric current to flow in a specific direction in accordance with the conventional current flow (from a higher to a lower potential).*

2. *There must be a continuous (closed) external path for current to flow around this path (mesh or loop).*

The external path is usually made of two parts: (a) the *metallic wires* and (b) the *load* to which the electric power is to be delivered in order to accomplish some useful purpose or effect. The load may be a resistive, an inductive, or a capacitive circuit, or a combination of these.

2.8 Single Mesh Circuit Analysis

We will use the following example to develop a step-by-step procedure for analyzing (finding current, voltage drops and power) in a circuit with a single mesh.

Example 2.1

For the series circuit shown in Figure 2.19, we want to find:

a. The current i which flows through each device

b. The voltage drop across each resistor

c. The power absorbed or delivered by each device

TABLE 2.1 Power delivered or absorbed by each device on the circuit of Figure 2.19

Device	***Power Delivered (watts)***	***Power Absorbed (watts)***
200 V Source	400	
64 V Source		128
80 V Source		160
4 Ω Resistor		16
6 Ω Resistor		24
8 Ω Resistor		32
10 Ω Resistor		40
Total	400	400

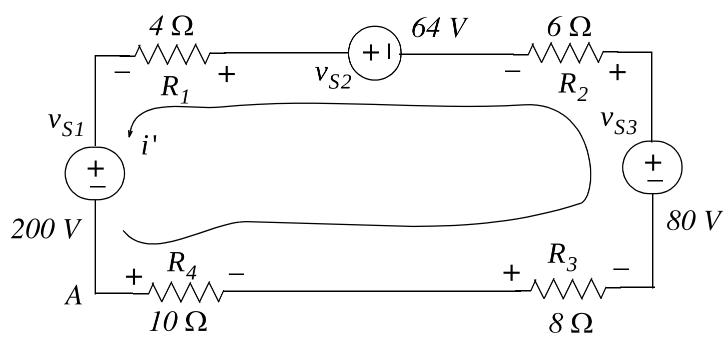


Figure 2.21. Circuit for Example 2.2

By substitution of given values, we get

$$200 + 4i' - 64 + 6i' - 80 + 8i' + 10i' = 0$$

or

$$28i' = -56$$

or

$$i' = -2A \tag{2.26}$$

Comparing (2.21) with (2.26) we see that $i' = -i$ as expected.

Temp (deg C)	Resistance (Ohms)
-250	-2.9328
-240	-1.0464
-230	0.84
-220	2.7264
-210	4.6128
-200	6.4992
-190	8.3856
-180	10.272
-170	12.1584
-160	14.0448
-150	15.9312
-140	17.8176

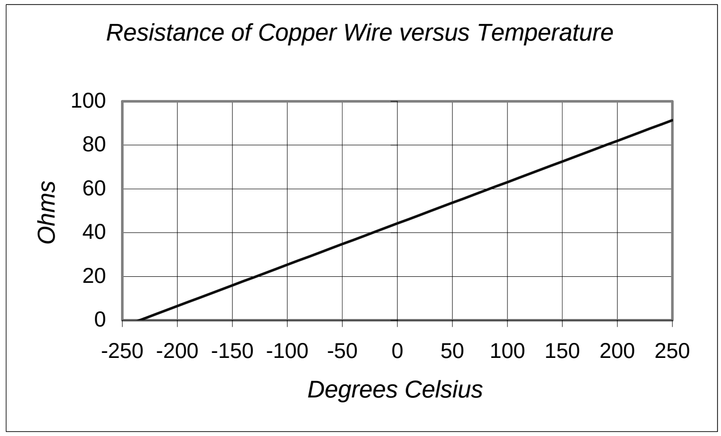


Figure 2.43. Spreadsheet for construction of equation (2.52)

2.18 Ampere Capacity of Wires

For public safety, electric power supply (mains) wiring is controlled by local, state and federal boards, primarily on the *National Electric Code* (NEC) and the *National Electric Safety Code*. Moreover, many products such as wire and cable, fuses, circuit breakers, outlet boxes and appliances are governed by *Underwriters Laboratories* (UL) Standards which approves consumer products such as motors, radios, television sets etc.

Table 2.4 shows the NEC allowable current-carrying capacities for copper conductors based on the type of insulation.

The ratings in Table 2.4 are for copper wires. The ratings for aluminum wires are typically *84%* of these values. Also, these rating are for not more than three conductors in a cable with temperature $30°C$ or $86°F$. The NEC contains tables with correction factors at higher temperatures.

2.19 Current Ratings for Electronic Equipment

There are also standards for the internal wiring of electronic equipment and chassis. Table 2.5 provides recommended current ratings for copper wire based on $45°C$ ($40°C$ for wires smaller than 22 AWG. Listed also, are the circular mils and these denote the area of the cross section of each wire size. A *circular mil* is the area of a circle whose diameter is 1 mil (one-thousandth of an inch). Since the area of a circle is proportional to the square of its diameter, and the area of a circle one mil in diameter is one circular mil, the area of any circle in circular mils is the square of its diameter in mils.

A *mil-foot wire* is a wire whose length is one foot and has a cross-sectional area of one circular mil.

b. The voltage drop throughout the electrical system must then be computed to ensure that it does not exceed certain specifications. For instance, in the lighting part of the system referred to as the *lighting load*, a variation of more than 5% in the voltage across each lamp causes an unpleasant variation in the illumination. Also, the voltage variation in the heating and air conditioning load must not exceed 10%.

Important! The requirements stated here are for instructional purposes only. They change from time to time. It is, therefore, imperative that the designer consults the latest publications of the applicable codes for compliance.

Example 2.12

Figure 2.44 shows a lighting load distribution diagram for an interior electric installation.

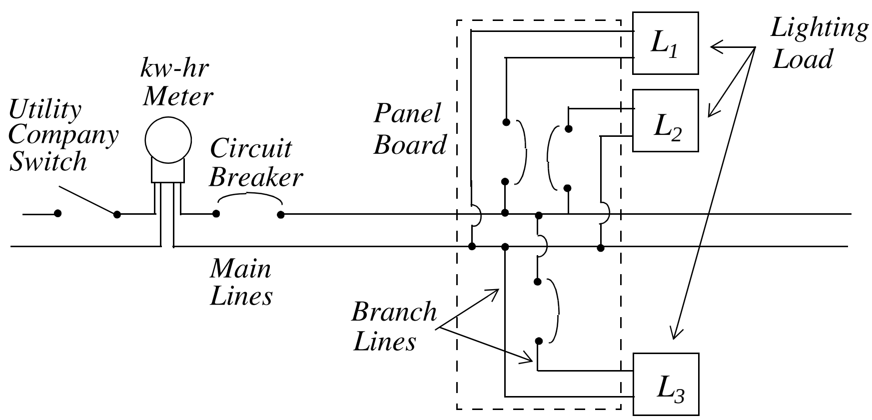


Figure 2.44. Load distribution for an interior electric installation

The panel board is 200 feet from the meter. Each of the three branches has 12 outlets for 75 w, 120 volt lamps. The *load center* is that point on the branch line at which all lighting loads may be considered to be concentrated. For this example, assume that the distance from the panel to the load center is 60 ft. Compute the size of the main lines. Use T (thermoplastic insulation) type copper conductor and base your calculations on $25°C$ temperature environment.

Solution:

It is best to use a spreadsheet for the calculations so that we can compute sizes for more and different branches if need be.

The computations for Parts I and II are shown on the spreadsheet of Figure 2.45 where from the last line of Part II we see that the percent line drop is 12.29 and this is more than twice the allowable 5% drop. With the 12.29% voltage variation the brightness of the lamps would vary through wide ranges, depending on how many lamps were in use at one time.

Part I	Sizing in Accordance with NEC (Table 2.2)				
Step	*Description*	*Calculation*	*Value*	*Units*	
1	Meter to Panel Board Distance		200	ft	
2	No. of Branches from Panel Board		3		
3	Outlets per branch		12		
4	Outlet Voltage Rating		120	V	
5	Power drawn from each outlet		75	w	
6	Power required by each branch	Step 3 x Step 5	900	w	
7	Current drawn by each branch	Step 6 / Step 4	7.50	A	
8	Required wire size for branches *	Table 2.4	14	AWG	Carries up to 15 A
9	Current required by the main line	Step 2 x Step 7	22.50	A	
10	Required wire size for main line	Table 2.4	12	AWG	Carries up to 20 A
Part II	**Check for Voltage Drops**				
11	Distance from panel to load center		60	ft	
12	Number of wires in each branch		2		
13	Total length of wire in each branch	Step 11 x Step 12	120	ft	
14	Specified Temperature in ^0C		25	^0C	
15	Resistance of 1000 ft 14 AWG wire at 100 ^0C	Table 2.5	3.20	Ω	
16	Resistance of 1000 ft 14 AWG wire at Spec. Temp	Equation (2.54)	2.24	Ω	
17	Resistance of actual length of wire	(Step 13 x Step 16) / 1000	0.269	Ω	
18	Voltage drop in each branch	Step 7 x Step 17	2.02	V	
19	Number of wires in each main		2		
20	Total length of wire in main line	Step 1 x Step 19	400	ft	
21	Resistance of 1000 ft 12 AWG wire at 100 ^0C	Table 2.5	2.02	Ω	
22	Resistance of 1000 ft 12 AWG wire at Spec. Temp	Equation (2.54)	1.41	Ω	
23	Resistance of actual length of wire	(Step 20 x Step 22) /1000	0.57	Ω	
24	Voltage drop in main line	Step 9 x Step 23	12.73	V	
25	Voltage drop from meter to load center	Step 18 + Step 24	14.74	V	
26	Percent line drop	(Step 25 / Step 4) x 100	12.29	%	

* No size smaller can be installed; smaller sizes have insufficient mechanical strength

Figure 2.45. Spreadsheet for Example 2.12, Parts I and II

We can use Cramer's rule or Gauss's elimination method as discussed in Appendix A, to solve (3.7) for the unknowns. Simultaneous solution yields $v_1 = 12\ V$, $v_2 = -60\ V$, and $v_3 = 30\ V$. With these values we can determine the current in each resistor, and the power absorbed or delivered by each device.

Check with MATLAB®:

```
G=[3 –1 0; 5 –9 4; 0 1 –6]; I=[96 720 -240]'; V=G\I;...
fprintf(' \n'); fprintf('v1 = %5.2f volts \t', V(1)); ...
fprintf('v2 = %5.2f volts \t', V(2)); fprintf('v3 = %5.2f volts', V(3)); fprintf(' \n')
```

```
v1 = 12.00 volts   v2 = -60.00 volts   v3 = 30.00 volts
```

Check with Excel®:

The spreadsheet of Figure 3.3 shows the solution of the equations of (3.7). The procedure is discussed in Appendix A.

	A	B	C	D	E	F	G	H
1	Spreadsheet for Matrix Inversion and Matrix Multiplication							
2								
3		3	-1	0			96	
4	G=	5	-9	4		I=	720	
5		0	1	-6			-240	
6								
7		0.417	-0.050	-0.033			12	
8	G^{-1}=	0.250	-0.150	-0.100		V=	-60	
9		0.042	-0.025	-0.183			30	

Figure 3.3. Spreadsheet for the solution of (3.7)

Example 3.2

For the circuit of Figure 3.4, write nodal equations in matrix form and solve for the unknowns using matrix theory, Cramer's rule, or Gauss's elimination method. Verify your answers with Excel or MATLAB. Please refer to Appendix A for procedures and examples. Then construct a table showing the voltages across, the currents through and the power absorbed or delivered by each device.

Solution:

We observe that there are 4 nodes and we denote these as ①, ②, ③, and G (for ground) as shown in Figure 3.5.

Continuing, we observe that there is no voltage drop across the $4\ \Omega$ resistor since no current flows through it. The current now enters Mesh 2 where we encounter the $36\ V$ drop due to the voltage source there, and the voltage drops across the $8\ \Omega$ and $6\ \Omega$ resistors are $8i_2$ and $6i_2$ respectively since in Mesh 2 the current now is really i_2. The voltage drops across the $16\ \Omega$ and $10\ \Omega$ resistors are expressed as in the previous examples and thus our first mesh equation is

$$2i_1 + 36 + 8i_2 + 6i_2 + 16(i_2 - i_4) + 10(i_1 - i_3) - 12 = 0$$

or

$$12i_1 + 30i_2 - 10i_3 - 16i_4 = -24$$

or

$$\boxed{6i_1 + 15i_2 - 5i_3 - 8i_4 = -12} \tag{3.28}$$

Now, we reinsert the 5 A current source between Meshes 1 and 2 and we obtain our second equation as

$$\boxed{i_1 - i_2 = 5} \tag{3.29}$$

For meshes 3 and 4, the equations are

$$10(i_3 - i_1) + 12(i_3 - i_4) + 18i_3 - 12 = 0$$

or

$$\boxed{5i_1 - 20i_3 + 6i_4 = -6} \tag{3.30}$$

and

$$16(i_4 - i_2) + 20i_4 - 24 + 12(i_4 - i_3) = 0$$

or

$$\boxed{4i_2 + 3i_3 - 12i_4 = -6} \tag{3.31}$$

and in matrix form

$$\underbrace{\begin{bmatrix} 6 & 15 & -5 & -8 \\ 1 & -1 & 0 & 0 \\ 5 & 0 & -20 & 6 \\ 0 & 4 & 3 & -12 \end{bmatrix}}_{R} \underbrace{\begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix}}_{I} = \underbrace{\begin{bmatrix} -12 \\ 5 \\ -6 \\ -6 \end{bmatrix}}_{V} \tag{3.32}$$

We find the solution of (3.32) with the following MATLAB code.

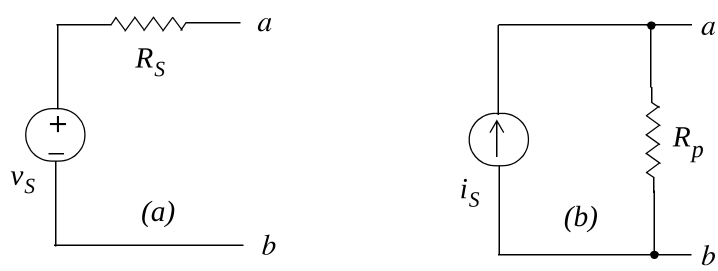


Figure 3.18. Practical voltage and current sources

In Figure 3.18 (a), the voltage of the source will always be v_S but the terminal voltage v_{ab} will be $v_{ab} = v_S - v_{R_s}$ if a load is connected at points a and b. Likewise, in Figure 3.18 (b) the current of the source will always be i_S but the terminal current i_{ab} will be $i_{ab} = i_S - i_{R_P}$ if a load is connected at points a and b.

Now, we will show that the networks of Figures 3.18 (a) and 3.18 (b) can be made equivalent to each other.

In the networks of Figures 3.19 (a) and 3.19 (b), the load resistor R_L is the same in both.

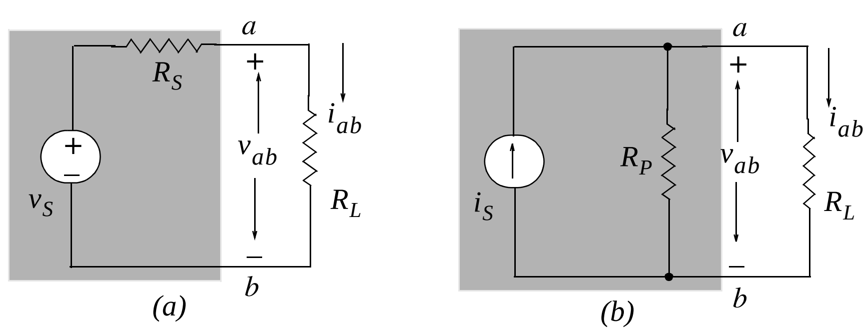


Figure 3.19. Equivalent sources

From the circuit of Figure 3.19 (a),

$$v_{ab} = \frac{R_L}{R_S + R_L} v_S \tag{3.33}$$

and

$$i_{ab} = \frac{v_S}{R_S + R_L} \tag{3.34}$$

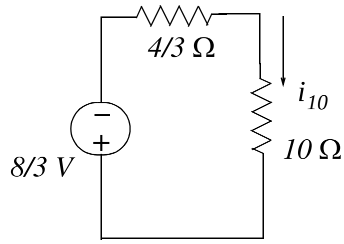


Figure 3.23. Circuit for Example 3.7 in its simplest form

Thus, the current through the $10\ \Omega$ resistor is

$$i_{10} = \frac{-8/3}{10 + 4/3} = -4/17\ A$$

3.5 Thevenin's Theorem

This theorem is perhaps the greatest time saver in circuit analysis, especially in electronic circuits. It states that we can replace a two terminal network by a voltage source v_{TH} in series with a resistance R_{TH} as shown in Figure 3.24.

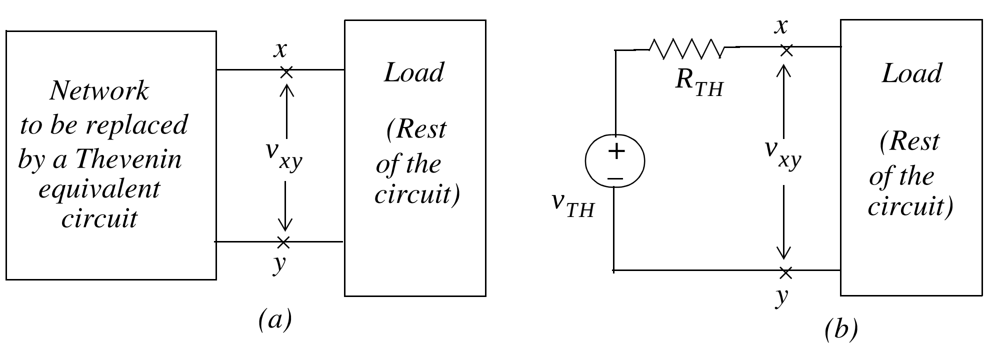


Figure 3.24. Replacement of a network by its Thevenin's equivalent

The network of Figure 3.24 (b) will be equivalent to the network of Figure 3.24 (a) if the load is removed in which case both networks will have the same open circuit voltages v_{xy} and consequently,

$$v_{TH} = v_{xy}$$

Therefore,

$$\boxed{v_{TH} = v_{xy\ open}} \tag{3.41}$$

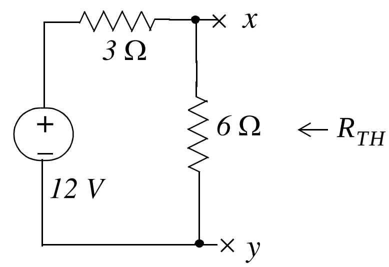


Figure 3.27. Second step in finding the Thevenin equivalent of the circuit of Example 3.8

Applying Thevenin's theorem at x and y and using the voltage division expression, we get

$$v_{TH} = v_{xy} = \frac{6}{3+6} \times 12 = 8\ V$$
$$R_{TH}\Big|_{V_S = 0} = \frac{3 \times 6}{3+6} = 2\ \Omega \tag{3.43}$$

and thus the equivalent circuit to the left of points x and x is as shown in Figure 3.28.

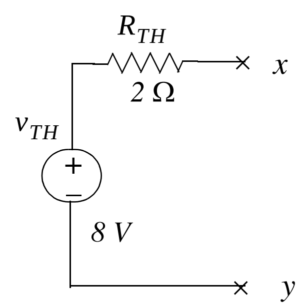


Figure 3.28. First Thevenin equivalent for the circuit of Example 3.8

Next, we attach the remaining part of the given circuit to the Thevenin equivalent of Figure 3.28, and the new circuit now is as shown in Figure 3.29.

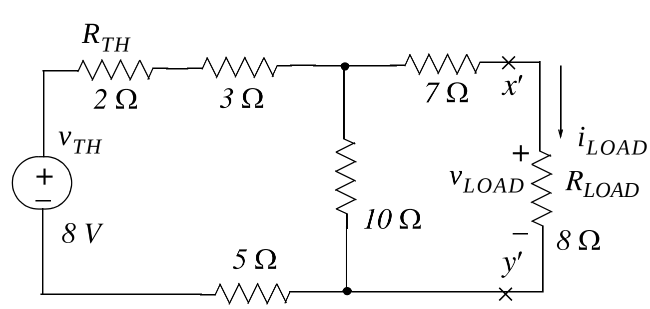


Figure 3.29. Circuit for Example 3.8 with first Thevenin equivalent

Chapter 4

Introduction to Operational Amplifiers

This chapter is an introduction to amplifiers. It discusses amplifier gain in terms of decibels *(dB)* and provides an overview of operational amplifiers, their characteristics and applications. Numerous formulas for the computation of the gain are derived and several practical examples are provided.

4.1 Signals

A *signal* is any waveform that serves as a means of communication. It represents a fluctuating electric quantity, such as voltage, current, electric or magnetic field strength, sound, image, or any message transmitted or received in telegraphy, telephony, radio, television, or radar. A typical signal which varies with time is shown in figure 4.1 where $f(t)$ can be any physical quantity such as voltage, current, temperature, pressure, and so on.

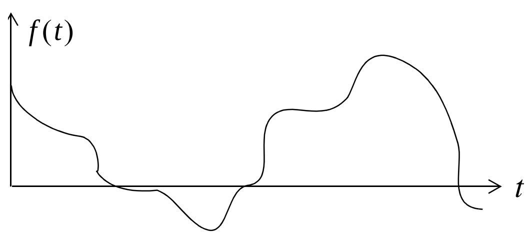
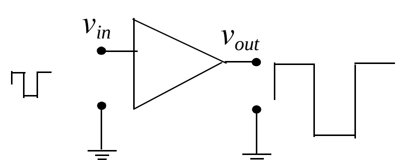


Figure 4.1. A signal that changes with time

4.2 Amplifiers

An *amplifier* is an electronic circuit which increases the magnitude of the input signal. The symbol of a typical amplifier is a triangle as shown in Figure 4.2.



Electronic Amplifier

Figure 4.2. Symbol for electronic amplifier

An electronic (or electric) circuit which produces an output that is smaller than the input is called an *attenuator*. A resistive voltage divider is a typical attenuator.

be also indicated with a large plus (+) symbol inside the circle. The positive (+) sign below the summing point implies *positive feedback* which means that the output, or portion of it, is added to the input. On the other hand, the negative (–) sign implies *negative feedback* which means that the output, or portion of it, is subtracted from the input. Practically, all amplifiers use used with negative feedback since positive feedback causes circuit instability.

4.5 The Operational Amplifier

The *operational amplifier* or simply *op amp* is the most versatile electronic amplifier. It derives it name from the fact that it is capable of performing many mathematical operations such as addition, multiplication, differentiation, integration, analog-to-digital conversion or vice versa. It can also be used as a comparator and electronic filter. It is also the basic block in analog computer design. Its symbol is shown in Figure 4.7.

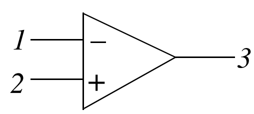


Figure 4.7. Symbol for operational amplifier

As shown above the op amp has two inputs but only one output. For this reason it is referred to as *differential input, single ended output* amplifier. Figure 4.8 shows the internal construction of a typical op amp. This figure also shows terminals V_{CC} and V_{EE}. These are the voltage sources required to power up the op amp. Typically, V_{CC} is +15 volts and V_{EE} is –15 volts. These terminals are not shown in op amp circuits since they just provide power, and do not reveal any other useful information for the op amp's circuit analysis.

4.6 An Overview of the Op Amp

The op amp has the following important characteristics:

1. Very high input impedance (resistance)
2. Very low output impedance (resistance)
3. Capable of producing a very large gain that can be set to any value by connection of external resistors of appropriate values
4. Frequency response from DC to frequencies in the MHz range
5. Very good stability
6. Operation to be performed, i.e., addition, integration etc. is done externally with proper selection of passive devices such as resistors, capacitors, diodes, and so on.

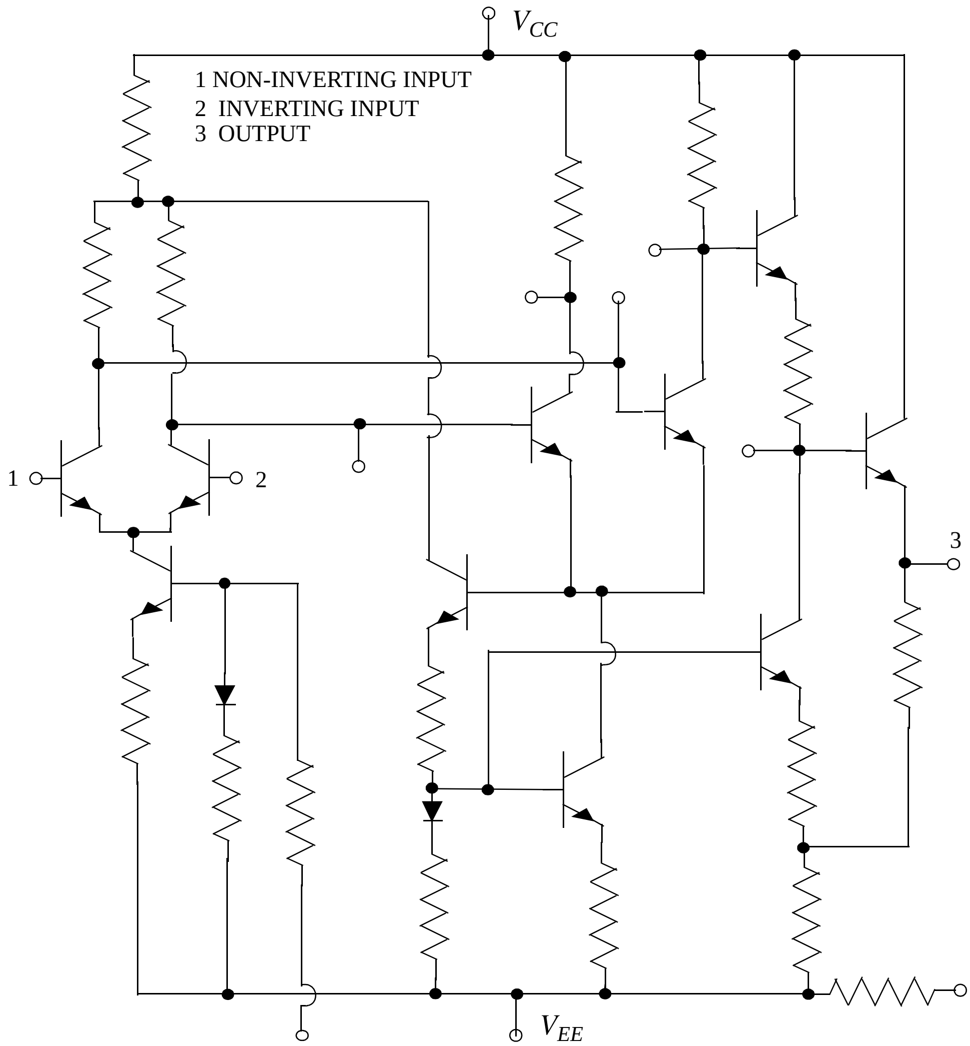


Figure 4.8. Internal Devices of a Typical Op Amp

An op amp is said to be connected in the *inverting mode* when an input signal is connected to the inverting (–) input through an external resistor R_{in} whose value along with the feedback resistor R_f determine the op amp's gain. The non-inverting (+) input is grounded through an external resistor R as shown in Figure 4.9.

For the circuit of Figure 4.9, the voltage gain G_v is

$$G_v = \frac{v_{out}}{v_{in}} = -\frac{R_f}{R_{in}} \tag{4.5}$$

Chapter 5

Inductance and Capacitance

This chapter is an introduction to inductance and capacitance, their voltage-current relationships, power absorbed, and energy stored in inductors and capacitors. Procedures for analyzing circuits with inductors and capacitors are presented along with several examples.

5.1 Energy Storage Devices

In the first four chapters we considered resistive circuits only, that is, circuits with resistors and constant voltage and current sources. However, resistance is not the only property that an electric circuit possesses; in every circuit there are two other properties present and these are the inductance and the capacitance. We will see through some examples that will be presented later in this chapter, that inductance and capacitance have an effect on an electric circuit as long as there are changes in the voltages and currents in the circuit.

The effects of the inductance and capacitance properties can best be stated in simple differential equations since they involve the changes in voltage or current with time. We will study inductance first.

5.2 Inductance

Inductance is associated with the magnetic field which is always present when there is an electric current. Thus, when current flows in an electric circuit the conductors (wires) connecting the devices in the circuit are surrounded by a magnetic field. Figure 5.1 shows a simple loop of wire and its magnetic field represented by the small loops.

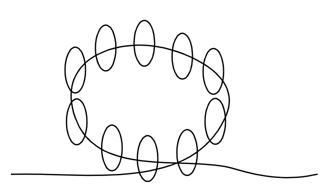


Figure 5.1. Magnetic field around a loop of wire

The direction of the magnetic field (not shown) can be determined by the left-hand rule if conventional current flow is assumed, or by the right-hand rule if electron current flow is assumed. The magnetic field loops are circular in form and are referred to as *lines of magnetic flux*. The unit of magnetic flux is the *weber* (Wb).

Chapter 6

Sinusoidal Circuit Analysis

This chapter is an introduction to circuits in which the applied voltage or current are sinusoidal. The time and frequency domains are defined and phasor relationships are developed for resistive, inductive and capacitive circuits. Reactance, susceptance, impedance and admittance are also defined. It is assumed that the reader is familiar with sinusoids and complex numbers. If not, it is strongly recommended that Appendix B is reviewed thoroughly before reading this chapter.

6.1 Excitation Functions

The applied voltages and currents in electric circuits are generally referred to as *excitations* or *driving functions*, that is, we say that a circuit is "excited" or "driven" by a constant, or a sinusoidal, or an exponential function of time. Another term used in circuit analysis is the word *response*; this may be the voltage or current in the "load" part of the circuit or any other part of it. Thus the response may be anything we define it as a response. Generally, the response is the voltage or current at the output of a circuit, but we need to specify what the output of a circuit is.

In Chapters 1 through 4 we considered circuits that consisted of excitations (active sources) and resistors only as the passive devices. We used various methods such as nodal and mesh analyses, superposition, Thevenin's and Norton's theorems to find the desired response such as the voltage and/or current in any particular branch. The circuit analysis procedure for these circuits is the same for DC and AC circuits. Thus, if the excitation is a constant voltage or current, the response will also be some constant value; if the excitation is a sinusoidal voltage or current, the response will also be sinusoidal with the same frequency but different amplitude and phase.

In Chapter 5 we learned that when the excitation is a constant and steady-state conditions are reached, an inductor behaves like a short circuit and a capacitor behaves like an open circuit. However, when the excitation is a time-varying function such as a sinusoid, inductors and capacitors behave entirely different as we will see in our subsequent discussion.

6.2 Circuit Response to Sinusoidal Inputs

We can apply the circuit analysis methods which we have learned in previous chapters to circuits where the voltage or current sources are sinusoidal. To find out how easy (or how difficult) the procedure becomes, we will consider the simple series circuit of Example 6.1.

Example 6.1

For the circuit shown in Figure 6.1, derive an expression for $v_C(t)$ in terms of V_p, R, C, and ω where the subscript p is used to denote the *peak* or *maximum* value of a time varying function, and the

Chapter 7

Phasor Circuit Analysis

This chapter begins with the application of nodal analysis, mesh analysis, superposition, and Thevenin's and Norton's theorems in phasor circuits. Then, phasor diagrams are introduced, and the input-output relationships for an RC low-pass filter and an RC high-pass filter are developed.

7.1 Nodal Analysis

The procedure of analyzing a phasor* circuit is the same as in Chapter 3, except that in this chapter we will be using phasor quantities. The following example illustrates the procedure.

Example 7.1

Use nodal analysis to compute the phasor voltage $\mathbf{V}_{AB} = \mathbf{V}_A - \mathbf{V}_B$ for the circuit of Figure 7.1.

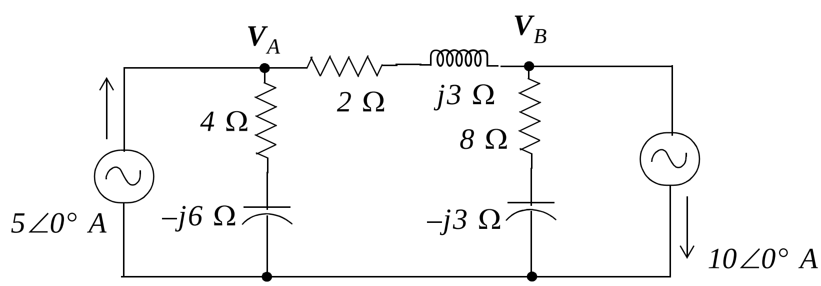


Figure 7.1. Circuit for Example 7.1

Solution:

As before, we choose a reference node as shown in Figure 7.2, and we write nodal equations at the other two nodes A and B. Also, for convenience, we designate the devices in series as Z_1, Z_2, and Z_3 as shown, and then we write the nodal equations in terms of these impedances.

$$Z_1 = 4 - j6 = 7.211\angle -56.3°$$

$$Z_2 = 2 + j3 = 3.606\angle 56.3°$$

$$Z_3 = 8 - j3 = 8.544\angle -20.6°$$

* *A phasor is a rotating vector*

Chapter 8

Average and RMS Values, Complex Power, and Instruments

This chapter defines average and effective values of voltages and currents, instantaneous and average power, power factor, the power triangle, and complex power. It also discusses electrical instruments that are used to measure current, voltage, resistance, power, and energy.

8.1 Periodic Time Functions

A periodic time function satisfies the expression

$$f(t) = f(t + nT) \tag{8.1}$$

where n is a positive integer and T is the period of the periodic time function. The sinusoidal and sawtooth waveforms of Figure 8.1 are examples of periodic functions of time.

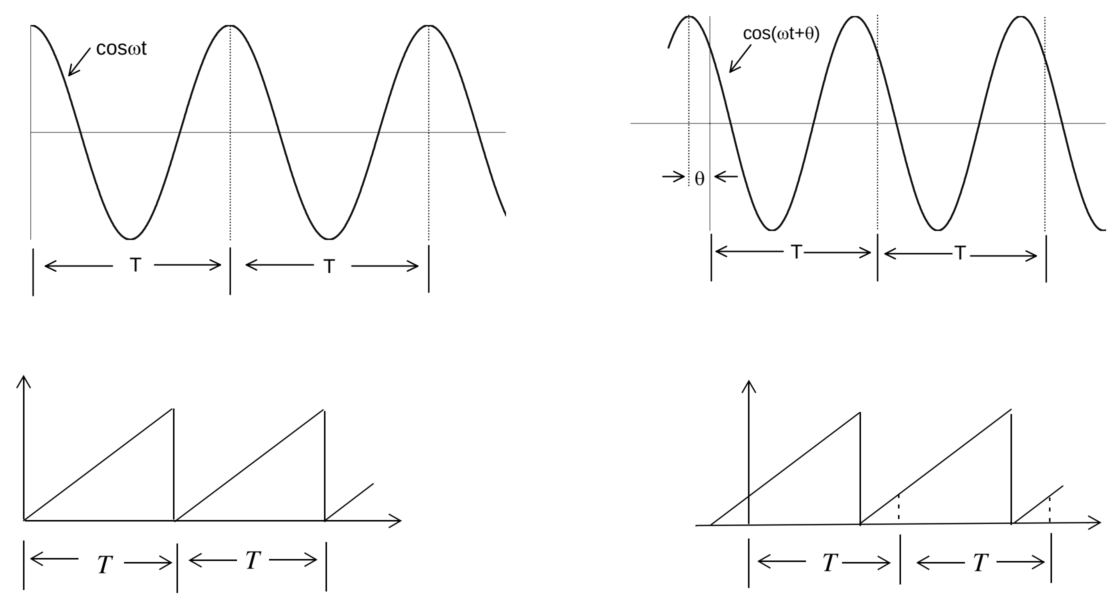


Figure 8.1. Examples of periodic functions of time

Other periodic functions of interest are the square and the triangular waveforms.

Chapter 9

Natural Response

This chapter discusses *the natural response* of electric circuits.The term natural implies that there is no excitation in the circuit, that is, the circuit is source-free, and we seek the circuit's natural response. The natural response is also referred to as the *transient response.*

9.1 The Natural Response of a Series RL circuit

Let us find the natural response of the circuit of Figure 9.1 where the desired response is the current *i*, and it is given that at $t = 0$, $i = I_0$, that is, the initial condition is $i(0) = I_0$.

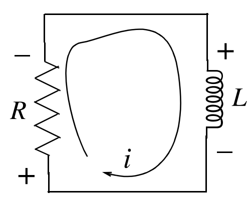


Figure 9.1. Circuit for determining the natural response of a series RL circuit

Application of KVL yields

$$v_L + v_R = 0$$

or

$$L\frac{di}{dt} + Ri = 0 \tag{9.1}$$

Here, we seek a value of *i* which satisfies the differential equation of (9.1), that is, we need to find the natural response which in differential equations terminology is the *complementary function.* As we know, two common methods are the separation of variables method and the assumed solution method. We will consider both.

1. Separation of Variables Method

Rearranging (9.1), so that the variables *i* and *t* are separated, we get

$$\frac{di}{i} = -\frac{R}{L}dt$$

Next, integrating both sides and using the initial condition, we get

Chapter 10

Forced and Total Response in RL and RC Circuits

This chapter discusses the forced response of electric circuits.The term “forced” here implies that the circuit is excited by a voltage or current source, and its response to that excitation is analyzed. Then, the forced response is added to the natural response to form the total response.

10.1 The Unit Step Function $u_0(t)$

A function is said to be *discontinuous* if it exhibits points of discontinuity, that is, if the function jumps from one value to another without taking on any intermediate values.

A well-known discontinuous function is the *unit step function* $u_0(t)$* that is defined as

$$\boxed{u_0(t) = \begin{cases} 0 & t < 0 \\ 1 & t > 0 \end{cases}} \tag{10.1}$$

It is also represented by the waveform of Figure 10.1.

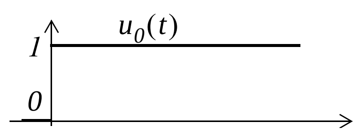
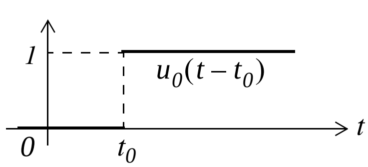


Figure 10.1. Waveform for $u_0(t)$

In the waveform of Figure 10.1, the unit step function $u_0(t)$ changes abruptly from *0* to *1* at $t = 0$. But if it changes at $t = t_0$ instead, its waveform and definition are as shown in Figure 10.2.



$$u_0(t-t_0) = \begin{cases} 0 & t < t_0 \\ 1 & t > t_0 \end{cases}$$

Figure 10.2. Waveform and definition of $u_0(t-t_0)$

* *In some books, the unit step function is denoted as $u(t)$,that is, without the subscript 0. In this text we will reserve this designation for any input.*

Appendix A

Introduction to MATLAB®

This appendix serves as an introduction to the basic MATLAB commands and functions, procedures for naming and saving the user generated files, comment lines, access to MATLAB's Editor/Debugger, finding the roots of a polynomial, and making plots. Several examples are provided with detailed explanations.

A.1 MATLAB® and Simulink®

MATLAB and Simulink are products of The MathWorks, Inc. These are two outstanding software packages for scientific and engineering computations and are used in educational institutions and in industries including automotive, aerospace, electronics, telecommunications, and environmental applications. MATLAB enables us to solve many advanced numerical problems fast and efficiently. Simulink is a block diagram tool used for modeling and simulating dynamic systems such as controls, signal processing, and communications. In this appendix we will discuss MATLAB only.

A.2 Command Window

To distinguish the screen displays from the user commands, important terms, and MATLAB functions, we will use the following conventions:

Click: Click the left button of the mouse

`Courier Font:` Screen displays

Helvetica Font: User inputs at MATLAB's command window prompt >> or `EDU>>`*

Helvetica Bold: MATLAB functions

Times Bold Italic: Important terms and facts, notes and file names

When we first start MATLAB, we see the toolbar on top of the *command screen* and the prompt `EDU>>`. This prompt is displayed also after execution of a command; MATLAB now waits for a new command from the user. It is highly recommended that we use the *Editor/Debugger* to write our program, save it, and return to the command screen to execute the program as explained below.

To use the Editor/Debugger:

1. From the *File* menu on the toolbar, we choose *New* and click on *M-File*. This takes us to the *Editor*

* `EDU>>` *is the MATLAB prompt in the Student Version*

Appendix B

A Review of Complex Numbers

This appendix is a review of the algebra of complex numbers. The basic operations are defined and illustrated by several examples. Applications using Euler's identities are presented, and the exponential and polar forms are discussed and illustrated with examples.

B.1 Definition of a Complex Number

In the language of mathematics, the square root of minus one is denoted as i, that is, $i = \sqrt{-1}$. In the electrical engineering field, we denote i as j to avoid confusion with current i. Essentially, j is an operator that produces a 90-degree counterclockwise rotation to any vector to which it is applied as a multiplying factor. Thus, if it is given that a vector A has the direction along the right side of the x-axis as shown in Figure B.1, multiplication of this vector by the operator j will result in a new vector jA whose magnitude remains the same, but it has been rotated counterclockwise by $90°$. Also, another multiplication of the new vector jA by j will produce another $90°$ counterclockwise direction. In this case, the vector A has rotated $180°$ and its new value now is $-A$. When this vector is rotated by another $90°$ for a total of $270°$, its value becomes $j(-A) = -jA$. A fourth $90°$ rotation returns the vector to its original position, and thus its value is again A. Therefore, we conclude that $j^2 = -1$, $j^3 = -j$, and $j^4 = 1$.

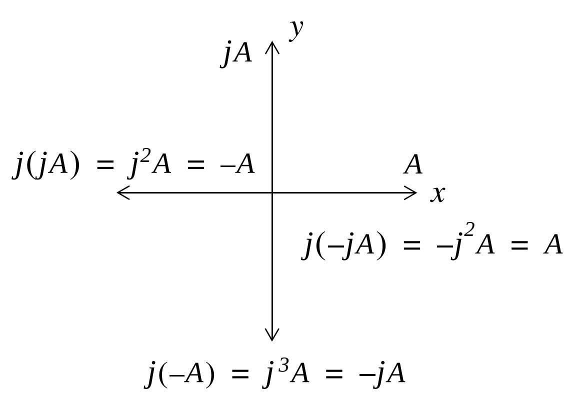


Figure B.1. The j operator

Appendix C

Matrices and Determinants

This chapter is an introduction to matrices and matrix operations. Determinants, Cramer's rule, and Gauss's elimination method are reviewed. Some definitions and examples are not applicable to subsequent material presented in this text, but are included for subject continuity, and reference to more advance topics in matrix theory. These are denoted with a dagger (†) and may be skipped.

C.1 Matrix Definition

A *matrix* is a rectangular array of numbers such as those shown below.

$$\begin{bmatrix} 2 & 3 & 7 \\ 1 & -1 & 5 \end{bmatrix} \qquad or \qquad \begin{bmatrix} 1 & 3 & 1 \\ -2 & 1 & -5 \\ 4 & -7 & 6 \end{bmatrix}$$

In general form, a matrix A is denoted as

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \dots & a_{3n} \\ \dots & \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & a_{m3} & \dots & a_{mn} \end{bmatrix} \tag{C.1}$$

The numbers a_{ij} are the *elements* of the matrix where the index *i* indicates the row, and *j* indicates the column in which each element is positioned. Thus, a_{43} indicates the element positioned in the fourth row and third column.

A matrix of *m* rows and *n* columns is said to be of $m \times n$ *order matrix*.

If $m = n$, the matrix is said to be a *square matrix of order m* (or *n*). Thus, if a matrix has five rows and five columns, it is said to be a square matrix of order 5.

In a square matrix, the elements $a_{11}, a_{22}, a_{33}, \dots, a_{nn}$ are called the *main diagonal elements*. Alternately, we say that the matrix elements $a_{11}, a_{22}, a_{33}, \dots, a_{nn}$, are located on the *main diagonal*.

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